

Neumann domains of Laplace eigenfunctions

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In this talk, we introduce the Neumann domains of Laplace eigenfunctions and describe various aspects of their structure. In addition to partitioning a planar domain using the nodal domains of a Laplace eigenfunction, one can also consider a natural partition formed by its gradient flow lines. Elements of such partition are known as Neumann domains and their boundaries are called Neumann lines. While earlier studies focused on Morse eigenfunctions (eigenfunctions with only non-degenerate critical points), many eigenfunctions exhibit more complex behavior. In this talk, we discuss a framework for defining and analyzing Neumann domains even when the eigenfunction is not Morse (i.e., it has degenerate critical points)—by relying on real analyticity rather than non-degeneracy. Our approach builds on earlier works in this direction and is inspired in part by Weinberger’s idea of the “*effectless cut*.” If time permits, we will also present some asymptotics about the number of Neumann domains connecting it with the Pleijel constant.

This talk will be based on the article [1], a joint work with T.V. Anoop (IIT Madras, India) and V. Bobkov (UFA Federal Research Centre, Russia).

Reference

[1] Anoop, T. V., Bobkov, V., and Ghosh, M. *Neumann domains of planar analytic eigenfunctions*. arXiv:2410.07811, 2024, <https://arxiv.org/abs/2410.07811>.