Time evolution of superoscillations

Peter Schlosser
Institute of Applied Mathematics
Graz University of Technology

joint work with: Y. Aharonov, J. Behrndt, F. Colombo

Abstract

In physics, superoscillations are superpositions of plane waves which (locally) oscillate faster than each of its components. In mathematics, this phenomenon is described by a sequence of functions of the form

$$F_n(x) = \sum_{l=0}^{n} C_l(n) e^{ik_l(n)x} \stackrel{n\to\infty}{\longrightarrow} e^{ikx},$$

which are linear combinations of exponentials with frequencies $|k_l(n)| \leq 1$, but converge to some exponential with frequency $k > 1$. The topic of this talk is now the persistence of superoscillations during the time evolution with respect to the Schrödinger equation. Or in other words, let $F_n(x)$ be the initial condition of the time dependent Schrödinger equation, does the resulting wave function $\Psi_n(t,x)$ have similar properties at some time $t > 0$? This question makes it necessary to generalize superoscillations to functions having the so called supershift property. Until now only specific potentials were investigated, for which in particular the corresponding Green's function is known explicitely. The main novelty of our unified approach is, that it only assumes suitable estimates and regularity properties on this Green's function.