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Guest Editors
J. Novotná & D. Pitta-Pantazi

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Introduction

JARMILA NOVOTNÁ: Department of Mathematics and Mathematical Education, Charles University in Prague, Czech Republic

DEMETRA PITTA-PANTAZI: Department of Education, University of Cyprus, Cyprus

Symposium on Elementary Maths Teaching (SEMT)

The conference *Symposium on Elementary Maths Teaching (SEMT)* is focused on the teaching of mathematics to children within the age-range 5–12 years. Its genesis started with the common belief of colleagues involved in elementary mathematics teaching and research in this domain that there was a lack of a platform for discussing the items of interest in mathematics for this age range. All other international scientific events were not specialised to this age range and it resulted in the lack of space for discussing deeply specific questions related to it.

SEMT is a biannual conference. The 9th Symposium being in August 2007, we can easily calculate that the 1st SEMT took place in August 1991. When conceived, a child has two parents. Our child, SEMT, also has two parents – two colleagues from Charles University in Prague, Michaela Kaslová and Jarmila Novotná. Conceived in 1990, SEMT was born in 1991 as the only conference focusing on teaching and learning of elementary mathematics. For SEMT, it were not only the parents who helped the child to grow up, it has had a number of aunts, uncles and friends who helped him to develop. The child grew up, developed, and adopted new ideas. But all the time, the main orientation towards teaching and learning of elementary mathematics was maintained. Getting older, it met new people who helped its development. They were Czechs as well as colleagues from abroad and they all influenced its face.

Each SEMT focuses of one important main topic of elementary mathematics teaching. The development from general topics towards more specific problems of elementary mathematics teaching can be easily recognised from the main topics of all nine SEMTs:

- 1991: The teaching of mathematics to elementary mathematics pupils
- 1993: The changing face of elementary mathematics
- 1995: Geometry and word problems for elementary mathematics
- 1997: Assessment and evaluation



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- 1999: How the world of mathematics emerges from everyday experiences of children
- 2001: What is meant by the competence and confidence of people involved in the teaching of elementary mathematics
- 2003: Knowledge starts with pre-conceptions
- 2005: Understanding the environment of the classroom
- 2007: Approaches to teaching Mathematics at the Elementary level

SEMT brings together elementary teachers, student teachers and teacher educators and researchers from all parts of the world. This is reflected in both the participants attending and those giving lectures and workshops. Among participants we can find colleagues from most countries in Europe, the Middle East, Japan, Australia and America. This mix of nationalities and the return of many old friends contribute considerably to the warmth and friendliness, which epitomises the SEMT conferences.

The multi-nationality basis of SEMT conferences helps to clear any ideas we might have that all the problems in the teaching of mathematics is in an individual country or even institution. Brief discussions with fellow participants quickly make the participants aware that many of the most difficult matters are common in many countries and that talking about them with each other helps and sometimes provides a solution.

Many new ideas for helping teachers to make mathematics both a meaningful and enjoyable subject have been exchanged during SEMTs. It has also helped new colleagues/post-graduate students to present their first papers to a discerning international audience.

SEMT has become an important international event with a high scientific as well as social standard. It has become an important junction where much international cooperation and a number of friendships have their origin.

It is not sure how many times an event has to occur before it can be called traditional (it seems that the media think that if something happens once that the second time it occurs a tradition has been set), however we can properly say that the biannual conference of the Symposium on Elementary Mathematics Teaching is now an established tradition since in 2009 there will be the tenth such conference. Its topic is: *The development of mathematical understanding*. We cordially invite all colleagues interested in elementary mathematics teaching to come to Prague in August 2009 and to participate in SEMT 09 scientific as well as social activities.

Papers published in this issue

The papers published in this Special Issue have one common feature: All authors were plenary speakers of SEMT 05 or 07. Their articles in the issue are closely connected with their plenaries.

N. Bednarz: Analysis of a Collaborative Research Project. A Researcher and a Teacher Confronted to Teaching Mathematics to Students Presenting Difficulties (SEMT 05, plenary: Bednarz, N.: A Mathematics Teaching Approach in “Weak Classes”: A Passage from Elementary to Secondary Level Rooted in Meaning Making Construal)

This article deals with an important issue – teaching mathematics in weak classes, more particularly in underprivileged schools. The research shows that students encounter important difficulties in mathematics, live repeated failure, which helps to explain their negative relation towards mathematics and their learning. These difficulties are particularly present during the transition between primary to secondary school. In the paper, the importance of the work in context, manipulations and changes of registers is documented. A collaborative research run by both teachers and the researcher was conducted with a group of weak students in the perspective of the possible significant learning of mathematics and a different relation to mathematical knowledge for these students. In the paper, the roles of participants in such a collaborative work, the fields of their separated responsibilities as well as the domain of common decisions are documented.

P. Scherer: Low Achievers Solving Context Problems – Opportunities and Challenges (SEMT 07, plenary: Scherer, P.: Investigating Children Learning Mathematical Argumentation in the Classroom Environment)

This article focuses on the problems that low achievers have while solving context problems. It suggests that low achievers do not simply face difficulties with calculations. They often have difficulties in understanding a situation, a given text, a picture or finding an appropriate representation for their ideas. These problems are illustrated through specific examples presented in the article. The article then addresses the different requirements that are necessary to solve context problems and emphasises the importance of modelling and mathematizing. Petra Scherer suggests that in order to facilitate low-achievers a variety of problems should be presented to them, their own strategies should be supported, basic mathematical knowledge should be emphasised and more conscious selection should be made to the numerical data used.

B. Sarrazy: Analysis of the Didactical Effects of Conception of the Wording of Arithmetic Problems. (SEMT 05, plenary: Sarrazy, B. & Novotná, J.: Didactical contract: Theoretical Frame for the Analysis of Phenomena of Teaching Mathematics)

This article offers the anthro-didactical approach as a powerful tool for developing phenomena from the theory of didactical situations in mathematics (TDSM). The basic TDSM concept dealt with is the didactical contract. The responsiveness to it is

presented as a factor influencing significantly the success of teaching/learning process. The relations to choosing sets of problems appropriate for an individual student, for a group of students or generally are documented by the case of arithmetic problems. The anthropo-didactical approach enables the author to bring in also external factors (family, society, socio-cultural environment) for both students and teachers.

H. Steinbring: Children's Ways of Mathematical Argumentation in the Classroom Environment. (SEMT 05, plenary: Steinbring, H.: Children's Ways of Mathematical Argumentation in the Classroom Environment).

The article by Heinz Steinbring is based on his book "The Construction of new mathematical knowledge in classroom interaction – An epistemological perspective". It explores the specific forms and conditions of mathematical communication in the primary school mathematics classroom. The author suggests that mathematical discourse among researching mathematicians and among learning students is very different. While mathematical argumentation in science is explicit, unambiguous and strict, the mathematical communication among learning students is based on concrete examples, every day descriptions and their personal judgements. To illustrate this point, two case studies of two school children are presented. The article concludes that irrespective of these differences between researching mathematicians and learning students, students are able to construct new mathematical knowledge with their own explanations, interpretations and argumentations.

R. Zazkis: Number Theory in Mathematics Education: Queen and Servant (SEMT 07, plenary: Zazkis, R.: Number Theory in Mathematics Education: Queen and Servant)

The unique position of the number theory in mathematics is well known from the whole human history. The role of a "queen" was explicitly expressed by important mathematicians like e.g. K.F. Gauss but implicitly it is present in most of mathematical "big" ideas. Being a queen does not mean only a dominant position through the whole history of the development of mathematics but it includes also the role of a "servant" for many fields of mathematics. In the paper, the elementary number theory is presented from both queen and servant perspectives. Findings are illustrated by examples.

J.M. Watson: The Development of Statistical Understanding at the Elementary School Level. (SEMT 07, plenary: Watson, J.M.: The Development of Statistical Understanding at the Elementary School Level)

This article presents a developmental picture of statistical understanding from early childhood across to the middle school years. This developmental picture can be useful to educators, curriculum planners and researchers in two fundamental ways. Firstly, it offers the possibility to pin point where students are at the developmental sequence, and secondly, it indicates the kind of activities that are necessary to facilitate their movement to higher levels of understanding. In this article, Jane Watson offers examples of tasks reflecting the development of understanding and suggests that exposure to these or similar tasks offer pathways for students' development in statistical understanding.

K. Ruthven: Towards a Calculator-Aware Number Curriculum. (SEMT 07, plenary: Ruthven, K.: Towards a Calculator-Aware Number Curriculum)

The article by Kenneth Ruthven examines what educators and policy-makers may learn from a pioneering effort to develop a “calculator aware” number curriculum. It considers pedagogical strategies in which calculators can contribute and then discusses pieces of research which examined the long-term impact of such curricula. It suggests that the most important long-term impact of calculator-aware curricula was on students’ attitudes and abilities in mental calculations. It was found that students, who participated in calculator-aware curricula, were more inclined to calculate mentally and use powerful strategies. The article concludes that a calculator-aware curriculum is not simply a curriculum which makes use of calculators, nor it is completely “calculator-based”. It is also stressed that such a curriculum cannot be designed around a conventional curriculum. Careful planning, well-thought sequences of activities are necessary to support the progression of children’s learning.