TEACHERS' VIEWS AND USE OF EXPLANATION IN TEACHING MATHEMATICS

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Abstract

This study analyses teachers' of mathematics views on explications in teaching mathematics. Various types of explanations are characterised and investigated from the point of view of their pedagogical, cognitive and social impact on learners. The theoretical frame for the research is based in the Theory of didactical situations (Brousseau, 1997). Based on questionnaires filled in by Czech teachers, the reality of teachers' views and perception of their own practice concerning the use and forms of explanation are studied.

1. Explanation in teaching/learning process

Using explanation in a mathematics classroom is a normal procedure[1], but its roles and forms vary. Predominantly explanation is seen as a tool for describing relevant phenomena, developing students' logical thinking, and guiding students by inductive judgement to generalising. It leads to clarifying interrelations, demonstrating and justifying (Skalková, 1999, p. 172). Although explanation is not often explicitly studied in literature, it is present in the background of most papers dealing with communication and reasoning.

In this paper the concept of explanation is developed in the sense of (Mopondi, 1995). Explanation is characterized by its function as "a tool that is used by a speaker for understanding or 'giving a sense' to the object of communication, of a debate, or a discussion … The role of an explanation is to make clearer the meaning of an object (method, term, assignment) maintaining formally the necessary distance between the object of the action or study and the tools." (p. 12). In the learning/teaching process, explanation is a tool used by both, teacher and students. Its goal is to manifest comprehension.

Varieties of explanation can be classified using a number of variables[2] from which we will concentrate on the following (1 refers to the general **situatio**n, 2 the **individuals** involved, 3 to the academic/ intellectual **contex**t, 4 to the **resul**t, 5 refers to the **formal structure** of the explanation):

- <u>(1) Type</u> of situation (teaching/learning, communication, discussion)

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[1] Traditionally, explanation belongs to monological teaching methods where the information is transmitted in the direction teacher to students (together with e.g. narrative, description or lecture). (Skalková, 1999) states that in practice, individual forms of explanation often pervade. In this perspective, explanation is seen as the task fulfilled by the teacher with students passively receiving what is presented. We see explanation in a much broader sense: Communication in school is a mutual interchange of information among participants of the educational process, i.e. students have an active role in the whole process (Mareš & Křivohlavý, 1995).

[2] In (Mopondi, 1995, p. 13-14) the most important variable is "transmitter – receiver", the subordinate ones are its goals, nature of the demand, object, type, form and type of occurrence.

- (1) Type of occurrence of the explanation (necessary for the progress of a didactical action, arising naturally from the situation, accidental, etc.)
- (2) Transmitter (author) of the explanation (teacher, student(s)) and receiver of the explanation (teacher, one student, group of students) (one-one; one-many; many-one)
- (3) Nature of the demand for explanation (proposed by the transmitter, demanded by the receiver, generally spontaneous); linked with the activity of students (active – student produces an explanation during a discussion, passive – student receives an explanation as a piece of information)
- (3) Objects of the explanation (explanation of an order, a mistake, a statement, an algorithm etc.); linked with the purpose of the explanation (for the teacher: to obtain information about students, e.g. to detect the location a misunderstanding/mistake, to eliminate a misunderstanding/ mistake; for students: to build up knowledge, to find an appropriate solving procedure, etc.)
- (4) Results of the use of explanation (for a teacher: facilitation of students' comprehension, feedback about the level of students' comprehension, etc.; for students: inclusion of a piece of knowledge into an existing knowledge structure, building a new structure, conversion of knowledge into a tool for problems solving, etc.)
- (5) Character of the explanation (a range of levels from purely formal to a deep involvement in the explanation object)
- (5) Form of the explanation (e.g. description, reformulation, visualisation, presentation of examples or counterexamples; for more details see chapter 3); linked closely with the age of participants
- (5) Language of explaining (a range of levels from the exact language using special terminology to the natural language without any "scientific" precision)
- (5) Frequency of the use of explication

From the variables proposed above, the vast diversity of explanation is obvious. This diversity leads to difficulties in detecting each of them in teaching/learning situations.

2. Our research

The work presented here is a part of a longitudinal study of the deliberate implementation of modes of explanation in teaching/learning situations in school mathematics. It focuses on teachers' theoretical perception of explanation and teachers' own school practice. The study represents an introductory part of a broad set of problems, for example the consequences of the use of different forms of explanation and the frequency of their and contexts in which they are used.

2.1. Research questions and methods

The research questions for the study are:

- Theoretical perspective: What modes of activity do mathematics teachers accept as explanations in their teaching (in cases of both types of transmitters)? What differences do they perceive and employ in relation to the age of the students?
- Practical perspective: How do teachers project the theoretical perspective into their own teaching practice?

For the initial part of this study, questionnaires were used which cover both the cases of transmitters – a) a teacher, b) a student. It was divided into two main sections each of which was devoted to one of the research questions: the "Theoretical part" with questions covering the forms and implementation of explanation; and the "Practical part" with questions reflecting teachers' own practice in both cases a) and b).

The main goal of the study was to map the situation with mathematics teachers. In order to be able to detect more clearly which views are more general and what is specific to the teaching/learning of mathematics, three groups of teachers participated in the investigation: Group A were 12 primary school teachers and there were two control groups: Group B of 11 secondary mathematics teachers (to reinforce detection of age factors), and Group C were 7 secondary teachers of non-mathematical subjects (to reinforce detection of what is specific for teaching/learning of mathematics). The comparison of the theoretical and practical parts was previewed to detect the distance between theory and school reality.

2.2. Analysis of teachers' responses to the questionnaire

2.2.1. Theoretical perspective

Forms of explanation

Question: What do you consider as explanation? (Underline types that you consider to be an explanation, possibly add further that are missing.) Description, rephrasing, definition, proof, presentation of a counterexample, commentary, illustration, use in another situation, analogy, decomposition into sub-cases, model.

Group A: All respondents underlined Presentation of counterexample, Illustration and Model.

Definition was not underlined as an explanation suitable for primary level children). This can possibly be ascribed to the influence of the age of students.

Group B: The mostly underlined form was illustration (10/11[3]), the less Definition (3/11).

[3] b/c expresses that b respondents from c choose the answer.

Added forms: Demonstration of context, reminding previous experiences

Group C: All proposed types were underlined by at least 2 respondents. The items mostly underlined were Description, Illustration, Use in another situation and Analogy, the less one Decomposition into sub-cases. It suggests that Decomposition into sub-cases is specific for mathematics (science) teaching. Three respondents (3/7) underlined Definition. This suggested that definitions in mathematics have a special status in comparison with non-mathematical domains.

Four questions related to two cases of the *transmitter* were posed:

(i) Which goals does explanation fulfil?
(ii) Which pitfalls does explanation bring?
(iii) In what phases of the teaching process do you consider explanation to be most effective?
(iv) How does explanation differ in relation to the age of students?

In the following text we summarise answers of Group A. Stock answers in Groups B and C follow (their frequency is not discussed). The answers are categorised according to about whom the answer speaks: T – teacher, S – student(s). The answers that occurred in more than one of Groups A, B and C are underlined.

a) ***Teacher as the transmitter***

Question (i), goals

Group A: In the answers, the possibility of better understanding children's reactions and the subsequent more appropriate action of the teacher (T) and understanding relationships and conclusions (U) were emphasized. In comparison with Groups B and C, the answers were much more student than subject matter oriented.

Typical answers:

T: Faster introducing of students into the subject matter (Groups B, C), more effective (Groups A, B)
T: Higher number of possibilities to address students with different levels of mathematics abilities (Groups A, B, C)
T: Presentation of interdisciplinary and intradisciplinary relations (Group B)
T: Locating students' difficulties (Groups A, B, C)
T: Delivery of knowledge to students with understanding (Groups A, B)
S: Illuminating concepts and processes that are not clear to students (Groups A, B, C)
S: Help when solving problems (Groups A, B)
S: Achieving precision of concepts (Group B)
S: Relating new knowledge to previous ones (Groups B, C)

S + T: Mediation of contacts between the teacher and students (Group A, C)

Question (ii), Pitfalls

Groups A and C

In these groups, the danger of using inappropriate language and lack of understanding the terms used was emphasized. It did not occur in Group B. It looks to be the age effect in Group A and the precise nature of mathematical language with older students which is not so strong in other school subjects.

Typical answers:

T: Extra work for the teacher – more preparation (Group B)

T: Excessive teacher's use of monologue, transmissive way of teaching (Group B)

T: Unsatisfactory focus on individual differences of students (Groups B, C)

S: Unsatisfactory involvement of students in the teaching/learning process, increase of students' passivity (Groups B, C)

S: Decrease of students attention (Groups A, B)

S: Decrease of students' responsibility for their education (Group B)

S: Insufficient use of students' previous knowledge (Group B)

Question (iii), Phases of teaching process

In all groups, answers belong to three domains: phases of the lesson – time, phases of the lesson – activity, (individual or group) needs. As no essential differences in the answers occurred, all groups are summarised together. The answers do not look to be significantly age or subject matter dependent. The constructivist nature of the use of explanation was significantly emphasized.

Phases of the lesson - time:

- Most answers concerning this domain proposed the opening of the lesson; in one answer it was stated that the appropriate period is the first half of the week
- After the phase of discovering

Phases of the lesson - activity:

- Introduction of new subject matter
- Revision
- Problem solving
- Commenting on results of assessment
- When students do not understand, or make too many mistakes

Needs:

S: When a student needs a help

S: When a group of students do not understand

T: When the teacher needs to get the feedback about students' understanding

Question (iv), Age

In all groups, answers were similar. They belong to four domains: form, length, language and frequency of explanations. Respondents' common opinion was that with younger children, the explanation should be more concrete, shorter but more frequent. It is necessary to use more illustrative forms. It was noted that explanation based on examples from real life situations are more important for younger students. Proving was not thought to be as acceptable for younger children. The precision and technical level of language was considered as more important for older students.

One answer in Group B was: The only difference which depends on students' age is in the level of complexity of solved problems, there are no differences in any other aspect.

b) ***Student(s) as the transmitter***

In all groups the answers were similar. We present the common ones. Single occurrences in one of Groups A, B or C are mentioned separately.

Question (i), Goals

T: Easier opportunity to find students' misunderstandings and location of mistakes
T: Possibility to follow students' argumentation in details, understanding of it
T: Involving students in the work, increasing of motivation
T: Learning students' preferred language
S: Development of students' creativity, argumentation, independence, ability to draw conclusions
S: Forming working skills
S: Development of interaction and communication skills
S: Increasing students' self-confidence

Only in Group B were the consolidation and verification of knowledge explicitly expressed. It looks to be typical for mathematics.

Note: The role in institutionalisation (Brousseau, 1997) of knowledge did not occur.

Question (ii), Pitfalls

T: Considerable time requirements, difficult (impossible) lesson planning and guiding students which demands the necessity to improvise
T: Necessity to handle individual mistakes and inaccuracies, and the negative impact of it on others
T: Misunderstanding between students and the teacher concerning explained items
S: Imbalance in students' involvement, passivity of some of them
S: Negative impact of presented mistakes etc.
S: Lower use of previous experiments and existing structures

In Groups B and C, there were two respondents who did not see any pitfall but stressed the positive sides only: "Even an incorrect explanation is an asset.", "Process is equally conductive, or even more, than the result."

Question (iii), Phases of teaching process

The answers copied the answers from the case a).

Question (iv), Age

The age differences concerned the level of using visualisation and modelling, the language and the use of concrete examples, all of them simpler with younger students. The use of the case b) (student(s) as transmitters) was strongly recommended only for older students. The independence in selecting the explanation form and the level of abstraction was considerably emphasised in Group B. In one case in group B, hints were proposed of a suitable form of explanation for all ages.

2.2.2. Practical perspective

Both cases a) (10 questions) and b) (7 questions) are dealt separately, one question concerns both of them. All questions ask for the respondent's own practice.

a) ***Teacher as the transmitter***

The first group of questions covers phases of lessons, of the educational process, and forms used by the respondent and the frequency of using them; these questions are directly linked with the theoretical perspective. The second group deal with planning the use of explanation, use of erroneous explanation, demanded precision of the language and consequences of too rare or too frequent use of explanations.

In the answers to the first group, theoretical perspective was repeated mainly in Groups A and C. It suggests that the answers in the Theoretical part were based on respondents' practical experience. The restriction of forms to visualisation, modelling and illustration, use in other situations, rephrasing and (surprisingly) use of hints occurred.

Answers to the second group of questions brought new information. All teachers except one (Group C) indicated that they choose the places for using an explanation when planning the lesson; their decision is based on their experience. The possibility of increasing the number of explanations according the situation in the class was explicitly expressed by two teachers in Group B. The need for precision of explanation by the teacher was commonly accepted. Only one teacher explicitly mentioned the benefits of lowering precision to help weaker students.

The diversity in answers concerning the deliberate use of explanations containing error was considerable. In Group A explanation containing error is used. In group B 3/11 respondents do not use any form of such explanation, the others use it as a tool for emphasizing more difficult places, for attracting students' attention, for breaking students' acceptance of facts without understanding; two teachers do not use such explanation before "sufficiently long practice" of the subject matter. In Group C this question was not answered. The deliberate use of explanation containing error looks to be specific for mathematics (and natural sciences).

As to the frequency of using explanation, all respondents mentioned the danger of students' passivity in the case of too frequent use and building incorrect knowledge if explanation is used only very rarely.

b) ***Student(s) as the transmitter***

The questions cover the nature of the demand for explanation (spontaneous or on the teacher's invitation), erroneous explanations (handling of them or the possibility of their further use) and the frequency of incorporating explanations in the lessons.

Teachers use both spontaneous and invited explanations; all of them praised the advantages of spontaneously proposed explanations. (Typical answer: "Let the person who has something to say speak, everybody will learn from mistakes that occur.") For correcting mistakes, discussion about mistakes (mediated by the teacher if necessary) is taken as the correct solution. (Typical answer: "We all learn from mistakes, and why it is a mistake.") As to the frequency, the danger of too frequent use of explanation was mentioned in all answers ("only for being listened to" was mentioned in various forms); in case that the use of explanation is too rare, the danger of passivity of students was emphasized in Group A and the danger of underdeveloped communication skills in Groups B and C. The frequency of incorporating explanation showed to be age dependent in the answers.

3. Concluding remarks

The study confirmed the diversity in approaches to explanation in teaching/learning of mathematics. It focused on teachers' views and approaches to explanation. It will continue by studying situations of explanations from the theoretical perspective and by observing and analysing the school reality. Our main interests are positive and negative impacts of the use of explanation in teaching/learning processes.

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