RESEARCH FORA

Classroom research in mathematics education as a collaborative enterprise for the international research community:
The learner’s perspective study
David Clarke and Jarmila Novotná - Coordinators

Examining teachers’ use of (non-routine) mathematical tasks in classrooms from three complementary perspectives:
Teacher, teacher educator, researcher
Ron Tzur, Orit Zaslavsky, and Peter Sullivan - Coordinators

Pursuing excellence in mathematics classroom instruction in East Asia
Yeping Li and Gabriele Kaiser - Coordinators
CLASSROOM RESEARCH IN MATHEMATICS EDUCATION
AS A COLLABORATIVE ENTERPRISE FOR THE
INTERNATIONAL RESEARCH COMMUNITY:
THE LEARNER´S PERSPECTIVE STUDY

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The key features of the Learner’s Perspective Study (LPS) methodology are the use of multiple video cameras to capture sequences of lessons, supplemented by post-lesson video-stimulated reconstructive interviews, and collaborative analysis of the resultant data set by an international team of researchers employing different theoretical perspectives. The multiple, parallel analyses being undertaken on the data, once generated, have fueled lively discussion regarding the complementarity and commensurability of the various theories being employed. The process and outcomes of undertaking international classroom research as a collaborative enterprise are evident in all the following contributions.

BACKGROUND

The Learner’s Perspective Study was designed to examine the practices of eighth grade mathematics classrooms in a more integrated and comprehensive fashion than had been attempted in previous international studies. The project was originally designed to complement research studies reporting national norms of student achievement and teaching practices with an in-depth analysis of mathematics classrooms in Australia, Germany, Japan and the USA. The project was initiated by David Clarke, Christine Keitel and Yoshinori Shimizu. Since its inception, research teams from other countries have continued to join the Learner’s Perspective Study. The title of the project (The Learner’s Perspective Study) was intended to complement teacher-focused studies by foregrounding the learner’s perspective. As the project grew, its purpose was progressively reinterpreted and expanded. Students, teachers and researchers can all be considered to be learners: partners in an international collaboration to develop new knowledge and to understand and improve the practices and outcomes of our classrooms. It is an essential thesis of the Learner’s Perspective Study (LPS) that international comparative research offers unique opportunities to interrogate established practice, existing theories and entrenched assumptions.

General structure and topics

The Research Forum is structured around the following topics:

i. The challenge of international comparative classroom research
ii. Approaches to researching lesson structure
iii. Connecting the Learner's and the Teacher's perspectives
iv. Contrasting theoretical approaches to the analysis of classroom data (including the reflexive association between theory and methodology)

v. The results of international classroom research: Finding structure in diversity

vi. The capacity of international classroom research to inform practice

This choice of topics reflects the progression in the activities of the international LPS research community over the past nine years, but also sets out the challenges confronting anyone contemplating international comparative classroom research, through the essential methodological, technical and theoretical considerations, to the production of substantive research findings and the consequent question of how such international research might inform classroom practice.
THE CHALLENGE OF INTERNATIONAL COMPARATIVE CLASSROOM RESEARCH

Logistical and technical challenges in international research collaborations

David Clarke

It is imperative that research in mathematics education makes optimal use of available technology. International comparative classroom research, in particular, poses methodological and technical challenges that are only now being adequately addressed through advances in:

- techniques and equipment for the generation of audio-visual data in classrooms;
- tools for the compression, editing and storage of digitised video and other data;
- storage facilities that support networked access to large complex databases;
- data distribution systems that support secure, remote access for data entry and retrieval on an international scale; and
- analytical tools capable of supporting sophisticated analyses of such complex databases.

The LPS community has addressed each of these challenges.

All too often it is forgotten that any use of technology in a research setting implies the existence of an underlying theory on which the type of data, the means of data generation, and the anticipated method of analysis are all predicated. Clarke (2001 and 2006) has argued that since a classroom takes on a different aspect according to how you are positioned within it or in relation to it, our research methodology must be sufficiently sophisticated to accommodate and represent the multiple perspectives of the many participants in complex social settings such as classrooms.

The LPS data have been generated for sequences of at least ten consecutive lessons occurring in the “well-taught” eighth grade mathematics classrooms of three teachers in fourteen of the participating countries (Australia, China, the Czech Republic, Germany, Israel, Japan, Korea, Norway, The Philippines, Portugal, Singapore, South Africa, Sweden and the USA). This combination of countries gives good representation to European and Asian educational traditions, affluent and less affluent school systems, and mono-cultural and multi-cultural societies.

Each participating country used the same research design to collect videotaped classroom data for at least ten consecutive math lessons and post-lesson video-stimulated interviews with at least twenty students in each of three 8th grade mathematics classrooms. The three mathematics teachers in each country were identified for their locally-defined ‘teaching competence’ and for their situation in demographically diverse government schools in major urban settings. The three
lesson sequences were spread across the academic year in order to gain maximum
diversity of local curricular content. Post-lesson student interviews were conducted,
in which a split-screen video record was used as stimulus for student reconstructions
of classroom events. Students were given control of the video replay and asked to
identify and comment upon classroom events of personal importance. Each teacher
was interviewed at least three times using a similar protocol.

The Learner’s Perspective Study is committed to (i) adequate recognition of the
perspectives of all participants and specific embodiment in the data generation of
those perspectives, (ii) deliberate utilisation of both primary and secondary analyses
to provide a wide range of theoretical perspectives on the social setting and situations
being studied, (iii) the synthesis of the subsequent primary and secondary analyses
into an integrative amalgam of interrelated complementary accounts (Clarke, 2006),
and (iv) the development of “practical explanatory theory” (Nuthall, 2004, p. 295) by
which classroom activity is connected to learning outcomes.
THE QUESTIONABLE LEGITIMACY OF INTERNATIONAL COMPARATIVE CLASSROOM RESEARCH

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The questionable legitimacy of comparison based on common sense approaches to culture

The ICMI launched its 13th study with the title “Mathematics education in different cultural traditions: A comparative study of East Asia and the West”. The discussion document (ICMI, 2001) stated: “For this study, culture refers essentially to values and beliefs, especially those values and beliefs which are related to education, mathematics or mathematics education.” The West is, in this ICMI-study, identified with the Greek, Latin and Christian tradition. Such a framing of comparative research runs the risk of oversimplifying the situation by appearing to assume that school systems or classrooms can easily be aligned with one of these traditions. Wong and Wong (2002) point to the fact that the “examination culture”, which was designed for governance purposes in China, might have a much greater impact on achievement orientation than Confucianism.

There might be a lot of other appropriate definitions of regions to compare. The content of school curricula is linked to political and economic characteristics, including colonial history. A case could be made for grouping former British colonies or for grouping Islamic countries. In some studies a simplistic interpretation of “culture” as synonym for nation is adopted. This identification of culture with nation or geographical region is only reasonable if it refers to the use of the same language or of socially significant different forms of it, or to the commonalities of the institutional setting and its tradition. Categorizing school systems and the environment in which they are embedded by country or by geographic location, does in any case not take into account that in many countries, classrooms are inhomogeneous in terms of ethnic affiliation. In addition, taking countries as units of analysis conceals differences within provinces or states, for example in countries like Germany and the United States. It has to be acknowledged that cultural phenomena do not occur in a social and economic vacuum.

Representativeness versus typification

One goal of comparative classroom research has been typification of elements of practices that are interpreted as being representative of mathematics teaching in a distinct cultural context. Such a research can produce valuable insights about differences between contexts and the amount of variation within one context. Based on a representative sample of mathematics lessons from Finland and Iceland, Savola (2008), for example, found a common lesson structure in the lessons from Finland. In Iceland, on the other hand, half of the lessons exhibited variations of a Review-
Lesson-Practice structure, whereas the other half followed a totally different pattern. While the notion of representativeness refers to statistical practice, typification is linked to ethnographic methodologies and phenomenology. However, describing data from classroom observations in terms that make the data comparable is a shared goal. For the LPS, the identification of patterns that are representative of the teaching in a nation was not the goal, but rather to compare and contrast different elements of classroom practices in a variety of school systems. The selection of classrooms was based on identifying competent teachers based on local criteria of what being a competent teacher might mean. Whether any national pattern identified on a base of a representative sample of lessons in a country manifests itself in the classrooms chosen for the LPS, has been analysed for Germany, Japan and the United States (Clarke et al, 2007). It turned out that the variation between the lessons of different teachers in each country was considerable. There was no evidence of the lesson patterns reported for these nations by Stigler and Hiebert (1999). The LPS classrooms cannot be seen as representative in a statistical sense. Still, the classrooms are typical for practices, in which some pedagogical values that define the notion of “competency” of a mathematics teacher are in operation.

The data produced in the LPS have the potential of identifying the lesson elements not only on the base of classroom observation, but include the insiders’ perspective with the help of the post-lesson student interviews. Unfamiliar forms can be more easily identified by the outsider, but are harder to be interpreted. In turn, it is harder for the insider to look for alternative interpretations. This remains a general issue for qualitative research in cross-cultural settings (cf. Hoonaard, 1997; Udy, 1964).
The contention of Stigler and Hiebert was that at the level of the lesson, teaching in each of Germany, Japan and the USA could be described by a “simple, common pattern” (Stigler & Hiebert, 1999, p. 82). This proposal was based on analysis of a “nationally representative” sample of single lessons. By contrast, the Learner’s Perspective Study (LPS) conducted a fine-grained study of sequences of ten lessons, informed by the reconstructive accounts of the participants. Such a study has the capacity to identify any recurrent pedagogical elements in a teacher’s classroom practice and any evidence of regularity in the sequencing of those elements. Such regularities and recurrent elements have the potential to serve as the basis for comparative analysis.

Lesson structure can be interpreted in three senses:

i. At the level of the whole lesson – regularity in the presence and sequence of instructional units of which lessons are composed;

ii. At the level of the topic – regularity in the occurrence of lesson elements at points in the instructional sequence associated with a curriculum topic, typically lasting several lessons;

iii. At the level of the constituent lesson events – regularity in the form and function of types of lesson events from which lessons are constituted.

In terms of international comparison, it is useful to consider which of these three forms of lesson structure are likely to prove useful as units of comparative analysis. The same three alternatives are available for the purposes of national typification, but the optimal unit of international comparison need not be the same as the optimal unit for national (or cultural) typification. We can conceive of the possibility of an idiosyncratic practice that might typify the classrooms of a nation, but be so unusual as not to constitute a legitimate basis for international comparison.

In terms of lesson structure, it might be that for one nation or culture there is no nationally characteristic structure to the lesson as a whole, but that particular types of idiosyncratic lesson events offer the most appropriate typification. For another nation or culture, there could be a high degree of regularity to the composition of lessons, or in the sequencing of particular types of instructional activity in the delivery of a topic. Such differences in the form of typification provide a basis for international comparison that reflects something more essential to each than the imposition of the same structural level as the basis for the comparison.

Incommensurability of the emergent typifications becomes relevant if the comparison is intended to be evaluative. However, in the case of the LPS, the identification of idiosyncratic practices, identified in one or a few classrooms but absent entirely in
other classrooms, offers the teachers of those other classrooms entirely new pedagogical tools, potentially valuable, since they derive from the practices of competent teachers elsewhere.

The teachers whose classrooms we had documented showed little evidence of a consistent lesson pattern, but instead appeared to vary the structure of their lessons purposefully across a topic sequence. The evident differences in the manner in which teachers structured their lessons, suggested that another unit of analysis was needed: one that corresponded more closely to the decisions made by each teacher regarding the structure of any particular lesson. Our analysis of the LPS lessons focused therefore on the form and function of recognizable activity conglomerates that we called ‘lesson events.’

Each individual lesson event had a fundamentally emergent character, suggested by the classroom data as having a form (visual features and social participants) sufficiently common to be identifiable within the classroom data from each of the countries studied. In each classroom, both within a culture and between cultures, there were idiosyncratic features that distinguished each teacher’s enactment of each lesson event, particularly with regard to the function of the particular event (intention, action, inferred meaning and outcome). The teacher and student post-lesson interviews offered insight into both the teacher’s intentions in the enactment of a particular lesson event and the significance and the meaning that the students associated with that event type.

Each lesson event required separate and distinct identification and definition from within the international data set. Lesson events included: Kikan-Shido (between-desks-instruction), beginning the lesson, the learning task, student(s) at the front, putting into practice, and Matome (summing up), and detailed analyses related to these lesson events can be found elsewhere (Clarke, Emanuelsson, Jablonka & Mok, 2006; Clarke, Keitel, & Shimizu, 2006; Clarke, Mesiti, O’Keefe, Xu, Jablonka, Mok & Shimizu, 2007).
AN ANALYSIS OF JAPANESE LESSONS ON LINEAR FUNCTIONS

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INTRODUCTION

Recent research studies have a common and persuasive vision of mathematics classroom as socioculturally mediated milieu. Different classroom cultures mediate different beliefs, attitudes, and contracts with respect to classroom interaction, and with respect to mathematical activity (Novotná & Hospesová, 2007). In everyday classroom practice, teacher and students coordinate the extent to which they participate in a particular mathematical activity, their role in accomplishing it, and the extent to which they take direct responsibility for accomplishing it (Clarke, 2003).

The purpose of this paper is to investigate “mathematical task structure” and “participation structure” in a Japanese (J1) classroom. By “mathematical task structure”, we mean the way the teacher elaborately organizes mathematical tasks throughout the unit. By “participation structure”, we mean the way the teacher coordinates the extent to which students participate in accomplishing mathematical tasks (Ohtani, 2002). The focus is on patterns of distribution of participation rights allocated for teacher and the students in accomplishing mathematical task. Transcripts of video-audio records of ten consecutive J1 lessons on linear equation were analysed.

Task Structure and Participation Structure

For “mathematical task structure”, we find three ordered components; “contextual tasks” (Day1-3), “transitional tasks” (Day 4-7), and “general tasks” (Day 8-10). Each component performs a unique role: setting mathematical motive; guided use of symbolic devices or cultural tools, and appropriation of mathematical object, respectively.

The first component involved establishing the motive for functional thinking. Contextual tasks in concrete situations serve as a continuous reference and model of quantitative relations. The teacher poses an open-ended contextual task and expects students with different perspectives to find many kinds of dependent and independent variables.

The second component consisted of the progressive transition from contextual task to referential tasks. The role of transitional tasks is to guide students to use symbolic devices such as table, graph, and algebraic expressions as cultural tools. During initial use, the symbolic devices have an operational aspect for computing particular values. The principal function of symbolic devices is making the transitory constantly present and, at the same time, providing tangible means of communicating their idea and
conjecture in particular concrete situations to others. This function is called “intermental” (Vygotskii, 1984).

In the third component, the teacher proceeds from referential to general tasks. The role of general tasks is to employ “linear function” as an abstract object, where the symbolic devices function not only as a means to solve decontextualized problems but also as objects representing the linear function itself. The teacher introduces problem conditions, which contain defining characters, and the mathematical terminology of linear functions. Students engage in solving these problems using symbolic devices in order to find invariant properties of linear functions. Such a use of symbolic devices is called “intramental” (Vygotskii, 1984).

**Teacher’s Strategy for Organising Lesson Structure**

For describing the teacher’s strategy for organizing “task structure” and “participation structure”, I draw on the concept of “revoicing” (O’Connor & Michaels, 1996). By “revoicing” we mean a particular kind of re-utterance of one’s contribution by another participant in a discussion.

Analysis of the data showed that two kinds of revicing were extensively and exclusively used by the teacher during classroom interaction. One was “public revicing” and the other was “measured revicing”. For public revicing, the teacher not only replied to nominated individual students, but also addressed all the students. This means that the teacher capitalized on particular students’ contributions to address the whole class in order to promote collective reflection. “Publicity of revicing” was obvious during student independent work. For “measured revicing”, the teacher expected a variety of student responses to the assigned task and had a plan to capitalize on their contribution in order to formulate challenging problems and elaborate their solutions through collective argumentation.
CONNECTING THE LEARNERS’ AND TEACHER’S PERSPECTIVES
Capturing Complex Classroom Interactions

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Background of participation in the LPS

Participating in the Learners’ Perspective Study (LPS) during PhD research provided research opportunities that would otherwise not have been available. The research design captured multiple perspectives on classrooms interactions that enabled study of what supported students during their creation of new (to the student) mathematical ideas. Taking a role in ensuring across country consistency of application of the design protocols gave a broad perspective of what it could mean to teach and learn mathematics. My studies were enriched by these opportunities.

The LPS team has at least one international meeting per year and in some years a Learners’ Perspective Conference in Melbourne, and a retreat to Wilson’s Promontory, which is a peaceful sanctuary in an isolated beach side area in Victoria, Australia. This provided many opportunities for researchers to get to know each other and discuss ideas. This study commenced with researchers from different countries sharing their research perspectives. During the study, we alerted each other to aspects of our country’s data that could be relevant to another’s focus. In addition, we probed interview responses further where we recognised the relevance of a response to another team member. Thus, I had a ‘research team’ who willingly alerted me to data that might be relevant, and sometimes even generated such data (Williams, 2005).

Team discussions about consistency of application of the study design across countries, and my participation in initiatives to gain this consistency, helped me to appreciate how differently a research design can be interpreted without such initiatives.

Visiting Year 8 classrooms in four countries (Germany, the USA, the Philippines and South Africa), focusing intently on classes in two other countries, discussing teaching and learning with research teams, and sharing my own observations about these classrooms from the perspective of my study, broadened my perspective, and helped me to communicate and crystallise my ideas.

LPS Team Membership: developing research rigor

In 2003, when I presented my findings at an LPS conference, I intended to show the importance of ‘spontaneity’ in the creative development of new knowledge. I was surprised to find this group of experts, who were empathetic to my research, did not understand what I was trying to communicate. The questions they asked me, and the intensity of the subsequent discussion, helped me to realise I needed to develop the construct of spontaneity more rigorously and illustrate it empirically.
The power of the LPS research design

In addition to the multiple perspectives enabling triangulation, the data collection techniques provided opportunities to ‘retrieve’ data when its significance later became apparent. Take the example where the student (Leon) stated:

When you look around the classroom and see how everyone else is doing it and you are doing it a completely different way- … and you think ooh! maybe my method isn’t the best and … you think about everyone’s … and then you think about your own and they all sort of piece together and you just sort of go oh! and it pops into your head (Williams, 2006a, p. 227).

What had Leon seen? Had other students already found what “popped into” Leon’s head? Or did he really develop this idea for himself by integrating what he could see on the pages with others with his own developing ideas? A search of the whole class, teacher, and focus-student videos suggested other students were undertaking the problem in less sophisticated ways, but not all student pages were visible on the videos. In her interview, the teacher confirmed that other students had used less sophisticated approaches. Knowing what was on other students’ pages assisted my analysis of how Leon developed his insight. Without access to another data source, the conclusions could not have been held with the same strength. Multiple secondary analyses of the data were employed to support interpretations made. The research design contributed to the insights developed.
THE USE OF WORK PLANS IN SIX NORWEGIAN 9TH GRADE MATHEMATICS CLASSROOMS

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Our empirical material was collected from six 9th grade classrooms in Norway. In all these classrooms, work plans were used as an organisational and didactical tool. The work plan is a document that describes what the students are supposed to do in the different subjects over a certain period of time, often two or three weeks. The idea behind work plans is to empower the students and give them the opportunity to make decisions related to their own work at school: what to do, how to do it, when to do it, and with whom. Work plans are in this way supposed to stimulate and facilitate self-regulated learning by making the students assume responsibility for their own learning processes (Klette 2007). In all the six mathematics classes, the content of the work plan in mathematics was decided by the mathematics teacher only. The students did not participate in composing the plan.

The work plans were not individual, but did contain some sort of level differentiation, usually three. It was up to the individual students to decide which one of these three levels to follow. The students could choose different levels from one work plan period to another. The three levels would usually cover the same mathematical themes, but were differentiated either by the amount of tasks connected to each level, by task difficulty, or by a combination of these two criteria. Common to all the schools was the practice of allocating time for the students’ handling of their work plans.

Observation of student behaviour during math lessons, especially the study/guidance-lessons, revealed that different strategies were being used in relation to the handling of the assignments on the periodical work plan. This was to a large extent confirmed in the interviews; through the students’ own explanations of how they strategically positioned themselves in the handling of this plan. Basically these strategies seemed to fall into three categories:

1. To postpone the work in mathematics to the end of the work plan period.
2. To finish the work in mathematics in one or two days at the beginning of the work plan period.
3. To apportion the work in mathematics throughout the work plan period.

Students’ reasoning for choosing these strategies varied quite a bit.

The first position is characterized by students who try to postpone the work until the very end of the work plan period. Especially at two of the schools this was a strategic positioning that the majority of the boys seemed to embrace. At these schools we observed that, while the majority of the girls were able to apportion their work and disperse it throughout the whole work plan period, nearly all the boys waited until the end of the last week to put any effort into completing their assignments.
The second strategic positioning involved students completing their math-assignments for the whole work plan period in just one or two days at the beginning of the period. The students presented two reasons for the choice of this strategy. The first one was connected to the pronounced wish of finishing the math assignments as fast as possible, because it was boring to work on. The second reason was that since mathematics was their favourite subject, they just couldn’t wait to work on the new assignments. Even if these stated reasons differed quite a lot, the consequences of both were quite similar; all the students in this group would finish their math assignments in just a couple of days at the beginning of the work plan period.

The third strategic positioning that seemed to attract certain students was to disperse the work throughout the period. Most of the students that consciously chose this strategy, and could account for it in the interviews, were high achievers. They were quite articulate in arguing that this was the best way of securing high grades. Many of them also had quite high ambitions for their future careers.

**Summary**

A central characteristic of the LPS study is the documentation of the teaching of sequences of lessons, rather than just single lessons. Using this research design, we have been able to document that the use of work plans in mathematics gives the students the opportunity to choose strategies that mean they will only work with mathematics one or two days during a work plan period of two/three weeks. For these students the consequences seem to be that there is little continual work in mathematics, it is all about completing a certain number of tasks. This is usually not regarded as an optimal way of working with mathematics. On the contrary, several theories of learning and instruction emphasize regular, step-by-step learning opportunities.

The results of our study can be used as a basis for discussion of the practice of using work plans as an organisational and didactical tool in mathematics classrooms and to help us to understand discrepancies between student and teacher perspectives.
Creative activity accompanied by high positive affect (‘flow’) can occur when people spontaneously set challenges and develop new skills in order to overcome them (Csikszentmihalyi, 1992). Creative thinking during flow specific to mathematical problem solving can occur when students discover a mathematical complexity of which they were previously unaware, and decide to explore it. Flow conditions include spontaneously setting an intellectually challenging question about the mathematical complexity, and exploring this question using non-routine mathematics (Williams, 2005). The thinking framework to study cognitive activity (Williams, 2005) was formulated by integrating aspects of thought processes identified by others (Krutetskii, 1976; Dreyfus, Hershkowitz, & Schwarz, 2001). The construct of student spontaneity was elaborated by subcategorising social elements of the abstracting process (Dreyfus, Hershkowitz, & Schwarz, 2001) into those from internal and external sources (Williams, 2005). Undertaking such activity involves moving from what is known to what is unknown, and there can be many failures before success is achieved. Some students are not inclined to undertake such activity (Seligman, 1995).

Flow activity during mathematical problem solving is illustrated through the activity of a Year 8 student, Eden (Williams, 2007), who found he could not position linear functions to ‘shoot’ ‘globs’ on a Cartesian plane in a computer game. Eden was not aware of connections between algebraic forms of linear functions and their positions as graphs. He observed and reflected on a dynamic visual display as it was generated by another student. This display showed a family of parallel lines appearing one after another on the screen as the student undertook a trial and error process to try to hit globs. Eden identified a pattern between the x and y values of co-ordinates of each point on the same line. He thought he saw a link between this pattern and the algebraic equation at the bottom of the screen. He returned to his own computer, and experimented. After seven minutes of intense activity, he left his computer screen and exclaimed softly to himself. He had confirmed that his patterns were expressed by the algebraic equation. He had developed new conceptual understanding that linked algebraic, numerical, verbal, and graphical expressions for linear functions (conceptual knowledge novel to Eden). Eden recognized patterns, and built with them by expressing them verbally and algebraically, and then experimenting to see whether there was always a link between the pattern and the equation. He synthesised to gain insight when he realised that all of the information he had found about the graph could be ‘held’ in the linear equation and ‘unpacked’ as needed.
In his interview, Eden described how he problem solved in mathematics. He perceived failure to understand as temporary and able to be overcome with effort.

You just have got to sort of think out the answers in your head (pause) occasionally you have gotta- got to write down on paper what you are thinking about (pause) and eventually get the answer (Williams, 2006b, p. 397).

My insights into Eden’s creative thinking relied upon analysis of the teacher video to find what the students were told at the start of the lesson, the focus student video to find whether other students provided mathematical input to Eden’s exploration, the whole class camera to determine that Eden was not interacting with others during his seven minutes of exploration, and all three videos in the next lesson to make sure others had not contributed to Eden’s understanding prior to his interview after that lesson. The video-stimulated interview assisted Eden to remember detail, and communicate his thoughts in class. This interview drew attention to the parts of the lesson when Eden had developed new knowledge, and provided indicators of his inclination to explore. The LPS data collection processes were crucial to this study.
One of the important questions currently discussed in the LPS community is “What are the related existing mathematics education research methodologies that can serve as the theoretical framework for drawing new results from the LPS resources?” In this contribution, the relationship between the LPS and the Theory of Didactical Situations in Mathematics (Brousseau, 1997) is presented.

Why just this theory? Laborde and Perrin-Glorian (2005, p. 2) state that “...classroom is the place of social interrelations between the teacher and students shaped by the difference of position of the two kinds of actors with respect to knowledge and giving rise to sociomathematical norms (Yackel & Cobb, 1996) or to a didactical contract” (Brousseau, 1997). Our analyses of sets of videotaped lessons (Binterová, Hošpesová, & Novotná, 2006; Novotná & Hošpesová, 2007; Novotná & Hošpesová, 2008) are based on the theory of didactical contract. The implicit nature of Brousseau’s concept of “didactical contract” is fundamental when explaining environment effects on learning mathematics (Sarrazy & Novotná, 2005).

Elements from the Theory of Didactical Situations in Mathematics (TDSM)

In TDSM, a learning process is characterized as a sequence of identifiable situations (natural or didactical), reproducible and leading regularly to the modification of a set of behaviours of the students, modifications that are characteristic of the acquisition of a particular collection of knowledge (Brousseau, 1975). Brousseau considers the conditions of a particular use of a piece of mathematical knowledge to form a system, which he calls a "didactical situation". (In non-didactical situations, the evolution of the learner is not submitted to any didactical intervention whatever.) Didactical situations which are partially liberated from direct teacher’s interventions are called a-didactical situations. In TDSM, situations are classified according to their structure (action, formulation, validation, institutionalization, etc.), which determines different types of knowledge (implicit models, languages, theorems, etc.).

The process by which the teacher manages a didactical situation by putting the learner in the position of a simple actor in an a-didactical situation is called devolution. Devolution does not only propose a situation to the learner which should provoke him/her to an activity not previously agreed, but also makes him feel responsible for obtaining a proposed result, and that the solution depends only on the use of knowledge which he/she already has.

Environmental effects on learning mathematics are explained using didactical contract, i.e. the set of the teacher’s behaviours (specific to the taught knowledge) expected by the student and the set of the student’s behaviour expected by the
teacher. This contract is not a real contract; in fact it has never been “contracted” either explicitly or implicitly between the teacher and students and its regulation and criteria of satisfaction can never be really expressed precisely by either of them. One of the fundamental teacher’s tasks in a didactical situation is institutionalisation, i.e. the passage of a piece of knowledge from its role as a means of resolving a problem or proof to a new role that of reference for future personal or collective uses.

**TDSM and LPS**

A significant distinguishing characteristic of LPS is its documentation of the teaching of sequences of lessons, rather than just single lessons. The main goal is to produce empirical analysis of pedagogical phenomena based on well recorded “reality”. In the following overview, the topics and related clusters of questions in the framework of TDSM that we tried to answer are summarized:

Didactical contract (Binterová, Hošpesová, & Novotná, 2006; Novotná & Hošpesová, 2007): Can we trace hidden didactical contract established in a particular classroom and illustrate it by suitable teaching episodes? What influence of the didactical contract on students' mathematical knowledge can be presupposed? How does it support or constrain learning? How does the teacher create a secure, confident work environment for the students in the classroom?

Topaze effect (Novotná & Hošpesová, 2007) How does Topaze effect reflect teacher’s beliefs? How does Topaze effect influence students’ work? What types of Topaze effect we can find in Czech lessons?

Linking in mathematics lessons (Novotná & Hošpesová, 2008): What are the prerequisites that the teacher refers to when solving new problems, developing new domains of school mathematics (these teachers’ actions are called “linking”)? What types of linking are used by teachers and how does their use influence students’ behaviour in the classroom and their understanding of mathematics?

**Endnote**

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THE RESULTS OF INTERNATIONAL CLASSROOM RESEARCH: FINDING STRUCTURE IN DIVERSITY
Similarities in students’ perspectives, classroom discourse and lesson elements

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Students’ motivations and the meanings they attribute to classroom activities

How students view their learning environment and why they choose to (or not to) participate in classroom activities has an impact on knowledge development. For the analysis, post-lesson interviews with 109 students referring to 60 lessons in LPS classrooms from Germany, Hong Kong and the U.S.A. were used. The findings have been compiled with a focus on similarities (Jablonka, 2005). The similarities found reflect how the students attribute meaning to distinct aspects of the classroom practices, which they regard as constitutive for their learning. The students’ motives were linked to the expectations they held and corresponded to the possibilities the classroom practice offered. Classroom practices obviously shape behavior and thought and can be taken as the premises on which the students’ (and the teachers’) ways of trying to succeed are based.

In total, 52 students talked about passing tests and examinations or about their grades in the interviews. A total of 13 students made statements in which they associated mathematical activities with “thinking”. (HK1: 8, HK3: 3; US1: 1, G3: 1). Some students referred to acquiring knowledge for everyday and professional practices. Everyday practices comprise managing a bank account, shopping, dealing with rents, salaries, fees, taxes, or buying and renovating a house, computer use, uncovering cheats. However, these examples do not refer to the topics of the lessons videotaped. Much more students from US2 than from the five other classrooms (that is 11 compared to 1-3) make reference to understanding why, finding patterns and establishing connections. This is entirely in accordance with the teacher’s goals. A total of 35 students from all six classrooms employed the metaphor of carrying out steps, ‘doing things’ and obtaining results.

Mathematical reasoning

Instances of a mathematical reasoning discourse have been identified on the basis of the lesson transcripts (Jablonka, 2004). It turns out that these were rare in all classrooms. This is not to say that interactive involvement of students was infrequent. Much of the interaction in which the teachers addressed the whole class involved the students interactively by posing questions. The questions often were a request to provide a reason. If counted as single events, the teacher asked the students to provide reasons many times in each lesson in US1. Similarly, in G1 the teacher asked the students to provide reasons a couple of times in most of the lessons. In US2 this happened only occasionally in half of the lessons. It is even less frequent in the Hong
Kong classrooms and in G3. Self-initiated reasoning on the part of the students in public talk was rare in all the classrooms except G3.

**Lesson structure**

In all the six classrooms teacher and students were frequently engaged in a pattern of interaction that was labeled ‘questioning-developing classroom talk’. This is a mode of whole class interaction in which the teacher interactively involves the class by asking a series of connected questions. The definition employed was intended to include a variety of forms (distinguished in terms of degrees of openness and closeness) and functions (setting a new task, review, application or developing new content). When the teacher’s questions are connected and aim at collectively developing new knowledge, these episodes can be interpreted as a form of a reasoning process. The teacher and the students collectively provide a chain of (minor) premises (‘reasons’) and implicit inferences, though the discourse contains at most a few utterances that can be interpreted as a request for, or a provision of, a reason. The teacher pre-structures the discourse by breaking down a chain of inferences into a series of questions, which - if answered accordingly - in sum warrant the resulting conclusion, that is the solution of the task. It can be argued that this form of interaction implies a systematic transformation of mathematical reasoning as found in the context of knowledge production, that is, the mathematician’s mathematics or ‘inquiry mathematics’. This transformation is linked to other features of classroom practice that are considered by the students as important for their learning.
The benefits of video data are obvious (in terms of the tool's ability to freeze, recapture and visualize learning situations). The analytical challenges and efforts this poses for the researchers are equally obvious. There is an obvious risk to get lost in the details in video data. In our analyses of the LPS data (LPSoslo) we have used coding categories and time scales as analytical tools for getting an overview of the material. In addition individual researcher(s) have made more specified and focused analyses based on delimited and fixed research interests.

**Coding categories/ Coding schemes as complexity reduction strategies**

We used coding categories/coding schemes operating on two levels as analytical devices for complexity reduction. The levels differ in focus. The first level (level 1) aims at capturing some central features of how the teachers design the classrooms as learning sites (in terms of instructional format, grouping arrangement, subject matter involved etc.) while the second level of analyses (level 2) concentrates on content driven teaching activities and interaction patterns across the same sites and classrooms (such as language features, dialogue initiatives and teachers' use of different didactic tools, such as reviewing, summarizing, developing new content knowledge, 'going over the do now' etc).

**Scales as analytical devices**

Levels of time scales are equally another important factor that impregnates the conclusions that can be drawn from our analytical endeavors. Interpreting meanings involved in the classrooms observed is closely linked to time scales of interpretations. For these interpretations the researcher, for example, analyzes short time interactions in order to then situate them within longer time segmentations such as episodes and themes, which themselves subsequently could be situated in larger scales like sessions, the whole teaching sequence etc. Lemke (1990) makes a distinction between macro, meso and micro analyses as three levels of time scale for analyzing classrooms activities. In our analyses, we have used meso level (teaching sessions) and micro level (teaching segments) as two units of analyses. By using a combinations of meso and micro analyses, different time scales open for multiple interpretations and findings.

**Findings**

In all classrooms, students were seated in groups or pairs. The instructional format was organised around teacher-led whole class instruction and individual seat work. Group activities were rare in the observed classrooms, and the students were given
few opportunities to discuss, analyse and talk mathematics with their peers, despite physical arrangements that could fuel such collaboration.

Teacher-led whole class instruction and individual seat work were the two most frequently used instructional activities in all observed math classrooms. The content-focused analyses of teacher activities (level 2) supported and elaborated on these findings and revealed “developing new canonical knowledge” and “offering seat work” as the two most recurrent teacher content driven activities.

There was ample room for student initiative in the mathematics lessons we observed. Despite a prevalence of teacher-led whole class instruction, students were given abundant space for initiatives and questions in the observed classrooms and, consequently, teacher-led whole class instruction did not equate to teacher monologues and recitation patterns.

**Looking across categories and scales - Findings taken together**

- The teacher repertoire was rather narrow, dominated by the two codes “developing new canonical knowledge” and “offering seat work”. This made the mathematics lessons quite monotonous.
- Students frequently initiated classroom discourse in the mathematics classroom. Across the six schools the percentage of classroom discourse that was student initiated varied between 35 and 65, which indicates that students' active participation was granted a significant space in terms of speech initiative and turn-taking. Compared with former studies addressing the issue of participation in classroom discourse, our studies indicate a significant change in classroom practice, from teacher dominated instruction (i.e. teacher monologues) to teacher-led discourses characterized by vast opportunities for student initiative.
AN ETHNOMETHODOLOGICAL ANALYSIS OF TEACHER’S STRATEGY FOR MANAGING LEARNERS’ DIFFERENT IDEAS

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INTRODUCTION
Recent research has a common and persuasive vision of mathematics classrooms as a discursive practice. This ethnomethodological study investigated how a Japanese mathematics teacher (J2) used strategy for managing learner’s different ideas.

Ethnomethodology investigates members’ accounting practices to attain the factual character of the social reality. The facticity of the sense is maintained by interpretive work. However, expressions are vague and equivocal, lending themselves to several meanings. The sense of these expressions is a product of the very way we look at something and talk about it. There is often a competition over the correct, appropriate or performed way of representing objects, events, or people. Proponents of various positions in conflicts waged in and through discourse attempt to capture or dominate modes of representation. The competition over the meaning of ambiguous events, people, and objects in the world has been called the “politics of representation” (Mehan, 1993: 241). It is tenable that a similar competition over the meaning of events and objects is played out in mathematics classroom discourse.

UNIT OF ANALYSIS
If the social formation of mathematical practice in the classroom is to be analyzed, social and mathematical dimensions as a whole have to be taken into account (Ohtani, 2000). Thus, we need a “unit of analysis” (Vygotskii, 1982). In his Proofs and Refutations, Lakatos (1976) portrays historical debates within mathematics about what a proof of a theorem represents by constructing a argumentation among a group of students that contains mixed within it many conceptual horizons among mathematicians through the last several centuries. In the midst of an argumentation, revised definitions and conditions are progressively introduced in light of refutations. It seems that formation and revision of definitions and introducing conditions are indispensable and essential components that constitute mathematical discourse.

RESULTS AND FINDINGS
My analysis of the Japanese (J2) data corpus found that mathematical definition and condition operate in classroom interaction as social and multi-consequential devices to coordinate and sustain interaction. Telling definitions and introducing conditions function as social resources widely used in order to negotiate certain representation of a problem situation rather than as cognitive resources used to analyze and describe the problem situation and to construct mathematical dialogue. Definitions and conditions are characterized by a monologic suppression by an authoritarian voice,
rather than by a dialogicality of voices. It is as if an invisible barrier has been placed around the topical space that is eligible for discussion. The formulation to which the student has privileged access is not motivated by the needs of an instructional activity. The condition functioned to regulate the student’s mathematical activity in ways that were appropriate for the classroom setting. The condition was characterized as directive. And by introducing the condition, the student was constrained to engage in a process sanctioned and regulated by the teacher.

CONCLUDING REMARKS

In sum, telling mathematical definitions and conditions involves the following social functions: to sanction and defend unexpected or insignificant interactions with students; as a means to defeat students’ ideas and proposals; to justify the teacher’s control over students; to show that the student's proposal is unrealistic; to terminate a student's request and to attain a degree of uniformity of what it transmits. Thus, in place of diversity or heterogeneity, the act of telling mathematical definitions is designed to get the student to participate in formulating the problem in particular way. It serves as a method of managing the teacher’s asymmetrical relationships with his students; and it tempers the teacher's obligation to be knowledgeable about the affairs of students.
I have made some contribution to math teacher education in China through sharing LPS methodologies and findings, conducting in-service and pre-service teacher education programs. In this presentation, I would like share my personal experience in the following aspects: (1) disseminating LPS methodology and findings; (2) comparing and contrasting teaching strategies in pre-service teacher education program; (3) demonstrating effective teaching in school-based teaching research projects.(4) complementing teaching ideas in summer courses; (5) sharing Chinese mathematics instruction internationally. I give a selection of examples below.

Comparing and Contrasting Teaching Strategies in Pre-Service Teacher Education Program

In my secondary prospective teacher education program, I use LPS methods and materials to compare the characteristics of mathematics classroom instruction in different cultures. On this basis, we encourage students to figures out what effective mathematics teaching looks like in China.

Shaping School-Based Teaching Research Project

Heavily influenced by LPS, I have participated in and led some school-based in-service teacher professional development.

Since 2003, I have participated in a national wide in-service professional development, called as Xingdong Jiaoyu (a teacher education programme using action research methods), in Shanghai, China. The LPS methodology has directly influenced the project (Huang & Bao, 2006). Later, the project was developed into Hypermedia Video Case Study (VCS) and popularized around China (Bao & Huang, 2007).

During 2006-2007, I led a school-based teacher professional development project in Macau. The aim of the project was to pursue effective teaching through reflecting and improving classroom instruction practice. This project included the following phases: Theory learning, classroom observation, field investigation, exemplary lesson study, and experience sharing and reflection. As a product of this project, we developed a multiple media Video Case. The LPS study has influenced this project in two ways: First, through showing some video clips from LPS and sharing some research findings, it helps teachers to reflect on characteristics of effective teaching; Secondly, the method of conducting the project was influenced by LPS methodology.
Table 1. Teacher’s perception of importance of relevant aspects

<table>
<thead>
<tr>
<th>Items</th>
<th>Mean</th>
</tr>
</thead>
<tbody>
<tr>
<td>Changing perspective on teaching and learning</td>
<td>3.4</td>
</tr>
<tr>
<td>Enhancing the understanding of mathematics content and pedagogical methods.</td>
<td>3.5</td>
</tr>
<tr>
<td>Fostering teaching design ability</td>
<td>3.4</td>
</tr>
<tr>
<td>Advancing classroom instruction skill</td>
<td>3.4</td>
</tr>
<tr>
<td>Enhancing awareness of reflection and improvement</td>
<td>3.2</td>
</tr>
<tr>
<td>Enhancing analyzing classroom from multiple perspectives.</td>
<td>3.0</td>
</tr>
<tr>
<td>Enhancing cooperation and exchanges among colleagues</td>
<td>3.1</td>
</tr>
<tr>
<td>Overall, enhancing mathematics professional development</td>
<td>3.3</td>
</tr>
</tbody>
</table>

Once completing this project, we conducted a survey on the effectiveness of the project. We asked participating teachers (17 teachers in one school) to rank the importance of the project for teacher’ professional development as not important, important, and very important. A numerical value (2, 3, 4) was then assigned in our data analysis for each scale, respectively. The means of the teachers’ responses are shown in Table 1. From participating teachers’ perspectives, this project has quite a positive impact on their professional development.

**Concluding Remark**

As described above, it is exciting and effective to adopt new ideas and findings from international research to inform normal teaching and teacher professional development. Thus, broad and updated views will benefit local teachers’ professional development. On the other hand, when we examine local classroom instruction and teacher professional development, we also identity some Chinese features to share with international audiences (Huang & Bao, 2006; Huang & Leung, 2004; Huang, Mok, & Leung, 2006).
The methodology of LPS can be successfully employed in cooperation with in-service teachers and in teacher training. The quality of students’ statements is to a great extent influenced by the student’s trust in the experimenter and his/her willingness to reflect on the preceding lesson and talk about it. Our experience with students’ post-lesson interviews from the Czech Republic has not been very favorable. What could be observed was certain students’ unwillingness to occupy their mind with an already-ended lesson they had already “successfully survived”, and perhaps also the novelty and even their fear of critical consideration of the teacher. Students seemed to regard themselves as the passive objects of pedagogical activity and did not appear to feel responsible for their own results.

Interviews with teachers were carried out simultaneously. What makes these interviews even more valuable is the fact that they are based on watching a video recording of the analysed lesson. This offers the researcher an uncommon insight into the teacher’s self-reflection. It can be presupposed that self-reflection is, on the intuitive level, present in all human activities. However, qualified pedagogical reflection is different; it considers description and analysis of key elements, evaluation or revaluation, ways of explanation, accepting decisions and determining a new strategy (Tichá & Hošpesová, 2006). In this sense, Jaworski (2003) speaks of reflection-in-action, reflection-on-action, and reflection-for-action. Reflection carried out within LPS naturally belongs to the second type of reflection. Nevertheless the use of video recordings brings in some features of the remaining two categories.

The data gained by the LPS methodology can be subjected to the methods of qualitative research; their structure is close to LPS methods comprising “direct” observation, in-depth interviews, and analysis of documents and materials. This data structure enables us to perceive classroom reality as socially constructed, to study its various complex variables that can otherwise be measured only with great difficulty, without at the same time neglecting the perspective of the subject matter. Our goal was in-depth understanding of the teachers’ and students’ behaviour and the motives that guide it.

Within our investigation, we compared a video recording of a lesson with its perception by the teacher and the students expressed in post-lesson interviews. Let us illustrate this idea with an episode from the Czech data (grade 8, topic: parametric equations, this episode has already been discussed from a different point of view in Binterová, Hošpesová, & Novotná, 2006). For homework, the students were asked to
solve the equation \( dx + 1 = 2(4x + 1) - 5x \) by substituting for \( d \) the day of their birth. The teacher expected that some pupils would substitute 3. Then the equation would have no solution. In the lesson, she tried to show why this had happened and she together with her students adjusted the equation to the form \( x(d - 3) = 1 \) (CZ1-L05, 00:16:42-00:17:56). In the post-lesson interviews with the pupils, the experimenter asked the student and the teacher how they perceived this episode.

(CZ1-L05, interview with Michal, 00:01:03):

Exp. 1: Would you like to tell me anything about the homework?
Michal 1: ... First of all, I didn’t know why the teacher had assigned it to us, but when we went through it together, I knew the result \( \frac{1}{3-d} \). We tried it before the lesson and one boy – Adam – showed it to me.

Exp.2: You knew why you had been given that homework?
Michal 2: I knew.
Exp 3: And you knew the explanation?
Michal 3: He (Adam) didn’t tell us. We knew the solution, but we couldn’t explain it.

(CZ1-L05 post-lesson interview with the teacher, 00:11:21):

T 1: Well, it’s certainly not easy. My feeling is that they didn’t quite cope with it. Every equation will be an exception. I will have to go over it again.
Exp 2: Those who substituted 3 got the result. They will have understood.
T 2: It seems they discussed it before the lesson.

Michal’s statement shows that he did not find the homework difficult. On the other hand, the teacher in her utterance (T1) seems to be expressing the belief that she is the one who is responsible for “passing knowledge over to her students” and seems to perceive any students’ difficulties as her own failure. This conclusion has not been drawn only from our observation of this episode. We have come to this conclusion after having analysed a sequence of lessons – which is another advantage of LPS methodology.

The LPS video materials are not only used in research. Selected interesting episodes are processed for joint-reflection in courses of in-service teacher training. The success of joint-reflection largely depends on the selection of the teaching episode. Discussion is best provoked by a short episode which involves a particular problem. When the teachers watch the episode together and jointly reflect upon it, they confront their ideas, which often results in changes in their beliefs. It is often very useful if the discussion is followed by a recording of the reflection of the teacher who had given the lesson.

Endnote

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