In the paper, Shimizu’s notion of explicit linking is used as the starting point for an analysis and classification of teachers’ repertoire of prerequisites that they employ to connect students’ previous knowledge to the didactical situation of linear equations. The method of data collection is based on the Learner’s Perspective Study framework. To illustrate the theoretical classification, the video recordings of two Czech classes are referred to. A comparison of the two different approaches of Czech teachers is presented. The findings are connected to Brousseau’s Theory of Didactical Situations in Mathematics.

INTRODUCTION

To learn why some mathematics teachers are more successful in their effort to create suitable climate for their students’ learning and understanding is still one of the most intriguing themes in research in the field of didactic of mathematics. Our understanding of teaching as relational (in agreement with Franke, Kazemi, Battey, 2007) is the reason why we have paid attention to discourse in mathematics classroom for a long time (Novotná, Hošpesová, 2007). It is through analyses of classroom discourse that we try to find out how teachers support students’ involvement and development of their knowledge.

Shimizu (1999) mentions Stigler’s study (Stigler et al., 1999, p. 117, in Shimizu 1999) on importance of explicit linking of knowledge across lessons and within a single lesson. In the study, linking is defined as an explicit verbal teacher’s reference to ideas or events from another lesson or part of the lesson. The reference is taken as linking to single events (i.e., referring to a particular time, not to some general idea) and the reference must be related to the current activity. Later Shimizu (2007) specifies the idea on the interrelations “in the way that mathematical ideas in the current lesson are connected to students’ experience in the previous or forthcoming lessons as well as part of the same lesson” (p. 177). It covers both, “looking back” as well as “looking ahead”.

In this paper, we focus on one aspect of Shimizu’s point of view, the connections to students’ previous experience – “looking back”: What are the prerequisites that the teacher refers to when solving new problems, developing new domains of school mathematics? In agreement with Shimizu (2007), we call these teachers’ actions “linking”. We consider both, linking within a single lesson and across lessons. Compared to Shimizu (2007), we include into linking also referring back to “general” ideas (see the example of multiple linking later). Our considerations are restricted to the domain of linear equations in the eighth grade in the Czech Republic.

In our study we come out from the belief that teachers have a certain repertoire of prerequisites (“pieces” of knowledge) that they see as unavoidable in the new didactical situation and that they have a pattern for recalling them.
We formulated our goals into the following questions:

- How can be linking uncovered in Czech lessons classified?
- What are the positive and negative aspects of frequent linking?
- What is the role of didactical contract (Brousseau, 1987) in developing links to students’ previous knowledge?

**DATA COLLECTION**

In the research reported we analyse linking in video recordings of ten consecutive lessons on the solution of linear equations and their systems in the 8th grade (students aged 14-15) of two lower secondary schools in the Czech Republic – in the following text, they will be labelled CZ1 and CZ2. Both schools are located in a county town with approximately 100,000 inhabitants. The method of data collection is based on the Learner’s Perspective Study (LPS) framework (Clarke, Keitel, Shimizu, 2006). We compare our findings with our long-time experience from observations of lessons in different research projects.

Both teachers are experienced and respected by parents, colleagues and educators, although their teaching strategies differ: CZ2 teacher mostly concentrates on the question “How?” She develops problem solving strategies in the perspective of each problem. CZ1 teacher pays more attention to the question “Why?” She tries to build the new knowledge on her students’ previous knowledge. On the other hand, CZ2 teacher trusts much more in her students’ independent discoveries.

**FORMS OF LINKING**

During the analysis of the lesson we detected two basic categories of linking characterized by the teacher’s prior intention to make linking (we call it a priori linking, shortly APL) or not to make linking (ad hoc linking, AHL). Our differentiation does not claim to be exhaustive, to cover all forms used by mathematics teachers in all educational settings. Nevertheless, our long time experience from our observations of many mathematics lessons in several countries lead us to the conviction that the main forms of linking are covered in the following text.

When presenting our classification of linking, we do not claim to give a hierarchy of their usefulness and influence on the level of students’ understanding of mathematics.

**a) A priori linking (APL)**

The teacher, using his/her experience from teaching the particular topic, plans to recall in advance the necessary knowledge that should be already known to his/her students. Bearing in mind the prerequisites for successful learning of the piece of knowledge, he/she recalls the part of mathematics that the students have already met. The newly introduced content is placed in the network of existing knowledge and skills with the aim to promote understanding.

APL may have the form of a “theoretical discourse” when the teacher mentions the knowledge from one of the previous lessons or can be presented as a solving procedure for a suitable problem:
And we will need one more thing today or in the next lesson. The difference of sets. We have already seen it. Do you remember? How do we draw it? ...

APL can be of two forms, linking across lessons (APL-Across) or linking within a single lesson (APL-Within). In both cases, APL might be used by the teacher before or inside a solving procedure of a problem or a proof (we will shortly speak about an “activity”).

When the teacher uses the “before activity” type of APL, the students often do not see the immediate use of the recalled piece(s) of knowledge. It is known to the teacher, not to the students (see also paradoxes of didactical contract in the TDSM, Brousseau, 1997). They are supposed to apply it at the appropriate moment later on.

The “before activity” type of APL is often used by the teacher as a reaction to a students’ common mistake. The teacher presents the linked knowledge related to a mistake before correcting it or before solving a similar problem. (For example CZ1-L04 starts with the correction of the common mistake in students’ homework.)

The teacher often uses the “inside activity” type of APL in the course of the work on a new piece of mathematics. Usually it is the case when the teacher expects the students not to remember an algorithmic step or to make a common mistake and plans to use it only in case that the necessity occurs.

Examples of the “inside activity” type of APL-Across:

1 CZ1-L02, 18:37 (linking to the previous lesson when solving the first equation with an unknown in the denominator, with infinitely many roots)

T: And yesterday we said that we could do this step if, if …

Student: If I do not multiply by zero.

1 CZ1-L02 means the second lesson in CZ1; 7:36 refers to the moment in the lesson when the dialogue/discussion started; similarly in the other excerpts. The excerpts are translated from authentic classroom discourse including all inaccuracies and clumsy expressions.

2 If the reference occurs after the activity, we see it as a form of institutionalisation (Brousseau, 1997), (Brousseau, Sarrazy, 2002, p. 6), not as a linking.

The “inside activity” type of APL can be successfully applied as motivation for a more complicated problem. For example in CZ1-L5 (10:40), students’ results when solving an equation with a parameter for various values of the parameter are used as motivation for solving the equation with the “general” parameter.
A special case of APL is linking a priori to the previous lesson (usually during a series of lessons developing the same topic). The teacher reminds the students what they were supposed to learn from the activities of the last lesson. The most frequent reason is the teacher’s belief that the students need more than one lesson to grasp the new piece of knowledge including the desired terminology.

*CZ1-L02, 43:10 (“before activity” linked to CZ1-L02, 15:35, see above)*

T: Let’s return to the equation where there were infinitely many solutions. If I meet a condition that I have to record, don’t forget about it in the end. ... In the problem that we went through together it was important to include the condition in the conclusion.

*CZ1-L02, 31:00 (“inside activity” before solving the second equation with the unknown in the denominator, the root is 5, the domain of the equation contains all real numbers different from 3)*

T: One more thing. Look here, you found that the root of this equation is 5, so what about it? We got around the condition and it’s not necessary to take it into account any more. But in this case (she points to the first equation) that we went through together when we got ...

CZ1 teacher often starts a new topic/procedure by “multiple linking” – linking to several items at the same time (e.g. *CZ1-L02, 15:30*).

T: We already know that it’s not always so straightforward – the solving of an equation. What has always happened so far?

Student: We isolated the unknown.

T: We isolated the unknown. We calculated always the one, the number, the root of the equation. It might also turn out differently ... Vítek, how?

Vítek: That it does not have any solution.

T: How can I recognize that the equation does not have any solution, Denisa?

Denisa: The left-hand side does not equal the right-hand side.

T: Yes, there will be a contradiction. For example what, Vasek?

Vasek: For example 2 equals 12.

b) *Ad hoc linking (AHL)*

By *ad hoc linking* we understand the linking integrated by the teacher as a reaction to what the students do or say when solving a new problem or when presented a new mathematical topic. Mostly it is a reaction to an incorrect answer or a step of the solving procedure requiring the use of knowledge that should already be known. It is the teacher’s reaction to the immediate situation in the classroom. The success of AHL depends on the teacher’s experience and pedagogical skills.

AHL can also occur as the teacher’s reaction to a student’s/students’ questions e.g. in case of their individual solving of problems. For example in CZ1-L10 (13:30), the AHL-Across to the properties of a tetrahedron is provoked by the context of the word problem chosen by one of the students for his solving.

AHL can again be of two forms, *linking across lessons (AHL-Across)* or *linking within a single lesson (AHL-Within)*. It is used by the teacher inside an activity. A
special case of AHL-Within is the individual help of the teacher during individual work in the lesson.³

a) AHL-Across CZ2-L03, 27:25 (solving the equation \(3(2 - x) - 4 = 1 - 2(x - 2)\))

T: Let’s start the solving process. Together, together, Lucka. If there are brackets in the equation, let’s try to get rid of them. As, apart from brackets, there are also other expressions there, we have to remove them by multiplying out the brackets. And we already know how to multiply a binomial by a number. Attention when multiplying by minus! …

b) AHL-Within CZ1-L02, 37:20 (creation of the equation for the “speed/distance” word problem, \(x = \text{time}\))

Student: \(x + x + 1\)

T: How do we calculate the distance, Michal? It was said a few minutes ago.

Michal: Speed times time.

THE TEACHING STYLES AND LINKING

Let us start by comparing incidence of linking in schools CZ1 and CZ2. This will later serve as the basis for comparing the teaching styles of the two teachers. Comments regarding “real life situations” done by the teacher are not included; the table is restricted to linking to school subject content.

Differences between the lessons

<table>
<thead>
<tr>
<th>Lesson</th>
<th>CZ1</th>
<th>APL/AHL</th>
<th>CZ2</th>
<th>APL/AHL</th>
</tr>
</thead>
<tbody>
<tr>
<td>L1</td>
<td>Presentation of new subject matter</td>
<td>8/2</td>
<td>Presentation of new subject matter through problems</td>
<td>5/3</td>
</tr>
<tr>
<td>L2</td>
<td>Presentation of new subject matter</td>
<td>13/4</td>
<td>Presentation of new subject matter through problems</td>
<td>5/2</td>
</tr>
<tr>
<td>L3</td>
<td>Practice and application</td>
<td>5/6</td>
<td>Practice and application</td>
<td>1/1</td>
</tr>
<tr>
<td>L4</td>
<td>Practice and application, students’ individual work</td>
<td>2/8</td>
<td>Practising prerequisites, development of the new subject matter through problem solving</td>
<td>3/7</td>
</tr>
<tr>
<td>L5</td>
<td>Correction of individual work, new subject matter</td>
<td>3/3</td>
<td>Development of the new subject matter through problem solving</td>
<td>3/3</td>
</tr>
<tr>
<td>L6</td>
<td>Practice and application, new subject matter</td>
<td>1/2</td>
<td>Test, practice</td>
<td>0/0</td>
</tr>
<tr>
<td>L7</td>
<td>Practice and application, students’ individual work, new subject matter</td>
<td>5/2</td>
<td>Word problems presented through problem types</td>
<td>6/0</td>
</tr>
<tr>
<td>L8</td>
<td>Correction of individual work, new subject matter</td>
<td>11/2</td>
<td>Special types of word problems presented through their solving</td>
<td>1/0</td>
</tr>
<tr>
<td>L9</td>
<td>Comprehension check, written test</td>
<td>1/2</td>
<td>Special types of word problems presented through their solving</td>
<td>4/1</td>
</tr>
<tr>
<td>L10</td>
<td>Correction of the test, summary of the learned subject matter</td>
<td>4/3</td>
<td>Special types of word problems presented through their solving</td>
<td>0/3</td>
</tr>
</tbody>
</table>

Table 1. Lesson orientation and number of APL/AHL

³AHL is often introduced similarly to the “inside activity” type of APL. The type can be recognized only when looking at it in the context of the students’ reactions and action or during the post-lesson interview with the teacher.
Differences observed in the two classes can be explained as the consequence of the lesson orientation. Tab. 1 indicates that in both classes, the number of APL decreases and AHL increases when the lesson focuses on practice.

**Differences between the classes**

There is a significant difference between the number and types of linking used in the two classes – see graph in Figure 1.

In order to create a consolidated net of knowledge, CZ1 teacher roots the new subject matter in the theory that her students are supposed to be familiar with. In most cases, she inserts it before solving problems, sometimes even far before the activity where it is really needed (e.g. in L2, 3:20, number sets are recalled but not used until L2, 22:10 and later). It is obvious from the students’ reactions that this type of linking became one part of the didactical contract. The number of APL applied inside the solving procedure of a problem is considerably lower. Earlier we (Novotná, Hošpesová, 2007) suggested that the reason for this teacher’s behaviour might be her distrust in students’ abilities resulting in students’ lack of self-confidence.

CZ2 teacher prefers the re-discovery of necessary facts and procedures through problem solving to linking across or within lessons. She believes that her students’ success in mathematics can be reached by their successful completion of assigned tasks. On the contrary, AHL occurs much more often. It is not only the reaction to her students’ mistake but often the teacher’s immediate reaction to her students’ performance and her attempt to prevent the occurrence of too many mistakes. The teacher’s experience plays an important role in her decision to use a certain linking type.

![Figure 1. Graph comparing CZ and CZ 2.](image)

**Discussion**

In some cases it could seem that linking within school mathematics is a special case of Topaze effect. In our previous work (Novotná, Hošpesová, 2007) we have studied the Topaze effect as a means of controlling students’ uncertainty. In the situation of Topaze effect, “the teacher begs for a sign that the student is following him, and steadily lowers the conditions under which the student will wind up producing the desired response. In the end the teacher has taken on everything important about the
work. The answer that the student is supposed to give is determined at the outset, and the teacher chooses questions to which this answer can be given. Obviously the knowledge required to produce the answer changes its meaning as well.” (Brousseau, Sarrazy, 2002, p. 9, item 16). In this paper, we focus on the “useful linking”; the teacher coherently or intuitively (experience-based) recalls previous pupils’ knowledge or experience that is useful for their successful work on the piece of mathematics. The aim is to build the nets of knowledge, structures, and links.

The teachers’ decision to include APL seems to correspond to their analysis a priori of the didactical situation. Predicting possible students’ solving strategies, previous knowledge needed for successful application of them etc. results in preparing the corresponding types and places of linking in the classroom.

Linking, especially AHL is susceptible to becoming a form of the Topaze effect. Used as a hint without checking students’ understanding of the relationship between the linked and the “new” knowledge, we see it as Topaze effect. In contrast to Topaze effect, linking is used as an aid to grasp the new knowledge and to place it into the existing grid of knowledge.

The presented analysis of linking indicates that linking can play different roles in mathematical activities. It might be used for example:

- as a means of recalling needed previous knowledge,
- as scaffolding applied when helping the students to overcome the failure in understanding the new knowledge due to lack of previous knowledge,
- as a tool for checking previous understanding,
- as a tool for clarifying something puzzling from previous activities.

Our analysis compared to the Stigler’s (in Shimizu, 1999) findings shows that linking could be an important feature of classroom culture. It significantly influences the quality of the grasped knowledge.

Endnote

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References


