INTENTIONALITY AND WORD PROBLEMS IN SCHOOL DIALOGUE

Alena Hošpesová & Jarmila Novotná
University of South Bohemia, Charles University Prague

Abstract. The aim of integration of word problems into mathematics education is teaching the pupils how to mathematize real situations. When solving a word problem, the pupil should go through: encoding, transformation, calculation stage and storage including verification. This requires employment of relatively demanding cognitive processes. However, what happens when solving the problem within a mathematics lesson led by a teacher is lowering of cognitive demands in consequence of the teacher’s stimuli and interference. In the contribution we will demonstrate which cognitive processes should be in play and how they can be distorted in school dialogue.

INTRODUCTION

In the conditions of institutionally constituted school education, it is the curricular materials that define the set of goals on various levels of explicitness, obligation, generality and comprehensibility. Their projection in the teaching and learning processes is hindered by a number of factors (the distractive influences in teaching, low pupils’ motivation, teacher’s competence etc.) It is often extremely difficult to uncover why the projected aims are not achieved in education. One of the possibilities is that the original aim disappears in the process of teaching and is replaced by other aims (e.g. the teacher must finish the lesson in time, he/she wants to involve more pupils in the dialogue, he/she wants to explain the solution of the problem in such a way that it is understood by more pupils etc.) This can happen even in the case that the teacher identifies with the curricular aims and is experienced enough for their realization.

Comparison of the didactic analysis of the curricular topic “solving word problems” and realization of a selected example in teaching serve as demonstration of the fact that the teacher unconsciously lowers the intellectual demandingness and thus he/she deviates from the original aim of inclusion of the topic in the curricula. The solving process of the word problem (planned and realized) is broken down into single steps which are assessed from the point of view of cognitive processes necessary for their solution. The comparison is based on the classification according to the revised Bloom’s taxonomy of cognitive processes (Anderson, Krathwohl, 2001, overview in Annex).

In the contribution we are going to discuss the following questions and statements: (a) Generally, when including a word problem in the teaching process we expect the pupil to apply the learned solving algorithms in a new situation (category 3 in the revised Bloom’s taxonomy – see below). However, the solving process presupposes
the execution of certain substeps, some of which may belong to different categories. In the contribution we ask which categories of cognitive processes the pupil should go through in completion of the individual phases of the solving process. (b) If the word problem is solved at school under the supervision of a teacher, the solving process is broken down by the teacher into smaller segments (tasks). The solution of these tasks may need only lower rated cognitive process than presupposed. It is a question what the consequences of lowering the category of cognitive processes are with respect to understanding mathematics and to pupils’ behaviour when solving a word problem.

**DATA COLLECTION**

In the research reported we analyse solutions of two word problems from a two videotaped lessons taught in different schools (CZ1, CZ2). The lessons were a part of video recordings of ten consecutive lessons on the solution of linear equations and their systems in the 8th grade (pupils aged 14-15) of two lower secondary schools in the Czech Republic. The method of data collection is based on the Learner’s Perspective Study (LPS) framework (Clarke, Keitel, Shimizu, 2006) and consists of the video recordings of a lesson and postlesson interviews with the teachers and a separately monitored pair of pupils. Both teachers are experienced and respected by parents, colleagues and educators, although their teaching strategies differ. In our previous analysis (Novotná, Hošpesová, 2008) we discovered that CZ2 teacher mostly focuses on the question “How?”; she approaches the solution of every problem as a new one and does not deal with it in the context of what the pupils may already know from solving other word problems. But she seems to trust her pupils’ independent discovery. CZ1 teacher pays more attention to the question “Why?” and tries to plant the new knowledge on her pupils’ previous knowledge.

**WORD PROBLEMS, THE AIM OF THEIR INTEGRATION INTO EDUCATION IN THE CR**

In mathematics education literature, the concept of a word problem is not formalized in a unified way. For the purposes of this research, we understand as word problem (in agreement with Verschaffel, Greer & De Corte, 2000) “verbal descriptions of problem situations wherein one or more questions are raised the answer to which can be obtained by the application of mathematical operations to numerical data available in the problem statement”. The solving process of a problem consists of a sequence of correctly ordered operations, decisive steps etc. starting from the data and relations among them and finishing by finding the unknown(s).

**Use of word problems in mathematical education**

Solution of word problems belong to one of the few domains of school mathematics necessitating mathematization of situations from the assignment and the return to the semantic context after solving the mathematical model (see Fig. 1 from Odvárko, 1990):
The tasks for the pupil is to discover or construct the mathematical model. The pupil’s available algorithms are of no use at this stage. The themes of the problems can but do not have to favour the solving process (see e.g. Sarrazy, 2002). Therefore, the didactical use of word problems appears to be a suitable means, often used by teachers, for developing their pupils’ mathematical competences. They allow them to develop their capacity to use their mathematical knowledge outside of the domain of mathematics and hence help to recognize, understand and memorize mathematical concepts, methods and results (Blum & Niss, 1991). Moreover, they contribute to their ability to select necessary information, work in a creative way and to develop their heuristic procedures (Verschaffel, Greer & De Corte, 2000).

**Word problem solving procedure – theoretical analysis**

In the paper the stages of word problem solving process (Novotná, 1997) are used: (a) encoding stage (grasping the assignment), (b) transformation stage (transfer to the language of mathematics and creation of a mathematical model), (c) calculation stage (mathematical solution of the problem including mathematical verification of the obtained results), (d) storage stage (transfer of mathematical results back into the context including contextual verification of the obtained results).

For the analysis of the cognitive processes necessary for overcoming the stages of word problem solution we used the categories (in the following text in italics) of the revised Bloom’s taxonomy (Anderson & Krathwohl, 2001): 1 Remember, 2 Understand, 3 Apply, 4 Analyze, 5 Evaluate, 6 Create.

**Encoding stage:** Coding of the word problem assignment is the transformation of the word problem text into a suitable system in which data, conditions and unknowns can be recorded in a more clearly organized and/or more economical form. Cognitive processes needed for this stage belong to the category 4 Analyze. It includes differentiating, organizing, and attributing.

**Transformation stage:** This stage covers mathematization of the assigned problem structure. It includes two categories: 4 Analyze, namely organizing and 3 Apply, namely implementing.

**Calculation stage:** This stage covers mathematical task solution and the (explicit or implicit) execution of verification of the obtained mathematical results. It is covered by the category 3 Apply, namely executing.
Storage stage: Mathematical results are transformed back to the semantic context of the assignment. They are confronted and verified in the context environment. This stage corresponds to the category 5 Evaluate including checking and critiquing.

The process of problem solving in school teaching

In the previous text, there is an analysis of an ideal progress of the word problem solving. However, the results of our experiments carried out within the frame of LPS and other long-term observations signal that in school teaching it may happen and often happens (be it consciously or unconsciously) that the process diverts to activities demanding other categories of cognitive processes, often lower than the anticipated ones.

Our discussion is based on examples of solutions of two word problems. The first sample – a more comprehensive one – is from the school CZ1 (L3 18:53 – 22.42). The following transcript of the dialogue proves that the teacher entered the solving process (T1- T17) with the intention of sustaining the understanding of the solution of a difficult problem. The assignment of the problem was: A 95 m long train is crossing a bridge at the speed 45 km/h. It takes 12 seconds from the moment when the locomotive drives onto the bridge until the moment when the last carriage leaves the bridge. How long is the bridge? The transcript includes all stages of the solving process: encoding (1 – 24), transformation (25 – 32), calculation (33 – 34), storage (36 – 39).

1 T1: We want to calculate how long the bridge is. What is the situation? Describe it, Lucka. A train drives onto the bridge and we know how ...
2 Lucka1: ... long it is.
3 T2: And we lose interest as soon as the end of the train leaves...
4 Lucka2: ...that bridge.
5 T3: Let’s make a drawing. The bridge (the teacher draws a line segment on the board). The train (the teacher sketches a locomotive) is driving onto it. Now we are becoming interested. And we lose our interest at this moment (the teacher is sketching a locomotive with carriages). Why is this drawing so important? Lenka.
6 Lenka1: Because we have to calculate the trajectory.
7 T4: Yes, the length of the distance covered. And what do we know about it, Michal?
8 Michal1: Only the length of the train. 95 meters.
9 T5: And there were some other data in the text.
10 M2: There was also the speed of the train 45 km/h.
11 T6: 45 km/h (she writes it down into the drawing on the board), ... This is also interesting because the units don’t correspond here. Well, let’s decide which we’ll change, Denisa.
12 D1: The speed.
13 T7: Yes, it’ll probably be most handy to change the speed. So how do we convert kilometers per hour to meters per second?
14 Pupil: Divided by 3.6.

Steps 15 – 20 were devoted to clarification of conversion of speed from km/h to m/s.
T8: What’s the result?
P: 12.5.

T9: Yes, 12.5. Let’s write it down here. (She writes it into the drawing on the board.) The unit? Hanka.
H: Meters per second.
T10: Great. So the units are OK now. Now it shouldn’t be a problem. How would you continue solving this problem?
P: The length of the train is x + 95.
T11: Not of the train, of the trajectory.
P: We know the speed, we know the time 12 seconds.
T12: How would you calculate it? The length of the bridge. The trajectory is …
Vítek3: Velocity times time
T13: And that’s it. Dictate it to me, David.
David1: X plus 95 equals 12.5 times 12.
T14: Calculate it, you have the calculator. … Peter, what’s the result?
David2: 55.
T15: Look at the question. What was the question? Peter.
P: How long is the bridge?
T16: We know how. But we haven’t verified the result. The result is neat, a whole number, but how shall we verify it? Adam.
P: I would calculate the time.
T17: Why not, yes. We won’t do it now. We were checking each other while solving.

The teacher paid most attention to encoding. That is understandable because this word problem is complicated by the conversion of units and by the necessity to realize what represents the “trajectory” in this problem. As stated above, this stage should focus on an analysis and evaluation of the situation described in the word problem (categories 4 Analyze and 5 Evaluate). In this phase the teacher intervenes in the solving process ten times. In some cases she uses intonation to signal that she expects the pupils to complete her statement (T1, T2, T4, T5). These signals belong to category 2 Understand, namely interpreting. Steps 15 – 20 that we have left out of the transcript and T8 - T10 are of similar nature. We regard the tasks in T3 and T7 as the most difficult moments of this stage. In step T3 the teacher herself sketches the drawing of the problem and asks the pupils to explain why the picture is important – category 2 Understand (explaining, constructing models). In T6 she asks the pupils to decide which units to change – category 4 Analyze (differentiating).

The most important stage of problem solving is transformation. It is necessary to state that the teacher initiated transformation already in the stage of encoding. Steps T10 and T12 are most demanding here (category 3 Apply – implementing). Pupils react fast although the wording of their answer needs to be refined (in T11 the teacher asks for reformulation – category 2 Understand, interpreting).
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The calculation itself was carried out very fast in just two steps that can be labeled as application of a known algorithm, a calculation in this case 3 Apply (executing).

The stage storage was again relatively brief. In T15 the teacher recalls the question from the word problem assignment (category 1. Remember - recalling). What could have been most difficult was the contextual verification of the obtained result, which belongs to category 5 Evaluate. However, in this case the pupil only traces new contexts (4 Analyze – differentiating) and proposes the procedure of the verification by verification by calculation, which was, however, not carried out.

We found an example of solving of a word problem dealing with movement also in the school CZ2. In this case it is a very straightforward problem from the introductory lesson to the topic “word problems dealing with movement”: The road from Adam to Eve is 5 km long. Adam has a date with Eve and sets off to meet her at the speed 6 km/h. At the same time Eve sets off to meet Adam at the speed 4 km/h. How long will it take before they meet and how many km will Adam have covered?

Due to the straightforwardness of the problem its solution was very fast, which can be clearly seen in the following transcript (CZ2 L8 22:20-24:31).

1 T1: Jirka, what do you remember of what I’ve just read?
2 Jirka1: There are Adam and Eve walking against each other.
3 T2: That this time they are walking against each other from different places. So let’s sketch it: here is Adam, here is Eve (the teacher draws a line segment with arrows in its end points on the board). I’ll read the following passage from the assignment: the distance between them is 5 km. There’s no reason why not to write at the top that the distance between them is 5 km. (She writes it on the board.) Let’s continue: Adam is walking at the speed 6 km/h, Eva 4 km/h. Try to record it into the drawing somehow. (The pupils make arrows into the drawings in their exercise books.) Radka, do you think that Adam will have covered shorter or longer distance?
4 Radka: Longer, because he is walking faster.
5 T3: Could anyone say at this point how long it’ll take before they meet?
6 Pupils: Half an hour.
7 T4: Some of you may have solved it using relative speed. What is the relative speed they are moving against each other?
8 Pupil: 10.
9 T5: Every hour 10 km. I add 6 and 4, ten. And because there are only 5 km between them, they will meet in half an hour.

In this case the teacher intervened into the pupils’ solving process only five times. Her every intervention can be perceived as the stimulus for transfer to the following stage of the solving process. In T1 she initiates encoding and she only asks for recollection of the problem situation (category 1 Remember – recalling). In T2 she finishes the encoding stage on her own and by the question in the end of T2 she transcends to the transformation stage. The question may be regarded as an appeal for a comparison of the two speeds (2 Understand – exemplifying) and to expression of a simple judgment (may be a hint of 5 Evaluate – critiquing). T3 at the same time
invites transformation and calculation stage (3 Apply – executing). As far as the cognitive processes needed for answering question T4 are concerned we can assume that they are an appeal to the use of algorithms that the pupils are already familiar with (3 Apply – executing). The storage stage is carried out in T5 by the teacher herself.

Discussion and conclusions
In the contribution we showed the difference between the suppositional level of the cognitive processes necessary for the completion of the individual stages of the solving process of a word and the reality in school teaching. We are fully aware of the fact that lowering the categories of cognitive processes happens whenever pupils do not solve the problem on their own but communicate with the teacher and/or their classmates during the solving process. The impact of such lowering may not necessarily be negative. In our point of view, it may serve as a solid base of understanding in early stages of presentation of the problem. It is however a question whether the teachers are always aware of this lowering and whether they do not stay in the position unnecessarily too long. When analyzing other solutions of word problems in our data, it became evident that especially the teacher CZ1 uses such approach whenever she communicates with her pupils about the solution of the problem. In the postlesson interview we were told that her goal is the state when all her pupils understand the solution. It turned out that the pupils expect her “help” because one of the pupils from a separately monitored pair said while analyzing the word problem: “I’m really glad that we are not alone in it (in the solving process).” The consequence of lowering the cognitive demandingness may be that the pupils rely on the teacher’s “help”. It may happen that they routinely repeat the learned process, often without deeper understanding. They do not attempt to find their own suitable solving strategies. The learning process fails to work with one of the key elements – mistake, its recognition and elimination.

When analyzing the dialogues we realized the connection with frequent inclusion of the Topaze effect (Novotná, Hošpesová, 2007), which is usually motivated by the teacher’s effort to have the pupils take active part in the dialogue with him/her.

Several questions arise in the context of the use of the Bloom’s taxonomy for evaluation of the cognitive level of the assigned problems in mathematics education. In our analyses we lacked a category which would demand the execution of a routine calculation – its placing into category 3 Apply (executing) does not correspond with its cognitive demandingness. We propose a new subcategory executing calculations in 2 Understanding. However, we believe that the use of the revised Bloom’s taxonomy is a suitable method, especially in qualitative research.

We find it useful to employ the taxonomy in pre-service and in-service training, e.g. a pre-service teacher first carries out the “theoretical analysis” of the solving process and then compares it with the reality. A group of in-service teachers uses the analysis
as the base for reflection on a taught lesson. In both groups we can anticipate a much higher degree of sensitivity in the subsequent self-reflection on one’s own teaching.

References
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