$$\lim_{n\to\infty} \frac{1}{n^2}\sum_{k=n^2}^{2n^2} \operatorname{arctg}\frac{k}{n^2}$$

Uvažujeme rozdělení (σ_n) intervalu $\langle 1,2\rangle$

$$\sigma_n = \left\{1+\frac{k}{n^2} : k=0,1,2,\dots,n^2\right\}.$$

Pak $$\frac{1}{n^2}\sum_{k=n^2+1}^{2n^2}\operatorname{arctg}\frac{k}{n^2} + \frac{1}{n^2}\operatorname{arctg}\frac{n^2}{n^2} =$$

$$\mathcal{J}(\sigma_n) + \frac{1}{n^2}\frac{\pi}{4}$$, kde $\mathcal{J}$ je integr. součet funkce $f(x)=\operatorname{arctg} x$

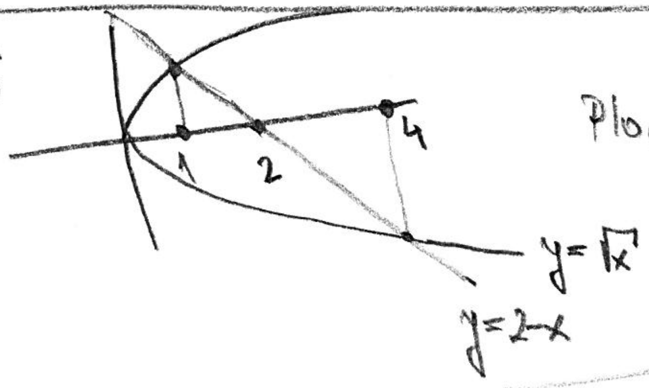
pro $n\to\infty$

dostaneme $\lim\limits_{n\to\infty} \dots = \int_1^2 \operatorname{arctg} x\,dx =$ per partes

$$= \left[x \operatorname{arctg} x\right]_1^2 - \int_1^2 \frac{x}{1+x^2}\,dx =$$

$$= \left[x \operatorname{arctg} x\right]_1^2 - \frac{1}{2}\left[\ln(1+x^2)\right]_1^2 = 2\operatorname{arctg} 2 - \frac{\pi}{4} + \ln\sqrt{\frac{2}{5}}$$

Plocha:



$$\text{Plocha} = 2\int_0^1 \sqrt{x} + \int_1^4 2-x+\sqrt{x} =$$

$$= 2\left[\frac{x^{3/2}}{3/2}\right]_0^1 + \left[2x - \frac{x^2}{2} + \frac{x^{3/2}}{3/2}\right]_1^4 = 4\tfrac{1}{2}$$

$$\int \left(\frac{x^2}{x^2+4x+5}\right)^2 = \int \left(1-\frac{4x+5}{x^2+4x+5}\right)^2 = \int \left(1-\frac{4(x+2)-3}{(x+2)^2+1}\right)^2 dx$$

substituce $y = x+2$

$$\int \left(1-\frac{4y-3}{y^2+1}\right)^2 dy$$

Upravme výraz: $\left(1-\frac{4y-3}{y^2+1}\right)^2 = 1-2\,\frac{4y-3}{y^2+1} + \frac{16y^2-24y+9}{(y^2+1)^2}$

$$= 1-4\,\frac{2y}{y^2+1}+6\,\frac{1}{y^2+1}+16\,\frac{1}{y^2+1}-7\,\frac{1}{(y^2+1)^2}-12\,\frac{2y}{(y^2+1)^2}$$

primit. funkce k jednotlivým členům

$$y-4\ln(y^2+1)+22\operatorname{arctg} y+12\,\frac{1}{y^2+1}-7\int\frac{1}{(y^2+1)^2}\,dy$$

→ to vyřešíme jako na přednášce pomocí per partes

a pak dosadíme do všeho $y = x+2$