How Pusheen uses computer to do mathematics

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May 21, 2015
1089 and 2178, the magic numbers!
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Reverse multiples

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We observed that if \( n \) is a palindromic number then

\[ \text{rev}_{10}(n) = 1 \times n \]

where \( \text{rev}_{10}(n) \) denotes the reverse order number \( n \) in the decimal base, i.e.,

if \( n = (\alpha_N \ldots \alpha_1\alpha_0)_{10} \), then \( \text{rev}_{10}(n) = (\alpha_0\alpha_1 \ldots \alpha_N)_{10} \).
Palindromic numbers

- Palindromic numbers represent another field of interest within the realm of $\mathbb{N}$. 

- People are interested particularly in:

  Palindromic primes:
  - 2, 3, 5, 7, 11, 101, 131, 151, 181, 191, 313, 353, 373, 383, 727, 757, 787, ...

  "It is not known if there are infinitely many of them."

  Palindromic squares:
  - 1, 4, 9, 121, 484, 676, 10201, 12321, 14641, 40804, 44944, 69696, 94249, ...

  Palindromic cubes and higher powers...

  Conjecture (G. J. Simons): "There is no palindrome of the form $n^\ell$ for $\ell > 4$.

  Conjecture (N. J. A. Sloane?): "If $k^4$ is a palindrome, then $k = 100...001$."
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Let $g \geq 2$ and $1 \leq k < g$. A number $n$ is called a $(g, k)$-reverse multiple iff

$$\text{rev}_g(n) = k \times n.$$
Back to reverse multiples

Formal definition

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- ... so, we write a program!
Between numbers 1 – 999 we find no \((10, k)\)-reverse multiples (with \(k > 1\)).
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9801 = 9 \times 1089 \quad \text{and} \quad 8712 = 4 \times 2178.
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For \( 10^4 \leq n < 10^8 \), we find:
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Can you see some pattern?
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If $n$ is a $(10, k)$-reverse multiple, then $k$ is 1, 4, or 9.
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Let's take a look to another numbers...
9-digit:
   219999978
   217802178

10-digit:
   2199999978
   2178002178
   2197821978

11-digit:
   21999999978
   21780002178

12-digit:
   219999999978
   217800002178
   21997800219978
   2199997821999978
   217821782178
9-digit:
219999978
217802178

10-digit:
2199999978
2178002178
2197821978

11-digit:
21999999978
21780002178
21978021978

12-digit:
219999999978
217800002178
21997800219978
2199997821999978
217821782178

Are you still able to follow the pattern?
9-digit:
- 219999978
- 217802178

10-digit:
- 2199999978
- 2178002178
- 2197821978

11-digit:
- 21999999978
- 21780002178
- 21997800219978
- 2199997821999978
- 217821782178

12-digit:
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  2199997821999978
  217821782178

- 12-digit:
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Are you still able to follow the pattern?
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**Case 1:**

\[
\underbrace{0 \ldots 0}_{m_1}
\]
To understand the pattern of general \((10, 4)\)-reverse multiple, it is better to start from the middle:

**Case 1:**

\[
\underbrace{219 \ldots 978}_k \quad \underbrace{0219 \ldots 978}_m
\]
To understand the pattern of general \((10, 4)\)-reverse multiple, it is better to start from the middle:

**Case 1:**

\[
\cdots 0 \ldots 0^{219} \ldots 9^{780} \ldots 0^{219} \ldots 9^{780} \ldots 0 \cdots
\]

\[m_2 \quad k_1 \quad m_1 \quad k_1 \quad m_2\]
To understand the pattern of general \((10, 4)\)-reverse multiple, it is better to start from the middle:

**Case 1:**

\[
\cdots 0 \ldots 0219 \ldots 9780 \ldots 0219 \ldots 9780 \ldots 0 \cdots
\]

\[
\underbrace{m_2}_{m_i} \quad \underbrace{k_1}_{m_1} \quad \underbrace{m_2}_{m_i}
\]

\[
\cdots 0 \ldots 0 \cdots
\]

\[
\underbrace{m_i}_{m_i}
\]
To understand the pattern of general \((10, 4)\)-reverse multiple, it is better to start from the middle:

**Case 1:**

\[
\cdots 0 \ldots 0219 \ldots 9780 \ldots 0219 \ldots 9780 \ldots 0 \cdots
\]

\[
\begin{array}{c}
\underbrace{219 \ldots 9780 \ldots 0 \cdots}_{k_i m_i} \\
\underbrace{m_2 k_1 m_1 k_1 m_2}_{m_2 k_1 m_1 k_1 m_2}
\end{array}
\]

\[
\begin{array}{c}
\underbrace{\cdots 0 \ldots 0219 \ldots 978}_{m_i k_i}
\end{array}
\]
The final result

To understand the pattern of general \((10, 4)\)-reverse multiple, it is better to start from the middle:

**Case 1:**

\[
\cdots 0 \cdots 0219 \cdots 9780 \cdots 0219 \cdots 9780 \cdots 0 \cdots
\]

\[
\underbrace{m_2}_{m_1} \underbrace{k_1}_{m_1} \underbrace{m_1}_{k_1} \underbrace{k_1}_{m_2}
\]

\[
219 \cdots 9780 \cdots 0 \cdots
\]

\[
\underbrace{k_i}_{m_i} \underbrace{m_i}_{k_i}
\]

**Case 2:**

\[
219 \cdots 978
\]

\[
\underbrace{k_1}_{m_i}
\]
To understand the pattern of general \((10, 4)\)-reverse multiple, it is better to start from the middle:

**Case 1:**

\[
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\]

\[
\underbrace{219 \cdots 9780 \cdots 0 \cdots}_{k_i} \quad \underbrace{\cdots 0 \cdots 0219 \cdots 9780 \cdots 0 \cdots}_{m_i}
\]

**Case 2:**

\[
0 \cdots 0219 \cdots 9780 \cdots 0
\]

\[
\underbrace{0 \cdots 0219 \cdots 9780 \cdots 0 \cdots}_{m_1} \quad \underbrace{\cdots 0 \cdots 0219 \cdots 9780 \cdots 0 \cdots}_{k_1}
\]
To understand the pattern of general \((10, 4)\)-reverse multiple, it is better to start from the middle:

**Case 1:**

\[
\begin{array}{cccccccc}
\cdots & 0 & \cdots & 0 & 219 & \cdots & 978 & 0 & \cdots & 219 & \cdots & 978 & 0 & \cdots \\
m_2 & & k_1 & & m_1 & & k_1 & & m_2 & & & & & \\
& 219 & \cdots & 978 & 0 & \cdots & & & & & & \cdots & \cdots & 0 & \cdots & 219 & \cdots & 978 \\
k_i & & m_i & & & & & & & & k_i & & m_i & & & \\
\end{array}
\]

**Case 2:**

\[
\begin{array}{cccccccc}
\cdots & 219 & \cdots & 978 & 0 & \cdots & 0 & 219 & \cdots & 978 & 0 & \cdots & 219 & \cdots & 978 & \cdots \\
k_2 & & m_1 & & k_1 & & m_1 & & k_2 & & & & & & & \\
& 219 & \cdots & 978 & 0 & \cdots & & & & & & \cdots & \cdots & 0 & \cdots & 219 & \cdots & 978 & \cdots \\
k_i & & m_i & & & & & & & & k_i & & m_i & & & & \\
\end{array}
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The final result

To understand the pattern of general (10, 4)-reverse multiple, it is better to start from the middle:

**Case 1:**

\[
\cdots \underbrace{0\ldots0}_{m_2} \underbrace{219\ldots978}_{k_1} \underbrace{0\ldots0}_{m_1} \underbrace{219\ldots978}_{k_1} \underbrace{0\ldots0}_{m_2} \cdots
\]

\[
\underbrace{219\ldots978}_{k_i} \underbrace{0\ldots0}_{m_i} \cdots
\]

**Case 2:**

\[
\cdots \underbrace{219\ldots978}_{k_2} \underbrace{0\ldots0}_{m_1} \underbrace{219\ldots978}_{k_1} \underbrace{0\ldots0}_{m_1} \underbrace{219\ldots978}_{k_2} \cdots
\]

\[
\underbrace{0\ldots0}_{m_{i-1}} \cdots
\]

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To understand the pattern of general \((10, 4)\)-reverse multiple, it is better to start from the middle:

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\[
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\cdots & 0 \cdots 219 \cdots 9780 \cdots 0219 \cdots 9780 \cdots 0 \cdots \\
& \underbrace{m_2}_{m_i} \underbrace{k_1}_{k_i} \underbrace{m_1}_{m_{i-1}} \underbrace{k_1}_{k_i} \underbrace{m_2}_{m_i} \\
219 \cdots 9780 \cdots 0 \cdots & \underbrace{m_i}_{m_i} \underbrace{k_i}_{k_i}
\end{align*}
\]

**Case 2:**

\[
\begin{align*}
\cdots & 219 \cdots 9780 \cdots 0219 \cdots 9780 \cdots 0219 \cdots 978 \cdots \\
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219 \cdots 9780 \cdots 0 \cdots & \underbrace{m_{i-1}}_{m_{i-1}} \underbrace{k_i}_{k_i}
\end{align*}
\]

...and similarly for the case of \((10, 9)\)-reverse multiples with numbers 1089.
The final result

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\[
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 & m_2 \quad k_1 \quad m_1 \quad k_1 \quad m_2
\end{align*}
\]

\[
\begin{align*}
& 219 \cdots 9780 \cdots 0 \cdots \\
 & k_i \quad m_i
\end{align*}
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\[
\begin{align*}
& \cdots 0 \cdots 0219 \cdots 978 \\
 & m_{i-1} \quad k_i
\end{align*}
\]

...and similarly for the case of \((10, 9)\)-reverse multiples with numbers 1089.
Theorem

An integer $n$ is a $(10, k)$ multiple if and only if one of the following condition holds:

1. $k = 1$ and $n$ is palindromic.
2. $k = 4$ and $n$ has the form as indicated in the Case 1 or Case 2.
3. $k = 9$ and $n$ has the form as indicated in the Case 1 or Case 2 where 2178 is replace by 1089.
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Well...they are magic!

Definition
A number is called magic if it is used by magicians to do their tricks.

1089 is magic, indeed!
Proof:
Write down a non-palindromic 3-digit number ABC.
Reverse the order of digits CBA.
Subtract the lower one from the bigger one getting DEF.
Reverse the order once more, FED.
Finally, compute DEF + FED = ...
and now you known why 1089 is magic!
A magic trick

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1089 and 2178, the magic numbers!

CoCon 2015
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Thus, there is an infinite number of solutions: numbers which are factorials of an integer.
Besides the above mentioned solutions \((m! = n - 1)\), is there any other solution of \(n! = m! \cdot k!\)?

There is, indeed, since \(10! = 6! \cdot 7!\).

**Question:** What can be said about the set
\[\mathcal{A} = \{n \in \mathbb{N} | \exists m, k \in \{2, 3, \ldots, n - 2\} \text{ such that } n! = m! \cdot k!\}\]

We do not know much about \(\mathcal{A}\). In fact, only \(10 \in \mathcal{A}\)

There are few more statements concerning the factors \(m\) and \(k\). These results, however, only slightly restrict the set of possible solutions. For example, it can be shown (and it is not very hard) that if \(n \in \mathcal{A}\), then \(m + k > n + 1\).
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\[ A = \{10\}. \]
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Can you do that better?

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Can you significantly improve the range of numbers that do (not) belong to $\mathcal{A}$?
Can you disprove the Satan conjecture? What is the respective formula $x!y! = z!$?
Apart from the computational properties, can you show something mathematically interesting about $\mathcal{A}$?
The End

Starring:
Starring:

Pusheen
The End

Starring:

![Pusheen](image1.png)
Pusheen

![TK](image2.png)TK

Thank you for your attention!
František Štampach (CTU Pusheen club)

How Pusheen uses computer to do mathematics
The End

Starring:

Pusheen

TK

Unhappy cat
Thank you for your attention!