| MIE-MPI - EXAM | December 19, 2019 |  |  |  |  |
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| Name | Q1-6 | Q7 | Q8 | Q9 | $\boldsymbol{\Sigma}$ |
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| Multiple choice question answer table |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Q1 | Q2 | Q3 | Q4 | Q5 | Q6 |
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Instructions: The questions $1-6$ have possible answers labelled A-E. There is always exactly one correct answer. Please, use the table above to mark your answer. If you make a mistake, correct your answer in the table (in a readable manner).

Other questions serve as a preparation for the oral part of the exam (nevertheless, your written preparation should be understandable). Don't forget to sign this sheet and all the sheets that you will hand in.

You can use only a paper, pen and your brain! Good luck!

Question 1 (5 points). Select the correct statement
(A) There exist at least two non-isomorphic cyclic groups of infinite order.
(B) There exist only two groups of order 4 up to isomorphism.
(C) The groups $\mathbb{Z}_{8}^{+}$and $\mathbb{Z}_{8}^{\times}$are isomorphic.
(D) Each finite group is isomorphic to the additive group $\mathbb{Z}_{n}^{+}$for some $n$.
(E) No other option is true.

Question 2 (5 points). Let us consider as domain $D$ the triangle with vertices the points $(1,0)$, $(2,0)$ and $(2,2)$. The value of the double integral

$$
\iint_{D} x+y \mathrm{~d} x \mathrm{~d} y
$$

is
(A) $-\frac{1}{8}$
(B) $\frac{7}{3}$
(C) 0 .
(D) -5
(E) No other option is true.

Question 3 (5 points). What is the derivative partial derivative of $f(x, y)=x^{2}-x y+y^{3}$ with respect to $x$ at the point $(1,-1)$ ?
(A) $2 x-y+3 y$
(B) 0
(C) -2
(D) 4
(E) No other option is true.

Question 4 (5 points). Let us consider the permutation $f=(34276511089) \in S_{10}$. The permutation $f^{47}$ is:
(A) (12358436710)
(B) (21436528910)
(C) (10 742156938 )
(D) (27415639108)
(E) No other option is true.

Question 5 (5 points). Consider $G F\left(3^{2}\right)$ with multiplication modulo $x^{2}+1$. Find the inverse of (12).
(A) 20
(B) 21
(C) 11
(D) 13
(E) No other option is true.

Question 6 (5 points). Let $A$ and $B$ be two fuzzy sets (over a universe U ) having membership functions $\mu_{A}$ and $\mu_{B}$, respectively. Using the product t-norm for intersection, give the formula for membership function of $A \cup B$.
(A) $\mu_{A \cup B}(x)=\max \left(\mu_{A}(x), \mu_{B}(x)\right)$
(B) $\mu_{A \cup B}(x)=\mu_{A}(x)+\mu_{B}(x)$
(C) $\mu_{A \cup B}(x)=1-\mu_{A}(x) \mu_{B}(x)$
(D) $\mu_{A \cup B}(x)=\mu_{A}(x)+\mu_{B}(x)-\mu_{A}(x) \mu_{B}(x)$
(E) No other option is true.
*** ORAL PART PREPARATION ${ }^{* * *}$
Question 7. (11 points) Let $f: \mathbb{R}^{3} \rightarrow \mathbb{R}$ and $(x, y, z) \in \mathbb{R}^{3}$. List sufficient conditions for $(x, y, z)$ to be
(a) a saddle point;
(b) a point of local strict maximum.

Question 8. (11 points)

1. Give a definition of machine number (consider single precision).
2. Give the representation of the largest number and the least normalized positive number in single precision.
3. Describe the problem of round-off error.

Question 9. (12 points)

1. Write down the definition of group, subgroup, ring, and field.
2. Write down the definition of cyclic group and give an example.
3. Write down the definition of order of a group. Write some statement connecting the order of a group and the order of its subgroup.
