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| MIE-MPI – EXAM | | | | JANUARY 20, 2020 | |
| Name | Q1–6 | Q7 | Q8 | Q9 | Σ |
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|---------------------------------------|----|----|----|----|----|
| Multiple choice question answer table | | | | | |
| Q1 | Q2 | Q3 | Q4 | Q5 | Q6 |
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Instructions: The questions 1 – 6 have possible answers labelled A–E. There is always exactly one correct answer. Please, use the table above to mark your answer. If you make a mistake, correct your answer in the table (in a readable manner).

Other questions serve as a preparation for the oral part of the exam (nevertheless, your written preparation should be understandable). Don't forget to sign this sheet and all the sheets that you will hand in.

*You can use only a paper, pen and **your** brain! Good luck!*

Question 1 (5 points). Let us consider the permutation $f = (3\ 8\ 7\ 9\ 4\ 5\ 2\ 1\ 6) \in S_9$. The permutation f^{44} is:

- (A) (2 3 5 8 4 1 7 9 6)
- (B) (8 7 1 4 5 6 3 2 9)
- (C) (1 2 3 9 4 5 7 8 6)
- (D) (7 1 2 6 9 4 8 3 5)
- (E) No other option is true.

Question 2 (5 points). Select the **correct** statement

- (A) Every group of order 15 is cyclic.
- (B) $P(x) \in K[x]$ is irreducible over a field K if and only if it cannot be decomposed into a product of two elements of $K[x]$.
- (C) Every group of order strictly less than 6 is cyclic.
- (D) The group \mathbb{Z}_{36}^\times contains a subgroup of order 12.
- (E) No other option is true.

Question 3 (5 points). In the field $GF(3^2)$ with multiplication modulo $x^2 + 1$, find the results of $(20 - 11) \cdot 12$.

- (A) 02
 - (B) 22
 - (C) 01
 - (D) 10
 - (E) No other option is true.
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Question 4 (5 points). Let $f(x, y) = x^3 + x^2y + y^2x - y$. What is the value of $\frac{\partial^2}{\partial x \partial y} f$ at the point $(1, -1)$?

- (A) 2
 - (B) 0
 - (C) -1
 - (D) $3x^2 + 2xy + y^2$
 - (E) No other option is true.
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Question 5 (5 points). Let us consider as domain D the triangle with vertices the points $(0, 0)$, $(0, 2)$ and $(1, 0)$. Select the value of the double integral

$$\iint_D x - y \, dx dy.$$

- (A) 1
 - (B) $\frac{1}{8}$
 - (C) $-\frac{7}{3}$
 - (D) 0
 - (E) No other option is true.
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Question 6 (5 points). Let us consider the set $M = \{\alpha + \beta\sqrt{5} \mid \alpha, \beta \in \mathbb{Z}\}$ with classical addition and multiplication.

- (A) $(M, +)$ is a ring.
- (B) $(M \setminus \{0\}, \cdot)$ is a group.
- (C) $(M, +, \cdot)$ is ring but not a field.
- (D) $(M, +, \cdot)$ is a field.
- (E) No other option is true.

*** ORAL PART PREPARATION ***

Question 7. (11 points) Let $f : \mathbb{R}^2 \rightarrow \mathbb{R}$ and $(x, y) \in \mathbb{R}^2$. List sufficient conditions for (x, y) to be

- (a) a saddle point;
 - (b) a point of local strict minimum;
 - (c) a point of local strict maximum.
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Question 8. (11 points)

1. Write down the definition of t-norm.
 2. Give an example of t-norm.
 3. How can we use t-norms in fuzzy logic?
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Question 9. (12 points)

1. Write down the definition of group and of subgroup.
2. What is the Cayley table of a finite group? Give an example.
3. Can two different groups have the same Cayley table (up to renaming the elements)?