MIE-MPI: Tutorial 1

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1.1 Functions

Exercise 1.1. Let $f(x) = sin(x)$ and $g(x) = (x-3)^3$. Find a formula for the following composite functions:

(a) $(f \circ g)(x)$,

(b) $(g \circ f)(x)$,

(c) $(f \circ g^{-1})(x)$,

(d) $(g^{-1} \circ f)(x)$.

1.2 Derivatives

Exercise 1.2. Find the derivative of the following functions:

(a) $(x^4 + 3x^3)x^8$,

(b) e^{2x},

(c) $\frac{x+3}{x^2}$,

(d) $\ln((x+4)^{15})$,

(e) $\sin^2 x + \cos^2 x$,

(f) xe^{2x},

(g) e^{x^2},

(h) x^x.

Exercise 1.3. Let P be the set of all real polynomials. Is the set P closed under differentiation? In other words: is is true that $p \in P \ \Rightarrow \ p' \in P$?

Exercise 1.4. Let $p(x) = \sum_{k=0}^{n} a_k x^k$ be a polynomial of degree n (i.e., $a_n \neq 0$), where $n \in \mathbb{N}$. Find the n-th derivative $p^{(n)}$.

Exercise 1.5. Find the n-th derivative of $\sin x$. Try to express the result as simply as possible.

1.3 Partial derivatives

Exercise 1.6. Find the following partial derivatives:

(a) $\frac{\partial f}{\partial x}$ and $\frac{\partial f}{\partial y}$ for

$$f(x,y) = xy + e^x \cos y,$$

(b) $\frac{\partial f}{\partial x}$ and $\frac{\partial f}{\partial y}$ for

$$f(x,y) = x^2y^3 + x^3y^4 - e^{xy^2},$$

(c) find the value of $\frac{\partial f}{\partial z}$ at point $(1,2,3)$ for

$$f(x,y,z) = \sin\left(\frac{xy}{z}\right),$$

(d) $\frac{\partial f}{\partial x}$ for

$$f(x,y) = e^{-x^2-y^2},$$

(e) $\frac{\partial f}{\partial x}$ for

$$f(x,y) = \ln\left(x^2+y^2+1\right),$$

(f) $\frac{\partial f}{\partial x}$ for

$$f(x,y) = \frac{1}{x^3+y^3}.$$

Exercise 1.7. Find $\frac{\partial^2 f}{\partial x^2}$ and $\frac{\partial^2 f}{\partial y^2}$ for:

(a) $f(x,y) = x^2y^2$,

(b) $f(x,y) = \sin(xy)$,

(c) $f(x,y) = xy^2 - ye^{-x} - \cos(x-y)$.

Exercise 1.8. Find the mixed partial derivatives $\frac{\partial^2 f}{\partial x \partial y}$ and $\frac{\partial^2 f}{\partial y \partial x}$ for:

(a) $f(x,y,z) = e^{xz} + y\cos x$,

(b) $f(x,y,z) = z\cos(xy) + x\sin(yz)$.

