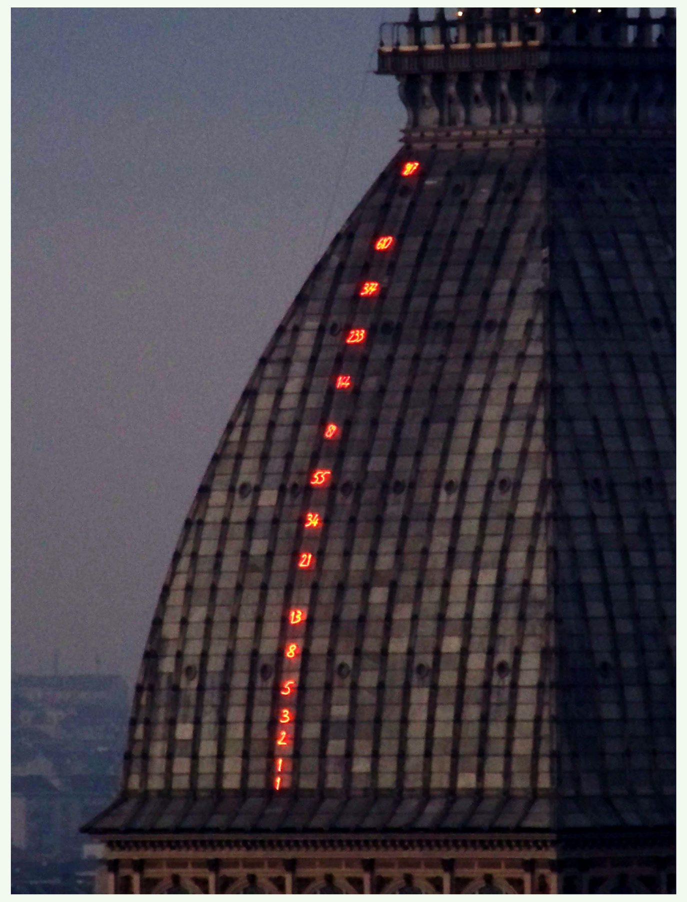
Sturmian words

Jacques Charles François Sturm

Sturmian words are infinite words over a binary alphabet that have exactly $n+1$ factors of length n for each $n \geq 0$ [1].
Sturmian words appear in different domains: Combinatorics on Words, Symbolic Dynamics, Theory of Fractals, Algebra, Theory of Codes, etc.
In 2011 we started a collective project to better understand these interactions.



Fibonacci sequence on the Mole Antonelliana (Turin, Italie).

The most well-know example of a Sturmian word is the Fibonacci word.
It can be computed in different ways as, for example:

- a fixed point of a primitive morphism φ,
- the limit of a sequence f_n [1].

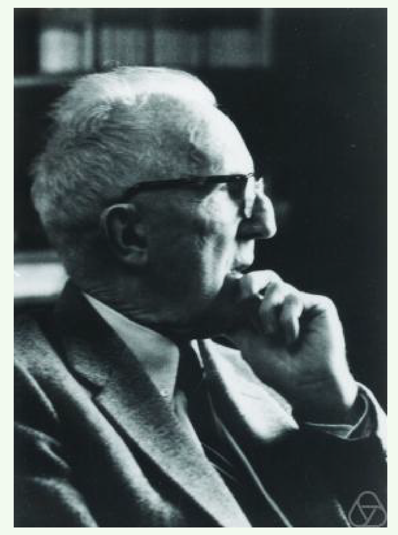
$$\varphi : A^* \to A^* \qquad a \mapsto ab \qquad b \mapsto a$$

$$\begin{cases} f_{-1} = b \\ f_0 = a \\ f_{n+2} = f_{n+1} f_n \end{cases}$$

$$x = \lim_{n \to \infty} f_n = \varphi^{\omega}(a) = abaababaabaababaababaabaababaabaab\ldots$$

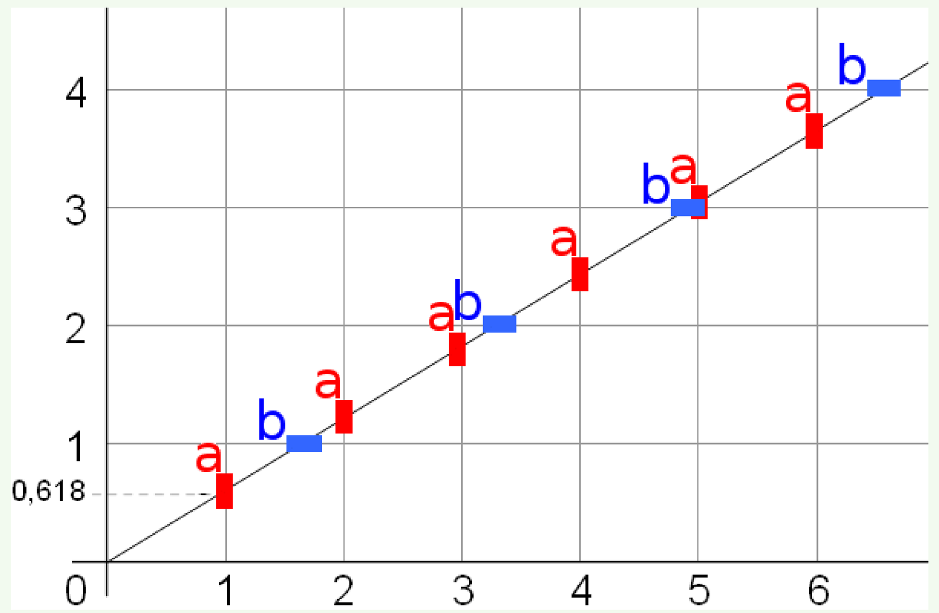
Leonardo Fibonacci

The term « Sturmian », after the French mathematician Charles Sturm, appears first in the work of Gustav Hedlund and Marston Morse [2].



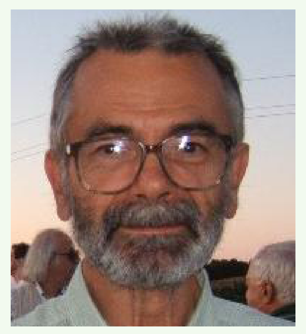
Marston Morse

Gustav Hedlund



Sturmian sequences can be defined geometrically as cutting sequences for lines of irrational slope, i.e. as a sequence of symbols whose elements correspond to the "cut" as the line crosses a square grid [1].

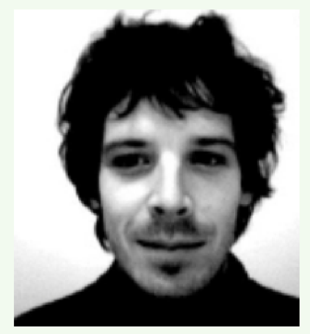
Using the factors of length n of the Fibonacci word we can obtain in the free group all words of length multiple of n [3] (special case of the Finite Index Basis Theorem [4]).



Jean Berstel

Valérie Berthé

Clelia De Felice



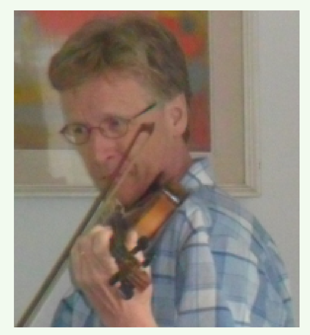
Vincent Delecroix

Francesco Dolce

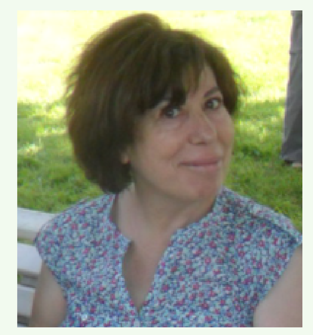
Julien Leroy

Dominique Perrin

Antonio Restivo



Christophe Reutenauer



Giuseppina Rindone

Bibliography

[1] M. Lothaire, *Algebraic Combinatorics on Words*, Cambridge University Press, 2002
[2] Marston Morse, Gustav A. Hedlund, Symbolic dynamics II. Sturmian trajectories, *Amer. J. Math.*, 62:1–42, 1940
[3] J. Berstel, C. De Felice, D. Perrin, C. Reutenauer, G. Rindone, Bifix codes and Sturmian words, *J. Algebra*, 369:146-202, 2012
[4] V. Berthé, C. De Felice, F. Dolce, J. Leroy, D. Perrin, C. Reutenauer, G. Rindone, The finite index basis property, *arxiv.org/abs/1305.0127*, 2013

Contact: Jean Berstel, Francesco Dolce, Dominique Perrin, Giuseppina Rindone

LIGM, fév. 2014