GEOMETRIC BOUNDS ON THE LOWEST MAGNETIC NEUMANN AND ROBIN EIGENVALUES

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We will first consider the two-dimensional magnetic Neumann Laplacian with homogeneous magnetic field. For its lowest eigenvalue on a convex domain and with moderate magnetic field, we obtain a geometric upper bound in terms of the torsion function and the lowest magnetic eigenvalue of the disk. Our analysis is inspired by the conjecture due to Fournais and Helffer stating that the lowest magnetic Neumann eigenvalue is maximized by the disk among all simply-connected domains of fixed area. Testing the obtained bound on ellipses, for which the torsion function is explicit, we get that, among all ellipses of fixed area, the disk is a unique maximizer of the lowest magnetic Neumann eigenvalue provided that the flux does not exceed an explicit constant. By continuity, our bound yields the conjectured isoperimetric inequality also for small deformations of ellipses.

For the magnetic Robin Laplacian with negative boundary parameter and moderate homogeneous magnetic field, we had a result that the lowest eigenvalue is maximized by the disk among all simply-connected planar domains of fixed perimeter and satisfying an additional geometric condition. This condition was first verified only for convex centrally symmetric domains. Recently, we managed to check this geometric condition for a significantly larger class of the domains. In the second part of the talk, we will discuss this improvement.

These results are obtained in collaborations with C. Dietze and A. Kachmar.