Contribution Title: A RIGOROUS PERSPECTIVE ON LIOUVILLE QUANTUM GRAVITY & KPZ

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Polyakov first understood in 1981 that the summation over random Riemannian metrics involved in transition amplitudes in gauge theory or string theory could be represented mathematically by the now celebrated *Liouville theory of quantum gravity.* The quantum gravity measure is formally defined by $d\mu_\gamma = e^{\gamma h(z)}dz$, where dz is the 2D Euclidean (i.e., Lebesgue) measure; $e^{\gamma h(z)}$ is the *random* conformal factor of the Riemannian metric, with h an instance of the Gaussian free field (GFF) on a bounded domain D; and γ is a constant, $0 \leq \gamma < 2$. Outstanding open problems include the relation of Liouville quantum gravity to discrete lattice models and their critical continuum limit — like the Stochastic Schramm-Loewner Evolution — through their embedding in random lattices.

In 1988, Knizhnik, Polyakov and Zamolodchikov predicted that corresponding critical exponents (x) of a conformally invariant statistical model in the Euclidean plane and in Liouville quantum gravity (Δ) would obey the the universal *"KPZ relation"*

$$x = \frac{\gamma^2}{4}\Delta^2 + \left(1 - \frac{\gamma^2}{4}\right)\Delta.$$

We present a (mathematically rigorous) probabilistic and geometrical proof of this relation. It uses the properly regularized quantum measure $d\mu_{\gamma,\varepsilon} := \varepsilon^{\gamma^2/2}e^{\gamma h_\varepsilon(z)}dz$, where $h_\varepsilon(z)$ denotes the mean value on the circle of radius ε centered at z of GFF h. When $\varepsilon \to 0$, this measure has both a limit, the Liouville quantum measure, and a Brownian representation in time $t = -\log \varepsilon$, of which KPZ appears as a martingale or large deviations property. The singular case $\gamma > 2$ is also shown to be related to the quantum measure $d\mu_{\gamma'}$, $\gamma' < 2$, by the fundamental duality $\gamma\gamma' = 4$.

This is joint work with Scott Sheffield (Department of Mathematics at MIT).