Contribution Title: VERTEX COUPLINGS IN QUANTUM GRAPHS
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(i) It is a well know fact that a general vertex coupling in a quantum graph vertex with n outgoing edges can be parametrized by n^2 real parameters. In 1999, Kostrykin and Schrader introduced a concrete parametrization, simple and clear, but with the defect of ambiguity. A year later, independently Harmer and Kostrykin & Schrader have shown that this parametrization can be made unique using a unitary matrix. However, in this unique form it is extremely difficult to see which role each of the n^2 real parameters plays, and moreover, even the couplings that are simple from the physical point of view, for example the δ coupling, have a very complicated expression here.

In this work we suggest an alternative way how to parametrize vertex couplings that combines the advantages of the both ways mentioned above. Namely, our parametrization is unique and at the same time it is relatively simple to understand the role of the involved parameters.

(ii) Although a general vertex coupling is well explored from the mathematical point of view, the explanation of the physical meaning of the full family of vertex couplings has not been given yet. Here we propose the following solution. We begin with a star graph with n arms and a *general* coupling in the vertex - the *approximated* system. Then we consider a system of n half lines whose endpoints are connected by lines of a length d such that there are δ couplings and δ interactions placed in the endpoints of the half lines and in the centers of the connecting lines; at the same time, the connecting lines are supposed to support vector potentials - the *approximating* system. Then we show that the approximated system can be considered as a limit of the family of approximating systems as $d \to 0$, and we support our claim by a proof that this convergence is in the norm-resolvent sense.