Contribution Title: RECURRENCES IN QUANTUM WALKS
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Recurrence in the dynamics of physical systems is an important phenomenon with many far reaching consequences. For random walks the recurrence is characterized by the so-called Pólya number which denotes the probability that the particle returns to the origin at least once during the whole time evolution. If the Pólya number equals one the walk is called recurrent, otherwise there is a non-zero probability that the walker never returns to its starting point. Such walks are transient. Pólya proved that one and two-dimensional walks are recurrent while for higher dimensions the random walks are transient and a unique Pólya number is associated to them in each dimension.

We extend the concept of Pólya number characterizing the recurrence of random walks to the quantum domain by a specific measurement scheme. We show that the Pólya number of a quantum walk depends in general on the choice of the coin and the initial coin state, in contrast to the classical walks, where the Pólya number depends on the dimension. Our analysis is based on the Fourier transformation and the method of stationary phase. We illustrate on several examples that one can achieve strikingly different recurrence behaviours for quantum walks. First, we show that for the class of quantum walks driven by tensor-product coins the Pólya number is independent of the initial conditions and the actual coin operator, thus resembling the property of the classical walks. Second, we analyze the 2-D Grover walk, which exhibits localization and thus is recurrent, except for a particular initial state. We employ the Grover walk to show that one can construct in arbitrary dimension a quantum walk which is recurrent. This is in great contrast with the classical walks. Finally, we analyze the recurrence of the 2-D Fourier walk. This quantum walk is recurrent except for a two-dimensional subspace of the initial state.