Contribution Title:	SPECTRAL ANALYSIS OF 1-D SCHRODINGER OPERATORS WITH LOCAL POINT INTERACTIONS
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Consider formal differential expressions

$$\ell_{X,\alpha} := -\frac{\mathrm{d}^2}{\mathrm{d}x^2} + \sum_{n\in\mathbb{Z}} \alpha_n \delta(x - x_n), \qquad \ell_{X,\beta} := -\frac{\mathrm{d}^2}{\mathrm{d}x^2} + \sum_{n\in\mathbb{Z}} \beta_n \delta'(x - x_n), \quad \alpha_n,\ \beta_n \in \mathbb{R}, \tag{1}$$

where $\delta(\cdot)$ is a Dirac delta-function and $\{x_n\}_{n\in\mathbb{Z}}$ is a strictly increasing sequence satisfying $x_n \to \pm\infty$ as $n \to \pm\infty$. Let $H_{X,\alpha}$ and $H_{X,\beta}$ be the minimal operators associated in $L^2(\mathbb{R})$ with the expressions $\ell_{X,\alpha}$ and $\ell_{X,\beta}$, respectively. The spectral properties of these operators are well studied in the case when the interactions sites are uniformly distributed, $d_* := \inf_{n\neq k} |x_n - x_k| > 0$ (numerous results as well as a comprehensive list of references might be found in the monograph of Albeverio, Gesztesy, Hoegh-Krohn, Holden, *Solvable Models in Quantum Mechanics*, AMS Chelsea Publ., 2005; see also a survey of recent result given by Exner in Appendix K).

If the assumption $d_* > 0$ is dropped, then the spectral analysis of operators $H_{X,\alpha}$ and $H_{X,\beta}$ becomes more complicated. For instance, for the operator $H_{X,\alpha}$ it is known only that it might be symmetric with nontrivial deficiency indices (the example of Shubin Christ and Stolz, *J. Math. Anal. Appl.* **184** (1994)).

The main aim of our talk is the spectral analysis of operators $H_{X,\alpha}$ and $H_{X,\beta}$ in the case $d_* = 0$. We show that spectral properties of the operators $H_{X,\alpha}$ and $H_{X,\beta}$ are closely related with the spectral properties of certain classes of unbounded Jacobi matrices. We exploit this connection to investigate self-adjointness, lower semiboundedness, and discreteness of operators with local point interactions.