Contribution Title:	CRITICAL POINTS AND LEVEL SETS OF SOLUTIONS TO ELLIPTIC PDES
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Invited speaker:	
YRS seminar:	NO

The study of the qualitative properties of the solutions to the equation

$$\Delta u = 0 \quad \text{in } \mathbf{R}^n \backslash \overline{\Omega}, \quad u|_{\partial\Omega} = 1,$$

which vanish at infinity was pioneered by Faraday and Maxwell in the 19th century. In physics, the level sets of the function u are equipotential surfaces, whereas its critical points correspond to the equilibrium positions for the motion of a charged particle in presence of the conducting surface $\partial\Omega$. The relevance of Faraday's lines of force, which are simply the integral curves of the gradient vector field $-\nabla u$, was unveiled in Maxwell's celebrated treatise, where they led to the first hints of what is nowadays known as Morse theory.

In this talk we will use related ideas to analyze some geometrical properties of the solutions to the above exterior boundary problem, Ω being a bounded domain with C^2 connected boundary. We prove that the critical set of u can be nonempty (in fact, of codimension 3) even when Ω is contractible, thereby settling a question posed by Kawohl, discuss sufficient geometric criteria for the absence of critical points in this problem and analyze the properties of the critical set for generic domains. Time permitting, related problems on Riemannian manifolds will be discussed as well. Our results hinge on a combination of classical potential theory, transversality techniques and the qualitative theory of dynamical systems, and are mainly extracted from the references

[1] A. Enciso, D. Peralta-Salas, Critical points and level sets in exterior boundary problems, Indiana Univ. Math. J., in press.

[2] A. Enciso, D. Peralta-Salas, Critical points and generic properties of Green functions on complete manifolds, submitted for publication.