

Contribution Title: LONG-TIME ASYMPTOTICS OF THE TODA LATTICE FOR ASYMPTOTICALLY PERIODIC INITIAL CONDITIONS

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Invited speaker:

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We consider the stability of periodic (and slightly more generally of algebro-geometric finite-gap) solutions of the doubly infinite Toda lattice under a short range perturbation. We prove that the perturbed lattice asymptotically approaches a modulated lattice. More precisely, let g be the genus of the hyperelliptic curve associated with the unperturbed solution. We show that, apart from the phenomenon of the solitons travelling on the quasi-periodic background, the n/t -plane contains $g+2$ areas where the perturbed solution is close to a finite-gap solution in the same isospectral torus. In between there are $g+1$ regions where the perturbed solution is asymptotically close to a modulated lattice which undergoes a continuous phase transition (in the Jacobian variety) and which interpolates between these isospectral solutions. In the special case of the free lattice ($g=0$) the isospectral torus consists of just one point and we recover the known result.

Both the solutions in the isospectral torus and the phase transition are explicitly characterized in terms of Abelian integrals on the underlying hyperelliptic curve.

Our method relies on the equivalence of the inverse spectral problem to a matrix Riemann-Hilbert problem defined on the hyperelliptic curve and generalizes the so-called nonlinear stationary phase/steepest descent method for Riemann-Hilbert problem deformations to Riemann surfaces.