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CHAPTER NINETEEN

Constitution of the Classroom Environment: A Case Study

INTRODUCTION

One of the central issues currently discussed is that of analysing the culture of the mathematics classroom (e.g. Seeger, Voigt, & Waschescio, 1998). The importance of the social dimension of learning is discussed in relation to an individual's ways of acquiring and using knowledge of mathematics. Laborde and Perrin-Glorian (2005, p. 2) stated

> … [the classroom] is the place of social interrelations between the teacher and students shaped by the difference of position of the two kinds of actors with respect to knowledge and giving rise to *sociomathematical norms* (Yackel & Cobb, 1996) or to a *didactical contract* (Brousseau, 1989, 1997).

Our analysis of the set of videotaped lessons in this chapter is based on the theory of didactical contract. The implicit nature of Brousseau's concept of 'didactical contract' is fundamental when explaining environment effects on learning mathematics (Sarrazy & Novotná, 2005).

BACKGROUND

Theory of Didactical Situations

In Brousseau's Theory of didactical situations (TDS) (Brousseau, 1989, 1997), teaching is seen as *devolution* of a learning situation from the teacher to the student. Brousseau (1975, in Warfield, 2005) described the learning process as follows:

> … a learning process can be characterized in a very general way (or even determined) by a sequence of identifiable situations (natural or didactical), reproducible and leading regularly to the modification of a set of behaviours of the students, modifications which are characteristic of the acquisition of a particular collection of knowledge.

D. J. Clarke, C. Keitel & Y. Shimizu (eds.), Mathematics Classrooms In Twelve Countries: The Insider's Perspective. 275–288.

Acquired knowledge can appear in many different forms. Knowing mathematics is not only knowing definitions and theorems and recognising where to apply them. Knowing also means doing mathematics, which includes solving problems, production and construction of models, formulation and justification of proofs and proving and so on.

Brousseau considers the conditions of a particular use of a piece of mathematical knowledge to form a system, which he calls a '*didactical situation*'. In *non-didactical situations*, the evolution of the learner is not submitted to any didactical intervention whatever. *Didactical situations* are situations in which an actor, for instance a teacher, organizes a plan for an action which is intended to modify or create new knowledge in another actor, the learner. Models of effective teaching combine the two approaches: didactical situations that are partially liberated from direct interventions are called *a-didactical situations.* In TDS, situations are classified according to their structure (action, formulation, validation, institutionalisation, etc.)[i] which determines different types of knowledge (implicit models, languages, theorems, etc.).

The process by which the teacher manages a didactical situation in putting the learner in the position of a simple actor in an a-didactical situation is called *devolution*. Devolution does not only propose a situation to learners which should provoke them to an activity not previously agreed, but also makes them feel responsible for obtaining a proposed result, and that they accept the idea that the solution depends only on the use of knowledge which they already have.

Environmental effects on learning mathematics are explained using Brousseau's concept of *didactical contract* presented in the 1980s, i.e. the set of the teacher's behaviours (specific to the taught knowledge) expected by the student and the set of the student's behaviour expected by the teacher. It equally concerns subjects of all didactical situations (students and teachers). This contract is not a real contract; in fact it has never been 'contracted' either explicitly or implicitly between the teacher and students and its regulation and criteria of satisfaction can never be really expressed precisely by either of them.

The interplay of relationships and constraints between the teacher and students may also produce certain unwanted effects and developments that can be observed (e.g. the Topaze effect, the Jourdain effect, metacognitive shift, the improper use of analogy). They are inappropriate for the learning (especially from the metacognitive point of view) but often inevitable. It is more their systematic use that is detrimental. In our analysis of videotaped lessons we will focus on the occurrence of two of these, the Topaze effect and the Jourdain effect (Brousseau, 1997). The occurrence of these effects in a teaching unit influences significantly the quality of the envisaged learning process.

The *Topaze effect*[ii] can be described as follows: When the teacher wants the students to be active (find themselves an answer) and they cannot, then the teacher disguises the expected answer or performance by different behaviours or attitudes without providing it directly. In order to help the student give the expected answer, the teacher 'suggests' the answer, hiding it behind progressively more transparent didactical coding. In this way, the teacher usually tries to achieve the optimum meaning to the maximum number of students. During this process, the knowledge, necessary to produce the answer changes.

The *Jourdain effect*[iii] is a form of Topaze effect. The teacher who, intentionally or unintentionally, does not want to admit student's lack of knowledge of an issue claims to recognize indications of scholarly knowledge in the behaviour or responses of a student, even though they are in fact motivated by trivial causes (such as analogy with a different problem, using lower mathematical knowledge, coincidence etc.).

DATA

In the Czech Republic data for this study were gathered in the eighth grade (students aged 14-15) of a junior secondary grammar school, the alternative to more academic education. The framework was based on the method used in Learner's Perspective Study (Clarke, 2001). The school is located in the county town České Budějovice (with approximately 100 000 inhabitants). The teacher was chosen on the recommendation of and by the agreement with the headmaster. This fact is of crucial importance for further analysis of her lessons. The observed teaching is rated as 'outstanding' in the school. Parents, teachers and professionals respect the teacher as one of the best mathematics teachers in the town. This is largely because her students are successful when passing entrance exams to institutions offering further education and parents do not have to help their children cope with the given homework and tasks. She is an experienced teacher approaching the end of her professional career. The fact that she agreed to being recorded reveals that she is confident in her professional skills.

THE CLASSROOM FROM A SOCIAL PERSPECTIVE

Let us now ask how the work of the students and their teacher can be characterized with regard to the theoretical framework described above. To put it simply, what kind of teacher's work is applauded by professionals, parents and students?

When evaluating the teacher's approach to students, the following questions were raised. Can we trace hidden didactical contract established in this particular classroom and illustrate it by suitable teaching episodes? What influence of the didactical contract on the pupils' mathematical knowledge can be presupposed? How does it support or constrain learning? How does the teacher create a secure, confident work environment for the students in the classroom?

TEACHING EPISODES

Episode 1: Didactical Contract and its Breach

The recognition of didactical contract in the classroom discourse is not easy. We can say that it can be best recognized at the moment when it is breached.

Let us illustrate this idea by the following episode. The main characteristic of the teacher's work is that she keeps returning to reasoning about rules that were taught and validated a long time ago. In our opinion, this is the source of students' confidence. The teacher even expressed that explicitly:

CZ1-L04, 00:37:57[iv]

T: It is important to have some system, step by step, and not, Vojta, to discover America again on your own! That's how you must cope with it. I also haven't come up with the procedure. It's not my invention. It has been tried and tested.

Students' expectations that the teacher would refer to previous knowledge that would be useful when solving the assigned problem sometimes led to mistakes. In Lesson 3, in assigning homework with equations containing mixed numbers (e.g. $5\frac{2}{3}$), the teacher did not refer to the students' former knowledge about mixed numbers. As a consequence of this, several students worked with them in an incorrect way. As a reaction to their mistakes the teacher explained again, in great detail, how to work with mixed numbers. We can observe that she took responsibility for the explanation herself, letting the pupils respond only to simple questions. She did this although the subject matter should have been well known:

CZ1-L04, 00:00:39

T 11: Let's start with the homework. You will have noticed that the equation written on the whiteboard is the equation from your homework. And some of you have made there a cardinal mistake! Not in the solution of the equation, I mean when using equivalent adjustments or anything like that, but a numerical mistake. And because I don't want you to make any similar mistake again, let's have a look at it again. It might be a good idea to open the exercise book and to check that the mistake isn't yours! [On the whiteboard, there is the equation from their homework $5\frac{2}{3}x-\frac{3}{2}=4\frac{1}{6}x+\frac{1}{2}$.] That you haven't made this mistake. Watch carefully [Points at the whiteboard.], do you know? Where the mistake has been made? Well? Those who have made the mistake have it marked in the exercise book. So they should know! Well here [Circles the fraction $4\frac{1}{6}x$.] and here [Circles the fraction $5\frac{2}{3}x$.]! What mistake could you have made?

Class 11: [Humming.]

T 12: Well! In that equivalent adjustment that you have written down here [Points at the beginning of the equation.], which could be done at once mentally, but I must take care doing it, you found the lowest common multiple, the common denominator, which is?

Class 12: Six.

T 13: Six. And you raised the whole equation by six. That was OK but! Five and two thirds doesn't equal five times two thirds and another cardinal mistake! That those of you who

	have made the mistake multiplied by six the wholes and also the fraction! If you remember when I first showed you equivalent adjustments I asked what the expression on the left side consisted of, Pavel? And you told me: it is a binomial, it is a trinomial. And how do terms differ from each other? Of the expressions? What is the difference? Hanka! How do I know what polynomial is on the left side?
Hanka 1:	It depends on whether there is a plus.
T 14:	Or minus, yes! So even if there was multiplication, it is still a monomial. But there is no multiplication, is there? It is neither five point two thirds, nor five times two thirds. What should you have done beforehand?
Class 13:	[Humming.]
T 15:	Say it out loud!
Jirka:	Transform it into fraction!
T 16:	Yes! To fraction! You needn't have written this down. You could have done it mentally, couldn't you? So how many thirds is this? [She points at the fraction $5\frac{2}{3}x$.]

Key to symbols used in transcripts in this chapter:

T12	Episode 1, the teacher's second utterance.
[text]	Comments and annotations, descriptions of non-verbal action.
…	A pause of 3 seconds or less.

Episode 2: Local Topaze Effect

The class were solving parametric equations. For their homework, the students had been assigned to solve the equation $dx + 1 = 2(4x + 1) – 5x$ by substituting the day of their birth for d. Some of them substituted 3 and then the equation had no solution. Together they tried to find a universal solution of such an equation. In the teaching episode, they adjusted the equation to the form $x(d – 3) = 1$.

CZ1-L05, 00:16:42

T 21:	What will I do with the equation now?
Class 21:	divide …
T 22:	Yes! I will divide it by that d – 3. And because there is a common number and I don't know its value, I must write down the condition because if the result by any chance was that I would have to divide …
Class 22:	by zero
T 23:	… by zero, I mustn't divide by zero, yes, you are already giving me the result [Hanka said 3 before], that wouldn't make sense, this adjustment would be nonsense, so, Hanka! What did you say?
Hanka:	d mustn't equal 3.
T 24	[Records this condition on the whiteboard.] That's why you got the result that you got [she means at home]. And I will finish it! So x must equal 1 over d – 3 [records on the whiteboard]. And now children! I will not only carry out the verification, but we will have to do one more step, which is called discussion. Because it is more complicated and because we are working with a parameter, the common number, so we have to carry out the discussion now. Which means we will have a look at how these two numbers work.

Let's try to substitute *d* by that 3, you already know it, you two have tested it already at home, but let's do it again, let's substitute it in the equation and see what will happen. So I will get here [points at the left side of the equation].
Class 23: 3*x*
T 25: 3*x* + 1. I will write that down; 3*x* + 1 should equal [points at the right side of the equation] here is no *d*, so I can do the calculation at once mentally as we have done before, so 8*x* + 2 – 5*x*, yes? We will solve it with what result, Michal?
Michal 21: … the result is …
T 26: Take your time with the calculations.
Michal 22: … 3*x*, the result is that zero doesn't equal one.
T 27: Yes, zero should equal one which isn't true, … so, we can write down the conclusion later and now! The other proof. So, the only number we eliminated was that 3. You tried some other numbers at home. You know what? Let's try whether in case we don't substitute only dates of birth but also for example decimal numbers or fractions, whether they could also be the result. Let's substitute *d* by for example what?
Class 24: 1.25
T 28: 1.25, why not, *d* = 1.25. So how will it work?

In the course of explanation the teacher provided the students with opportunities to fill in single words of her explanation, which they accepted, speaking in chorus (Class 21-24). The occurrence of the didactical contract mentioned earlier can be seen again in that the teacher, explaining the new procedure, kept referring to students´ earlier knowledge and relations to other topics (e.g. T 22, T 25).

In their homework, most students had forgotten to discuss the conditions for division by the expression $d - 3$. When checking the homework, the teacher tried to help them discover their mistake. In turns T 21 to T 23 the teacher used a local Topaze effect. For a certain period (locally), she replaced division by an algebraic expression by division by numbers which she discussed with her students. She drew their attention to the condition of a non-zero divisor. Then she returned to the original problem and related it to other content.

We can question whether the pupils were able to grasp the explanation albeit the discussion was influenced by the Topaze effect. In the post-lesson interviews with the pupils, the experimenter asked about that:

CZ1-L05 (post-lesson interview with Michal 00:01:03)

Exp. 1: Would you like to tell me anything about the homework?
Michal 1: … First of all, I did not know why the teacher assigned it to us, but when we went through it together, I knew the result $\frac{1}{3-d}$. We tried it before the lesson and one boy – Adam – showed it to me.
Exp.2: You knew why you were given that homework?
Michal 2: I knew.
Exp 3: And you knew the explanation?

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Michal 3:  He [Adam] didn’t tell us. We knew the solution, but we
           couldn’t explain it.
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The fact that a contract was at stake is confirmed in the post-lesson interviews with the teacher. Clearly, she feels responsible for her students’ grasping of the subject matter:

CZ1-L05 (post-lesson interview with the teacher, 00:11:21)

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T 1:       Well, it’s certainly not easy. My feeling is that they
           didn’t quite cope with it. Every equation will be an
           exception. I will have to go over it again.
Exp 2:     Those who substituted three got the result. They will have
           understood.
T 2:       It seems they discussed before the lesson.
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Episode 3: Creation of the Didactical Contract

In her lessons the teacher shows the importance of appropriate handling of the problem, its mathematisation and logical argumentation. She creates a belief that in mathematics all solution steps must be reasoned. To these ends she guides the students step by step giving clues to correct answers. This was manifested for example during the solution of the following problem from the textbook “Find two numbers whose sum is twenty and the difference of their squares is 120.”

CZ1-L07, 00:36:43

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T 31:      So … do they expect you to produce the result by heart? Not
           likely, is it? You will have to calculate that! So what?
           What shall we do with it?
Class      [Humming.]
T 32:      Luboš! I just don’t want to hear “I don’t know”! I won’t
           take an answer like that! So what should be done?
Luboš:     I would try it out! [Class laughs.]
T 33:      You would try it out! Oh my god! If you were, let’s say …
           six years younger, I would accept it! Such an answer.
           [Vojta puts up his hand.] Vojta!
Vojta 1:   A set of equations?
T 34:      Very good! That’s more mature. I will simply write it down
           and I will solve an equation! Even more, Vojta proposes a
           set of equations! How comes? Why?
Vojta 2:   There are two variables.
T 35:      Well, they could be there and still it needn’t be a set of
           equations! Denisa! Am I distracting you from something
           important? It needn’t be a set of equations! What is the
           value of the first number? I don’t know, x. So what is the
           other number?
Jirka:     20 – x
T 36:      Well done, Jirka! Twenty minus x. But you proposed a set of
           equations [points at Vojta], so let’s use a set of
           equations! Well, let’s write down the first number [writes
           on the whiteboard], now the other number. What is the value
           of the first number then? x, the other number therefore y.
           Lenka, dictate the first equation!
Lenka:     x + y = 20
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T 37: Read the rest of the assignment and dictate the other equation, Marek!
Marek: $x^2 - y^2 = 120$

One student, Luboš, proposed to solve the problem by trial and error[v]. It seemed as if the teacher was giving the students a free choice how to solve the problem, but actually what she expected was mathematisation via an algebraic expression, even with square numbers. She strongly refused the trial and error strategy (T 33). Students noticed it and did not propose it any more (even in the following lessons, this strategy was not used). If similar teacher behaviour occurs whenever a trial and error type of strategy is proposed or used by students, the unsuitability of the trial and error strategy becomes a part of didactical contract. Students will, as in this episode, use those strategies which are valued as ‘mathematically higher level’.

From the post-lesson interviews it could be seen that some pupils searched for their own ways of solution but respected the authority of the teacher:

CZ1-L07 (interview with Luboš)

Exp 1: You solved the problem with a system of equations. Is it convenient for you? Do you solve it in a different way?
Luboš 1: I tried it with the calculator. I knew the results within several seconds.
Exp. 2: Aha, you found it without an equation?
Luboš 2: Yes. Then I did it in the manner required by the teacher. It is simple.
Exp. 3: Would it be possible to solve it always with a calculator?
Luboš 3: Definitely not, an equation gives the whole result.

Other pupils did not hesitate about the correctness of the teacher’s statements:

CZ1-L07 (interview with Roman, 00:09:38)

Exp 1: You solved the problem with a system of equations, in the same way as you did it, or differently?
Roman 1: With a system
Exp. 2: Did you solve it by yourself or with the teacher?
Roman 2: By myself.
Exp. 3: Were you right?
Roman 3: Yes.

Episode 4: Topaze Effect

The Topaze effect can occur with different purposes:
(a) In order to avoid mistakes the teacher simplifies the tasks by
– recalling needed previous knowledge, or previous activities (e.g. T 13)
– breaking up the procedure into simpler steps (e.g. T 32-T 36)
– giving partial answers to the questions posed (e.g. T 25)
– posing ‘warning’ questions (e.g. T 401)
without always making explicit the relationship between the original task and the simplified one.
(b) In order to draw students’ attention to an occurrence of a mistake, the teacher ‘fails to notice’ the mistake when students perform it and continues with the

procedure in the form (a) until they reach a point where they recognise the false result. Then she draws their attention to the moment where the mistake was born.

CZ1-L03, 00:01:24

T401:	So write down! Square root 4*x* + 6 equals minus 4. No objections? … That's fine! You won't have any objections because you've had no experience with it so far. Otherwise some of you would object. And now you see, the variable, the *x* is under the radical. How can I get it from under there, how can I make it stand-alone? It would be easiest for us if the radical disappeared. If I could somehow remove it form the equation! And for those ends I could use the reverse mathematical …
Class 41:	operation
T 402:	… operation, which is? …
Class 42:	exponentiation …
T 403:	Exponentiation, and what do we do with the whole equation? Well? We say we raise it to the second power. The left side to the second power, the right side to the second power. So, what's the outcome on the left?
Michal:	The radical is cancelled.
T 404:	Well, the radical there is alone so if I raise it to the second power, I cancel it. And what remains, Michal?
Michal:	4*x* + 6
T 405:	Yes, equals the right side, also raised to the second power, Jirka!
Jirka:	16
T 406:	Speak up!
Jirka 2:	16
T 407:	16. Yes, that would be nifty, let's continue. … When we are doing these adjustments, what equivalent adjustment am I doing now? As an exception, let's write this adjustment down. Hanka?
Hanka:	We subtract 6
T 408:	Yes, only if you could speak up, we subtract six, from both the left and the right sides of the equation and so we get … Dominika!
Dominika:	4*x* = 10
T409:	Yes 10, that's it. And Dominika, finish it off …
Dominika:	… one *x* equals 10 quarters, which means …
T410:	five halves, yes? I will record that in the following manner, five halves, two and one half. So now you will think. All right, we have the root …
Pavel:	But it doesn't work!
T 411:	Excuse me?
Pavel:	But it doesn't work …
T 412:	What doesn't work? Speak up!
Pavel:	The verification doesn't work!
T 413:	You've already carried out the verification?
Pavel:	No, I haven't had time to do it but a radical can never have a negative result!
T 414:	Yes!!! This is what I expected you to object at the very beginning. But now, yes? Well, let's pretend nobody has noticed and let's carry out the verification, shall we? The left side of the equation equals, we substitute the result

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          into the equation! Four times five halves, how much is
          that?
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This unusual reaction to a mistake was confusing for the pupils as is again revealed in post-lesson interviews:

CZ1-L03 (post-lesson interview with Pavel 00:01:18):

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Pavel 1   I wanted to tell, that I objected to something, but I told
          it softly.
Exp 1:    What do you want to tell?
Pavel 2:  That the square, we said before, could not be … negative
          number … radix can not be minus …
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The teacher commented on the situation with surprise:

CZ1-L03 (post-lesson interview with the teacher 00:22:40)

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Exp 1:    One of the boys in the first bench said that he knew that …
T 1:      And why didn't he tell it?
Exp. 2:   He said he was afraid that it is not correct.
T 2:      You see. They should get it? I will return to it again.
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DISCUSSION AND CONCLUSIONS

Sarrazy and Novotná (2005) present three teaching styles (devolving, institutionalising and intermediary) which are in strong contrast with one another. The characterisation is based on the modes of teacher's actions described by the following three dimensions (variables v_1 to v_6):

- *Didactical structure of the lesson* (v_1. type of didactical dependence; v_2. place of institutionalisation; v_3. types of validation.
- *Forms of social organisation* (v_4. interaction modes; v_5. management with regard to the students' groupings)
- *Variability of the problem assignment* (v_6. teacher's 'capacity' to consider diverse modalities for the same didactical variable in editing the problem assignment)

Teachers with a *devolving style* use a strong variability in class management:

> These teachers regularly use group work without inevitably restricting to this form of students' grouping; generally speaking, the problems are complex; classroom work is very interactive (students interact spontaneously, 'choral' answers are not rare, …); in the lesson, institutionalisation is diverse. (Sarrazy & Novotná, 2005, p. 39).

Teachers with the *institutionalising style* mostly use the scheme 'show-remember-apply'.

> These teachers institutionalise very quickly one solving model and then present students with exercises of growing complexity. First, the exercises are corrected locally – the teacher passes through the rows and corrects them

> individually. Then the teacher gives the complete correction on the blackboard; here he gives details of the solution and, depending on the time he has, occasionally invites some students to the board either to make sure that they are paying attention, or to remind of certain knowledge. (Sarrazy & Novotná, 2005, p. 39)

The *intermediary style* combines features from both previous cases. "The students have more chances than those of 'institutionalising' teachers to encounter research situations, and debate, but markedly less than those exposed to the devolving style." (Sarrazy & Novotná, 2005, p. 40)

The teaching style of our teacher is intermediary. Referring to variables v_1 to v_6 we can say that:

v_1: In most cases, the teacher proceeds from simple to more complex tasks. If there are didactical reasons, she does not hesitate to insert a more difficult task in between.

v_2: She does not have a fixed place for institutionalisation in the meaning mentioned above. She uses her pedagogical and didactical experience to decide when institutionalisation is appropriate.

v_2: She tries to keep students informed about the validity of their results by using a stable means of evaluation. In the teaching sequence on solving linear equations the usual way of evaluating the results is by verification. Occasionally, she applies the validation by Topaze effect (see e.g. Episode 4) or by exchange (especially of written works) in pairs.

v_4: The prevailing interaction is between the teacher and one or more students. In this organisation, she offers students a wide space for participation in lessons.

v_5: The prevailing class organisation is whole class teaching. Individual work is included for practising of taught algorithms. Group work did not occur in any of the ten videotaped lessons. In order to avoid difficulties and mistakes, she offers a limited space for students' independent investigations.

v_6: She has sequences of problems where the mathematical formulation of the problem as well as the solution procedure needed for its solution are of increasing difficulty.

What can be said about the teacher's style and behaviour from this perspective?

– The selection of tasks and problems is the result of the teacher's rich, long term experience. Her ability to predict possible obstacles leads her to pose only questions that she expects students will be able to answer. She does not insist on individual justification. We can say that she concentrates on creating 'routine' skills. She leaves only a limited space for students' independent investigations.
– She creates a work atmosphere in the classroom. Most of the time, she works with the whole class at once. She asks questions and reacts to all student contributions. She also explicitly urges the students to be diligent and responsible in the dialogue, for example:

CZ1-L05, 00:10:45

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T:        The most important is that you want to. It won't just
          happen, Michael! Every success is hard work! Well, if you
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          are not willing to invest anything! To say ‘I can’t cope or
          I don’t get it’ is just an excuse. There must be some
          effort, some input!
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– If a student finds a mistake in the teacher’s calculation, reasoning or method, she is glad. She regards this correction as a proof of their attention; it shows that students are following her instruction, for example:

CZ1-L04, 00:41:17:

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Vojta:    There is a y missing.
T:        Where? I see! Here! Yes, thanks.
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– By asking certain questions, the teacher tries to eliminate students’ fear of the unknown world of ‘new’, non-standard tasks. In her lessons, she shows the importance of appropriate handling of the problem, its mathematisation and logical argumentation, for example:

CZ1-L04, 00:40:42:

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T:        Word problems. Let’s return to word problems again. We will
          again return to the word problems. How shall we solve using
          equation? How shall we begin? Denisa.
Denisa:   First we have to record it and set the unknown.
T:        And you will record it with the …
Student:  unknown …
T:        with the unknown. You express the relations with … if you
          solve it using an equation. Alternatively you might solve
          it using logical thinking. If you use an equation, it is
          this way.
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– From the interviews it was apparent that the students appreciated her approach:

CZ1-L10 (post-lesson interview 00:03:36):

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David:    I am quite good at those problems about cooperation, it is
          drill, they’re always the same.
I:        Even here where the assignment was not so unambiguously set
          as before? Have you solved this one also at once?
David:    About as fast as on the whiteboard, but I haven’t been
          copying it!
I:        I wonder if you needed help of the teacher.
David:    No. I checked from time to time with the whiteboard that I
          was proceeding well and I continued solving. … So, I
          checked this first part of the problem with the whiteboard,
          it was trouble. I did the second part on my own.
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– The teacher maintains students’ attention mainly by calling on any student in the class. When observing them, we found out that in reaction to this situation some students try to be a little in advance with their work, to ensure that when called out they will be able to respond without difficulty. However, it cannot be said that they do it in fear that their answer would be incorrect. The students called on do not seem to be afraid that their answer will be wrong or that they will not be able to solve the problem or individual work, because the atmosphere in the classroom is calm and the teacher’s reaction to a mistake is not rejecting or negative. The students accept positively even her gentle irony. For example:

CZ1-L07, 36:32:

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T:       Read it out, Hanka!
         [Hanka reads something else]
T:       Page 89, which means an 8 followed by a 9 [Other students
         laugh.]
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Sarrazy and Novotná (2005) raise the question of whether there is one 'best' teaching style. In this paper we have tried to demonstrate that it would be wrong to say that either the devolving or the institutionalising style produces the best results in developing students' knowledge of mathematics and their personalities. The recognition accorded to our teacher from among professional, parental and students communities confirms this conclusion.

The question of which of the six variables v_1 to v_6 has the biggest impact on the success of the teaching remains unsettled.

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NOTES

i *Situation of action*: the actor decides and acts on the milieu, it is of no importance whether the actor can or cannot identify, make explicit or explain the necessary knowledge
Situation of formulation: at least two actors are put into relationship with the milieu; their common success requires the formulation of the knowledge in question
Situation of validation: a situation whose solution requires that the actors establish together the validity of the characteristic knowledge of this situation; its effective realization thus depends on the capacity of the protagonists to establish this validity explicitly together
Situation of institutionalisation: a situation which reveals itself by the passage of a piece of knowledge from its role as a means of resolving a situation of action, formulation or validation to the new role of reference for future personal or collective uses. (Brousseau and Sarrazy, 2002, pp. 4-5)

ii The name comes from the first scene of Marcel Pagnol's play *Topaze* where Topaze is dictating to a weak student and suggests the spelling by concealing it in more and more transparent ways.

iii It is called so by reference to the scene in *Le Bourgeois Gentilhomme* by Molière, where the philosophy tutor reveals to Jourdain what prose and vowels are. The whole humour of the scene is based on the absurdity of repeatedly giving familiar activities the status of learned, scholarly discourse.

iv The transcripts from the classroom are labelled as follows: CZ1 (Czech school 1), L05 (Lesson 5), time of the start of the episode.

v By a trial and error strategy, the solution of the problem takes only a few steps, e. g. 10 + 10 (difference is unsuitable), 11 + 9 (difference unsuitable), 12 + 8 (difference unsuitable), 13 + 7 (difference 120).

REFERENCES

Brousseau, G. (1975). *Etude de l'influence des conditions de validation sur l'apprentissage d'un algorithme* [Study of the influence of the validation conditions on the learning of an algorithm]. Bordeaux: IREM de Bordeaux.

Brousseau, G. (1989). Le contrat didactique: le milieu [The didactic contract: The milieu]. *Recherches en Didactique des mathématiques*, *9*(3), 309-336.

Brousseau, G. (1997). *Theory of Didactical Situations in Mathematics*. (N. Balacheff, M. Cooper, R. Sutherland, V. Warfield, Eds. & Trans). Dordrecht: Kluwer.

Brousseau, G. & Sarrazy, B. (2002). *Glossaire de quelques concepts de la théorie des situations didactiques en mathématiques* [Glossary of some concepts of the theory of didactical situations in mathematics.]. Bordeaux: DAEST. Unpublished manuscript, (pp. 4-5).

Clarke, D. (2001). Complementary accounts methodology. In D. Clarke (Ed.), *Perspectives on practice and meaning in mathematics and science classrooms*. Dordrecht: Kluwer.

Laborde, C. & Perrin-Glorian, C. (Eds.). (2005). Teaching situations as object of research. Empirical studies within theoretical perspectives [Special Issue]. *Educational Studies in Mathematics*, 59, 1-3.

Sarrazy, B. & Novotná, J. (2005). Didactical contract: Theoretical frame for the analysis of phenomena of teaching mathematics. In J. Novotná (Ed.) *Symposium of elementary mathematics teaching* 05 (pp. 33-45). Prague: Univerzita Karlova v Praze, Pedagogická fakulta.

Seeger, F., Voigt, J. & Waschescio, U. (Eds.). (1998). *The culture of the mathematics classroom*. Cambridge: Cambridge University Press.

Warfield, V. (2005). Invitation to didactics. Unpublished Manuscript.

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