MULTIPLE PERSPECTIVES ON LANGUAGE AND MATHEMATICS: INTRODUCTION AND POST-SCRIPT

Candia Morgan, Institute of Education, University of London Konstantinos Tatsis, University of the Aegean Hana Moraová, Charles University in Prague, Faculty of Philosophy & Arts Jarmila Novotná, Charles University in Prague, Faculty of Education Margarida César, Universidade de Lisboa, Centro de Investigação em Educação FCUL

Birgit Brandt, Johann Wolfgang Goethe-Universität, Frankfurt a. Main Elmar Cohors-Fresenborg & Christa Kaune, Institut für Kognitive Mathematik, Universität Osnabrück, D-49069 Osnabrück

INTRODUCTION

Candia Morgan

The papers presented and the discussions of the Working Group on Language and Mathematics at CERME5 were marked by, on the one hand, diversity in the orientations and research foci of the various participants and, on the other hand, an interest in establishing dialogue and engaging with each other's questions, data and analyses. A concrete outcome of the opportunity to meet provided by the conference was an agreement to do some 'homework' resulting in new analyses, from several perspectives, of data presented in two of the conference papers. This introduction to the papers of the Working Group starts with an overview of the major themes emerging from the papers and from our discussions. It then presents the outcomes of the 'homework'.

In recent years, there has been increased recognition of the importance of language, not just as a means of communication but as a means by which we make sense of, or even construct, the world. This has led to a widening of the community of those within mathematics education who see language as a significant focus for their research and a consequent widening of the orientations of those choosing to participate in the Working Group on Language and Mathematics. Two main research orientations can be identified: study of the nature of language and its use in doing and learning mathematics and study of other issues, using language as a tool for addressing them. Within these two broad orientations there is also considerable diversity. For example, in studying the nature of language used in mathematics, choices must be made about the level of granularity at which the language is to be studied. The focus may vary from the nature and functioning of individual signs or small sets of signs, as seen in the work of, among others, Steinbring & Nührenbörger, Bloch and Farrugia, to consideration at a much more holistic level of the nature and function of writing in mathematical practices, as in Misfeldt's study of professional mathematicians or Stamou & Chronaki's analysis of the discourse of a magazine for school students. In between these two may be found studies of the functioning of language within spoken or written discourse and its contribution to the construction of mathematical meaning (e.g. Boero & Consogno). Where language is studied as a vehicle to address other issues of primary interest, there is perhaps even more space for diversity. Among the papers presented here, we encounter studies of student attitudes and beliefs (Perkkila & Aarnos), of the development of socio-mathematical norms (Edwards) and their influence on problem solving (Tatsis), and of assessment (Björklund Boistrup). A concern with the nature of learning environments that may facilitate learning is apparent in studies of linguistic activity and interaction in classrooms (e.g. Fetzer, Brandt) and in Ascione & Mellone's experimental study. The development of methodological approaches to the analysis of linguistic data were also offered in the papers by Koichu (for studying cognitive processes in problem solving) and Cohors-Fresenborg & Kaune (for identifying and categorising metacognitive activities).

The important role that linguistic activity (and that involving other sign systems) plays in the construction of mathematical meaning is widely recognised and many of the contributing authors present analyses of written texts and verbal interactions that contribute to our understanding of aspects of this. The social and collaborative aspect is of particular interest as we focus on interactions and learning that takes place in 'natural' classrooms rather than in laboratory or interview settings. At the same time, it must be recognised that participants in interactions do not always successfully collaborate to construct coherent meanings. Several contributions identify problems in communication that may constitute 'obstacles' to learning (Petrová & Novotná, Roubíček, Slezáková & Swoboda), while Farrugia identifies 'clarity' in teacher's speech as one of the keys to student learning and attempts an analysis of its characteristics. Students' acquisition of mathematical language is clearly an important aspect of communication that may support learning, yet, as Meaney demonstrates in her analysis of the role of authority in communications between children, teachers and parents, developing competence in use of the mathematics register itself raises difficult issues. The relationship between linguistic activity, linguistic competence and mathematical learning and competence is complex and as yet unresolved. When we consider students' mathematical competence, to what extent is their linguistic competence a part of this? And conversely, when we analyse interaction in a classroom or in an interview, how is this interaction affected by the mathematical aspects of the context?

An interesting development in the past few years has been the increasing attention to alternative, non-linguistic sign systems. We noted in the introduction to the proceedings of the Working Group at CERME4 (Morgan, Ferrari, Johnsen Høines, Duval, 2006) the importance of recognising and analysing the nature and roles of algebraic notation and geometric diagrams as well as 'natural' language. While work on these aspects continues, several contributions to CERME5 go further to consider other non-verbal sign systems such as gesture, body language and gaze. Björklund

Boistrup, in particular, develops a multi-modal approach to study interactions between students and teachers, taking all of these into account in addition to spoken language. As researchers develop this work with multi-modal data, the tools that are available for analysing the various modes need to be integrated and coordinated to ensure theoretical consistency.

Multi-modality is of increasing interest within mathematics education and elsewhere, especially in the context of new technologies that provide new types of signs and ways of interacting with them. It is perhaps surprising that none of the present contributions address this aspect of language, as influenced by new technology, though the papers by Back & Pratt and Pimm, Beatty & Moss consider the nature of interactions in text-based on-line environments. It may be that the multi-modal opportunities offered by technological developments are currently considered of specific interest to those concerned with the use of new technologies. As the field matures, providing more developed tools for analysis of multi-modal discourse, and as new technologies become more fully integrated into mathematical teaching and learning situations as well as into our everyday lives, it will become increasingly difficult to restrict our research focus to the more conventional and familiar mathematical sign systems.

Among other methodological issues discussed, the selection, status and treatment of data seemed particularly significant and in need of explicit clarification. Many of the papers present 'episodes' of data from classroom interactions. 'Episode', however, may be simply a fragment, perhaps chosen to illustrate a point, or it may be more 'logically' defined by its content, its interactional features or its crucial significance. The nature of episodes presented in papers is not always made explicit to the reader, yet must make a difference to the way in which the results of their analysis may be understood: as raising issues or hypotheses; as 'slices' of a developmental process that has been studied more extensively; as representative broader phenomena. It was suggested that there may be a case for complementing the use of detailed fine grained analysis of 'episodes' with larger scale quantitative approaches.

The fine grained analysis in many cases makes use of transcriptions, yet transcriptions do not necessarily provide a good representation of an episode of semiotic activity, often neglecting prosodic features as well as the coordination of linguistic with visual or physical modes. Researchers need to consider the rigour and scope of their methods of transcription. There are several well-developed sets of conventions employed by linguists for transcription. Some of these may help us to be more rigorous in representing speech but it is important to ensure that any conventions adopted are adequate to capture those features of speech considered to be significant and that the methods and conventions used match the theoretical assumptions of the research. When other modes of communication are also to be considered, the task of representing them is further complicated. Another role of technology discussed in the Working Group may provide one way of beginning to address this problem by making research tools available to us that enable us to have a

fuller view of an episode and, indeed, to ask new questions. For example, digitised video technology allows us to gather and examine more complex multi-modal data and allow us to analyse both temporal and spatial relationships between gestures, visual representations and speech. The work of Bjuland, Cestari & Borgersen begins to make use of such technology to analyse student reasoning during problem solving, as expressed through gesture and spoken language.

Perhaps as a consequence of the diversity of our backgrounds (both cultural and disciplinary), discussions were marked by simultaneous interest in the substantive research questions and findings reported by the presenters and in the methodological and theoretical issues raised. Thus, in considering the use of signs and language in meaning making, it was important to ask not only how students use signs in order to make mathematical meanings but also what linguistic and semiotic knowledge is useful to us as researchers in interpreting meaning making. By making use of different sets of theoretical constructs, different insights emerge. A shared interest in exploring these theoretical and methodological differences led to an agreement to continue working on this issue after the conference by preparing complementary analyses from different perspectives of some of the data presented. Episodes originally analysed and presented in the papers by Cohors-Fresenburg & Kaune and by Boero & Consogno were chosen for this treatment. The following sections of this paper include four brief complementary analyses by Tatsis, Moraová & Novotná, Margarida César and Birgit Brandt of an episode presented in the paper by Cohors-Fresenborg & Kaune. (For convenience, the episode in question is reproduced as an annex to this paper.) This is followed by a complementary analysis by Cohors-Fresenborg & Kaune of data from Boero & Consogno.

USING POLITENESS THEORY TO ANALYSE A CLASSROOM DISCUSSION

Konstantinos Tatsis

The linguistic analysis of classroom interactions can be used as a tool to better comprehend these interactions and then better organise the didactic approach. Cohors-Fresenborg & Kaune's analytic approach addresses the important question set by Candia Morgan during the Language and Mathematics Working Group meeting: When we analyse interaction in a classroom or in an interview, how is this interaction affected by the mathematical aspects of the context? In order to better comprehend the interactions involved in any setting (including classrooms) one needs to consider all aspects that influence in one way or another what is said and what is done. The most important aspect that affects people's behaviour is "face", i.e. "the positive social value a person effectively claims for himself by the line others assume he has taken during a particular contact" (Goffman, 1972, p. 5). Face is further categorised into positive and negative: positive face is related to a person's need for social approval, whereas negative face is related to a person's need for freedom of action.

Each person does not only have these wants her/himself, but recognises that others have them too; moreover, s/he recognises that the satisfaction of her/his own face wants is, in part, achieved by the acknowledgement of those of others. Indeed, the nature of positive face wants is such that they can only be satisfied by the attitudes of others. These views are in the core of "politeness theory" as expressed by Brown and Levinson (1987) and used by Rowland (2000) and will be the theoretical base for the analysis that follows.

Each verbal act can be categorised according to its effect on the speaker or the hearer's face. Some acts ("face threatening acts", or FTAs) intrinsically threaten the hearer's face. Orders and requests, for example, threaten negative face, whereas criticism and disagreement threaten positive face. Each person must avoid such acts altogether (which may be impossible for a host of reasons, including concern for her/his own face) or find ways of performing them whilst mitigating their FTA effect, i.e. making them less of a threat. Imagine, for example, that a student says something that the teacher believes to be factually incorrect; the teacher would like to correct him/her. Such an act would threaten the student's positive face; thus, the teacher has to employ a particular strategy in order to minimise the potential FTA effect.

The discussion contained in Cohors-Fresenborg & Kaune's paper is very interesting because it contains many instances of potential FTA acts, which are successfully resolved by the speakers. In 5-6 Mona supports her claim about the existence of a particular figure and the teacher, knowing that this figure does not really exists, asks for a numerical representation of it; she begins her request with the modal form "Could you please" in order to minimise the threat to Mona's negative face. Mona initially admits that it is not possible, but tries to support her view in two ways: she uses "you" on an attempt to make her claim impersonal (i.e. it is not her own inability, but a general one); then she utters that "logically it would be possible", which suggests that her claim is logical and reasonable (this utterance can refer to a possible sociomathematical norm established in the particular classroom, i.e. that a mathematical proposition is expected to be logical in order to be acceptable). In 12-13 the teacher tries to raise the others students' interest in Mona's claim; this is a FTA to Mona, that is why she immediately replies (although not asked) by using once again the impersonal "you" (14) in order to assign a general character to her claim. Suse (17-21) only repeats Mona's claim and the teacher utters "Yes" not as a sign of acceptance, but as a way to encourage more students to participate in the discussion; that is why she uses the first plural person ("let's") in her prompt. Suse (24-31) refers to Peter's and Mona's claims by using many times the impersonal "you" in order to distance herself from both of them; this is done in order to minimise the threat to her own positive face, in case they prove faulty. Mona eventually realises that her initial claim is not grounded; she begins by using the shield "Well" and gradually she admits this fact. It is interesting to observe that Mona was led to withdraw her initial claim without any interference on behalf of the teacher; this is a sign of a student who observes the sociomathematical norm of justification (for a more detailed discussion

on social and sociomathematical norms see Tatsis, this volume), which is important for a fruitful mathematical discussion.

What the above analysis demonstrates is an alternative way to look into mathematical discussions; students and teachers always adopt particular strategies to save their (or their hearer's) face. Moreover, we can use such an analysis to examine the teachers' and the students' attempts to generalise and to justify but with the minimum effect towards their own and the others' face. The educator who is aware of these strategies can better organise the discussions, and particularly his/her own verbal strategies towards smooth and productive mathematical interactions.

DISCOURSE ANALYSIS USING PRAGMATICS

Hana Moraová & Jarmila Novotná

Pragmatics is one of the three divisions of semiotics (together with semantics and syntax). It studies language from the point of view of the user, especially of the choices he/she makes, the constraints he/she encounters in using the language in social interaction, and the effects his/her use of language has on the participants of an act of communication. It focuses on language in use and relatively changing features of conversation. It studies continuous wholes (for more information see Leech 1983).

We believe that this approach is suitable for analyses of teaching episodes as it enables us to see why the participants of the communication behave in a particular manner and what the possible sources of misunderstanding may be or why individual contributions may seem "clumsy", illogical or confusing.

At the core of the analysis are major principles and their maxims, which in normal speech situation are expected not to be violated by the participants. If they are violated, it brings confusion or misunderstanding. Also, the different principles may be in opposition to each other which can cause that if one of the principles is obeyed the other violated. (E.g. the politeness principle is often in conflict with the cooperative principle – namely the quality and quantity maxims.)

In this contribution we only refer to those principles and maxims that are relevant to the particular transcript.

Analysis of the episode

Cooperative principle (for more information see Grice 1975)

- *Quality maxim* (try to make your contribution one that is true, do not say that for which you lack adequate evidence) is often violated; however this is not surprising as the conversation is from a lesson where students are expected to reason, deduce, search and will say things without having sufficient evidence for it; it happens that only after some time they realize their original assumption was wrong (Mona's assumption that a number between 0,99... and 1 exists is untrue, but progressive in the course of the lesson - 1. 5).

- *Quantity maxim* (make your contribution as informative as required for the purposes of the exchange, do not make it more informative than is required): Mona's only "valuable" contribution is on line 5-6, then she keeps repeating the same idea: "logically you can imagine but you cannot write it down" (l. 8, 10, 31, 33) and thus brings no new information into the exchange.
- *Relevance maxim* (make your contribution relevant): Again, Mona's later contributions become more or less irrelevant as they are not informative and do not move the communication forward. Also Juli's turn (1. 36-37) is irrelevant to the course of the communication as a whole. However, she reacts to the teacher's question which springs out from the non-verbal reality of the teaching episode.
- *Maxim of manner* (be perspicuous and specifically avoid obscurity, ambiguity, be brief, be orderly): The teacher thinks that Mona on l. 5-6 is violating this maxim and therefore she asks her to write what she means on the board to explain the ambiguity/unclearness. There is no doubt that Suse is violating this maxim. Her turns are very long, she needs many words to express one idea, and there are repetitions and it takes her a long time before she gets to the point. (l. 17-30) What she basically says in her 14 lines is: "Peter's solution is right because it works with different numbers and Mona's number cannot be recorded and therefore doesn't exist." However, her turns always move the conversation forward.

(*Implicature*, i.e. what is inferred as additional meaning but not worded): Suse is in the position of an "arbiter"; she evaluates Peter's and Mona's ideas, says who means what and why this or that should be correct; in a way she seems to be stepping in for the teacher, as if the teacher could not understand.

Politeness principle:

- *Tact maxim* (minimize cost to others): A typical example in speech is the teacher's use of questions (l. 7, 34-35) and indirect questions (l. 12-13) rather than imperatives. These statements are obviously meant as commands.
- Agreement maxim (minimize disagreement, agree at least in part): Suse often obeys these principles at the cost of cooperative principles. One of her turns begins "This is what I wanted to say ..." (l. 17) as if she agreed with Mona but ends "you cannot write it down" (l. 21) ... "Thus a figure doesn't really exist." (l. 29) Also on l. 30 she says "...this could be right" although she basically means "this is right". The conditional is used here not to hurt Mona.
- *Sympathy maxim* (minimize antipathy between self and others) is manifested by Mona, e.g "I <u>only</u> meant" (1. 31).

ANALYSIS OF CLASSROOM DISCUSSIONS

Margarida César

The first thing that strikes us is that these students are already used to participate in this type of general discussions. This is illuminated by the way they react to their peers' interventions, trying to (re)interpret them, or complete and/or clarify what they stated, and also by the few times the teacher chose to make her interventions. This discussion shows part of the didactic contract of this class. This teacher is giving the students time and space to participate as legitimate participants (Lave & Wenger, 1991) and she is trying to develop a learning community. But this general discussion also illuminates the existence of an intersubjectivity that was developed between this teacher and her students (e.g., they all talk about the figures inbetween, and they know what they are referring to).

In this discussion there are two groups of argumentations: (1) the ones who argue that 0.9(9) = 1 is true (Peter, Suse, Jens); and (2) those who argue that this should not be true (Mona). But the point of this discussion was not merely finding a solution to this mathematical task. If that was what this teacher had in mind, students would not be used to this kind of general discussion. What this teacher wanted to do was to explore students argumentations and to facilitate students' appropriation of mathematical knowledge through discussion, i.e., through the diverse argumentations and confrontations that were elaborated by the students. This is, in our interpretation, why there are no evaluative comments on her talks. Even when she is trying to control Juli and Judith's behaviour (Lines 34 and 35), she does not produce an evaluative comment, and she does not use an imperative verbal form. She just tells them that everyone needs to be able to hear them, which is a particular way of interacting with students and making them pay attention and participate.

This general discussion illuminates different levels of cognitive development and also different levels of mathematical argumentation. Although most students use formal reasoning in their statements, Mona is probably at an interface between concrete and formal reasoning. This is probably why she believes there is another figure between 0.9(9) and 1, but also why she needs to go back to a more concrete description of that figure ("many many zeros", instead of "zero point infinite zero and then one"), but also why she needs to make the distinction between what can be said/thought (the figure she imagined) and what can be written down/drawn (Lines 8 to 11). And for her there are mathematical (logical) entities that can be imagined, that exist logically, but which can not be written down. According to her Talk 6 (Lines 14 to 16) she does not seem to have recognised any error in her previous statements. Probably the laughter (Line 10) is more a nervous sign than the recognition of a mistake. She seems to be trapped because she can imagine that figure – and the figure is very clear to her, mentally – but she is not able to write it down and she knows the rules of their game: if a figure cannot be written, then it does not exist. But for her, that figure could have a logical explanation, and according to her argumentation logic should be accepted in Mathematics. This is why we interpret her laughter as confusion, disappointment, and not as the recognition of a mistake. Even after Suse's intervention in Talk 8 (Lines 23-30) Mona still thinks that figure exists logically, it just cannot be written down, and that is why it would not work. But she never claims that the figure she imagined would not be a periodic continued one.

Thus, Mona seems so taken by imagining the figure that would confirm her hypothesis that she forgot what is a periodic continued. She seems to be moving from concrete reasoning into formal one. As she is making an effort to imagine the figure between 0.9(9) and 1, she forgets the notion of periodic continued, that should be taken into consideration. But this way of reasoning – concentrating on one feature and not taking into consideration the others – is also very typical of concrete reasoning. It cannot be taken simply as a mathematical error, or lack of mathematical knowledge.

Suse is clearly using formal reasoning in her argumentations. She is able to make transitions between her own way of reasoning and Mona's argumentation; she is able to use Mona's language and then transforms it into more accurate mathematical language (Lines 23 to 30) and she is also able to use other examples to make her point clearer (Lines 24 to 26). She is also the one who explains to Mona that if we have a periodic continued, suddenly there is not a one in the middle of the zeros (Lines 39 to 41). Thus, she is the one who is able to argue in such a way that Mona will understand her point. And although Jens had also used a similar argumentation in his talks (Lines 1 to 4; and 38), he used his own argumentation and he did not relate directly to Mona's doubts/ difficulties. Thus, it was through Suse's interventions that Mona could be aware of some weaker arguments she used and replace them by more robust ones.

Just taking in consideration this small piece of interaction, I would say, if we wanted to use it for teacher evaluation, that her way of acting is very consistent and that she is able to develop students' participation, level of argumentation, respect towards each others' argumentations and autonomy. And these are competencies students need in order to succeed in evaluations (namely the most formal ones, like tests and exams) and also in their professional life. Moreover, she is able to facilitate students' mathematical development, as they do not merely repeat answers or rules they do not interpret, but they are developing their relational knowledge (Skemp, 1978).

THE PRODUCTION DESIGN OF "A FIGURE IN-BETWEEN"

Birgit Brandt

In Brandt (this volume) I outlined our concept of participation in mathematical classrooms (Krummheuer and Brandt 2001), which traces back to Goffman and Levinson. With respect to the interactional theory of learning mathematics, the main focus of our approach is the emerging process of 'taken as shared' meanings, which

includes the alternating of the active speakers and the interweaved emerging of the subject matter. Applying the production design to the transcript of Cohors-Fresenborg & Kaune, I will point out this interweaving for the interactive argumentation by the formulation "a figure in-between".

In the beginning, Jens refers to Peter, but he does not address him as a dialog partner – Peter is only one recipient of the broad listenership. Jens' contribution can be seen as a recapitulation and appreciation of Peters statements, but due to the presented extract it is not possible to decide about the production design of his utterance in detail. The argumentative *ideas* of Jens utterance are

- Between two digital numbers must be at least one figure.
- There is no figure between $0.\overline{9}...$ and 1.
- Therefore $0.\overline{9}=1$ is logical.

In the ongoing interaction, these *ideas* are linked to Peter (e.g. [24]). So, Jens is surely not an *author* of all aspects of his utterance, but probably for the evaluation of this argument as logical. In contrast to Jens, who stresses his conformity to Peter, Mona emphasizes her autonomy. She explicitly refers to her responsibility (I do think), but she links her utterance to Jens's *formulation* that there always has to be a figure in-between [2]. She takes this part as a *ghostee* (that here is a figure [5]), and as an *author* she supplements a figure in-between 0.9 and 1, that doesn't exist [9, 16]. With her construction zero point infinite zero and then a one, some time or other [6] (and [16] as a *spokesman* of herself) she describes her certain *idea* of "a figure inbetween", which she makes more explicit later as a *spokesman* of herself "I meant the figure that you would need in order to make zero point periodic continued nine a one" [14]. This *idea* of "a figure inbetween" refers to the conception of real numbers as length of lines. Summarizing her several statements, these are the *ideas* of her argumentation:

- There must be a figure in-between in the sense of $0.\overline{9}+x=1$.
- The (not existing) figure zero point infinite zero and then a one (at the end of the unlimited figure) can be thought as this figure in-between $0.\overline{9}$ and 1.

At first, Suse is a *spokesman* of Monas *ideas* [17]. Subsequently, she continues with an additional example for a number in-between (three is in-between two and five in [24]). This can be seen as an application of Monas *idea* as mentioned above, but as a *ghostee* she uses this for an extension of Peters argumentation: There is no number in-between $0.\overline{9}$ and 1 in this sense, because Mona figure doesn't exist [30] (this is amplified in [36-44]).

First of all, Jens uses the *formulation* "a figure in-between" for his summery of Peters argument, but without clarifying his concept of in-between. Taking this *formulation* for a counter-argument, Mona explains more and more precise her *idea* of in-between. At the end, Suse ties up to Monas *idea* as a backing for Peters argument.

This interweaving is retraced by the reciprocal referring as *spokesmen and ghosthees*. Overall, the interaction process features the criteria of an "Interaktionale Verdichtung" (Krummheuer and Brandt 2001; "condensed period of interaction" Krummheuer 2007) – hence this interaction process provides optimized conditions for the possibility of mathematical learning.

REMARKS ON BOERO & CONSOGNO

Elmar Cohors-Fresenborg & Christa Kaune

Boero & Consogno (this volume) show how increasing mathematical knowledge can be constructed by social interactions. The mechanisms described by them are especially promoted in a discursive teaching culture. Activities like monitoring and reflection play a particular role. It is therefore obvious to analyse their transcripts also by means of the category system, which has been developed by Cohors-Fresenborg & Kaune (this volume) for the analysis of discursive and metacognitive activities. The connections of differing theoretical frameworks is meant to show exemplarily how scientific development in mathematics education can be promoted by international co-operation.

The categorisation of the two following transcript extracts are visually supported by colours, i.e. discursive activities are green, monitoring activities red and reflective activities ochre. Statements which do not match any of the categories remain black.

	Elisa:	I agree with Mattia, as he considers the results.	<u>DS2a</u> rMS4a
2	Giulia:	Mattia has considered all possibilities, because he has considered the two dice and has put the results and (I think) has looked at all possibilities.	<u>DS2a</u> rMS4b
4	Teacher:	Is it the same thing to think of the result or to think of the two dice?	<u>DT1a</u>
	Mattia:	It is the same thing no yes!	MS8c
6 8	Giulia:	If you think of dice to the digit shown by your dice because the result is one digit plus another digit that makes a result. Before adding them, those two numbers are alone, they are not together because if one casts 3 and the other 4.	DS2c MS8b
10 12	Roberto:	for instance, 4 is a number and 3 is another number, as Giulia told , if you add them, they make 7, but before putting them together, 4 is a solitary number and 3 is another solitary number, then when they go together we get a number formed by smaller numbers.	DS2d DS2a
	Giulia:	yes, but before getting the result, the two numbers can be other numbers.	rMS4c

The meaning of discursive activities for the social construction of knowledge can primarily be recognised in our analysis by a high share of green colour of the pupils statements, especially because it is solely sub-categories of DS2 that appears. This points to an "embedding of discursive contributions". The only intervention by the

DOO

teacher (line 4) is to be considered as an educational action, i.e. the invitation to a discourse (DT1a).

The red colour, which is the only other colour apart from green, shows that all other pupils' contributions are monitoring activities, i.e. careful supervision of their own (MS8) or other argumentations (MS4). The high share in founded metacognitive activities - marked by a prefixed r (*reasoning*) - is striking.

In the second transcript as well, most of the assigned categories belong to discursive activities. The sub-categories are spread similarly to the first transcript extract. The monitoring activities often contain reasons. A reflecting evaluation of a proceeding (RS6a) is new (lines 9-12).

2	Anna:	I agree with Giulia that in the 2-paths labyrinth you get out earlier, in the case of the 6-paths labyrinth you must try all the paths and you spend a lot of time.	DS2a
4	Matteo:	But in the 6-ways labyrinth you do not need to try all the paths, because for instance the first time you fail the exit, but then at the second or third trial you may find the good way to escape You don't need to try all the paths!	rMS4c
6 8	Giovanni:	It is necessary to consider the condition posed by Giulia, namely that there is only one exit, otherwise all the paths might have an exit, and it would not be a labyrinth any more!	DS2c DS2d rMS4c
10 12	Mattia:	If a labyrinth would have more exits than paths with no exit, practically it would be very easy to escape, on the contrary if the labyrinth has the same number of paths and exits, it would be easier but the exits must be more than one half of the number of the paths.	RS6a
	voices:	less than one half!	MS2a
14	Teacher:	<u>I would like Mattia to repeats his sentence - please, listen to him, then we will discuss what he said.</u>	<u>DT1a</u>
16 18	Mattia:	Can I make an example? In the 2-paths labyrinth there is one exit, while in the 6-paths labyrinth there are 3 exits; in order to make the 6-paths labyrinth easier than the 2-path labyrinth, you must put exits to more than one half paths, because if in the other labyrinth there are two paths and one exit, it is one half.	DS2c

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APPENDIX

2	Jens:	but Peter says that zero point nine is the same as one and that otherwise there always has to be a figure in-between that there has to be at least one figure between two decimal numbers. And in this case, it isn't and therefore it is logical that this should actually be correct.	DS2d MS8b MS4a MS4b
6	Mona:	Well, I do think that there is a figure. It may, however, be zero point infinite zero and then a one, some time or other [she laughs]	DS2f DS5d MS8b
8	T.:	Could you please write it on the board, how you imagine this (figure)?	<u>dRT5e</u>
10	Mona:	No, not really, as a figure it doesn't exist you can't write it down in this way. But logically it would be possible. [The following 38 sec. (Mona's slip of tongue) have been deleted.]	MS4c rRS7
12	Т.:	<u>I would like to know if it is at all clear to everybody what Mona wanted to say, about what figure she has been talking.</u> She said: "I cannot write it down."	<u><i>d</i>DT2c</u> <u>DT2d</u>
14 16	Mona:	Well, I meant, the figure that you would need in order to make zero point periodic continued nine a one. That's the figure I have been talking about. If first there are many, many zeroes and then at some time or other a one, but this doesn't exist in principle.	DS2c MS8c
18 20	Suse:	This is what I also wanted to say: There is an infinite number of nines behind the zero, hm, periodic continued nine, and she thinks that, that there should be a figure which has exactly as many zeroes, which means infinite zeroes, well there is a one at the end so that, if you add it, you obtain one. That's the figure she is looking for. But you cannot write it down because there would have to be an infinite number of zeroes.	DS2d MS8b DS2d rMS4c
22	T.:	Yes, let's pick up some more ideas. You said, we have got two positions. () [No reaction of pupils during the next 14 seconds.]	DT5c
24 26 28 30	Suse:	I would say Peter's solution is correct, because, if you take different figures instead, if you take for example five and two instead of zero point periodic continued nine and one, you know that they are not the same, as three is between them. And regarding zero point period continued nine and one, there is no figure between them. You know you cannot write down a figure. Mona, however, thinks that there should be this "zero point period continued zero one"-figure, but you cannot write it down. Thus, it doesn't really exist a figure. And therefore this could be right. Mona.	rMS4b DS2b DS5d rMS4c DS1a
32 34	Mona:	Well, I only meant: the figure doesn't exist, but logically you could imagine it so [laughter] That it could exist. Therefore [murmuring] but this figure doesn't exist. Hm, that is clear. It doesn't work.	DS2d RS7 RS6a
	T.:	Say it loud, please, Judith and Juli?	
36	Juli:	Yes, Judith and I are just trying to imagine <u>the figure "zero point periodic continued</u> <u>zero one"</u> , but this is somehow weird.	RS7 DS2b
38	Jens:	I think that there cannot be a further figure behind a periodically continued figure.	MS2b
40	Suse:	Yes, that is right, yes, that is true, because the zero, hm, because the periodic line is above it, which means, it is the zero that always repeats. Thus there cannot suddenly be a one behind it.	rMS2b MS8c DS2c
42 44		Which means this figure doesn't exist. If at all, the periodic line would have to be above both figures, and then it would be continued in that way: zero point zero one zero one zero one. This would not be the figure <u>Mona meant</u> .	RS6a RS3a RS6a DS2c

2	Jens:	but Peter says that zero point nine is the same as one and that otherwise there always has to be a figure in-between that there has to be at least one figure between two decimal numbers. And in this case, it isn't and therefore it is logical that this should actually be correct.	DS2d MS8b MS4a MS4b
6	Mona:	Well, I do think that there is a figure. It may, however, be zero point infinite zero and then a one, some time or other <i>[she laughs]</i>	DS2f DS5d MS8b
8	T.:	Could you please write it on the board, how you imagine this (figure)?	<u>dRT5e</u>
10	Mona:	No, not really, as a figure it doesn't exist you can't write it down in this way. But logically it would be possible. [The following 38 sec. (Mona's slip of tongue) have been deleted.]	MS4c rRS7
12	Т.:	I would like to know if it is at all clear to everybody what Mona wanted to say, about what figure she has been talking. She said: "I cannot write it down."	<u>dDT2c</u> <u>DT2d</u>
14 16	Mona:	Well, I meant, the figure that you would need in order to make zero point periodic continued nine a one. That's the figure I have been talking about. If first there are many, many zeroes and then at some time or other a one, but this doesn't exist in principle.	DS2c MS8c
18	Suse:	This is what I also wanted to say: There is an infinite number of nines behind the zero, hm, periodic continued nine, and she thinks that, that there should be a figure which has exactly as many zeroes, which means infinite zeroes, well there is a one at the end so that, if you add it, you obtain one. That's the figure she is looking for But you	DS2d MS8b DS2d rMS4c
20	Т.:	cannot write it down because there would have to be an infinite number of zeroes. Yes, let's pick up some more ideas. You said, we have got two positions. ()	<u>DT5c</u>
		[No reaction of pupils during the next 14 seconds.]	
24 26 28 30	Suse:	I would say Peter's solution is correct, because, if you take different figures instead, if you take for example five and two instead of zero point periodic continued nine and one, you know that they are not the same, as three is between them. And regarding zero point period continued nine and one, there is no figure between them. You know you cannot write down a figure. Mona, however, thinks that there should be this "zero point period continued zero one"-figure, but you cannot write it down. Thus, it doesn't really exist a figure. And therefore this could be right. Mona.	rMS4b DS2b DS5d rMS4c DS1a
32	Mona:	Well, I only meant: the figure doesn't exist, but logically you could imagine it so [laughter] That it could exist. Therefore [murmuring] but this figure doesn't exist. Hum that is clean It doesn't work	DS2d RS7
34	т٠	Say it loud please Judith and Juli?	KS0a
36	Juli:	Yes, Judith and I are just trying to imagine the figure "zero point periodic continued zero one", but this is somehow weird.	RS7 DS2b
38	Jens:	I think that there cannot be a further figure behind a periodically continued figure.	MS2b
40 42	Suse:	Yes, that is right, yes, that is true, because the zero, hm , because the periodic line is above it , which means, it is the zero that always repeats. Thus there cannot suddenly be a one behind it. Which means this figure doesn't exist.	rMS2b MS8c DS2c RS6a
44		If at all, the periodic line would have to be above both figures, and then it would be continued in that way: zero point zero one zero one zero one. This would not be the figure <u>Mona meant</u> .	RS6a RS6a DS2c