MATHEMATICS IN A FOREIGN LANGUAGE LEARNING STRATEGIES

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ABSTRACT

The paper presents some of the results of a longitudinal research aimed at exploring cognitive aspects of learning mathematics through the medium of a foreign language. The authors believe that it is necessary to study learning strategies first to be able to later use the knowledge in teacher training. The effective choice of strategies constitutes the substantial part of the students’ intrinsic motivation. The findings can be applied in the work with talented students. The significance of the paper is in the transfer: understanding learning strategies has impact on the choice of teaching strategies.

1. INTRODUCTION

Several new trends related to foreign language teaching can be observed in the Czech schools of nowadays. One of them is Content and Language Integrated Learning (CLIL). Similar programs have a long tradition. European CLIL, however, seems to be an approach fundamentally different from bilingual programmes implemented overseas. Nowadays young Europeans in general have different goals than their peers in the USA or Canada. Making oneself understood

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does not suffice any more. Foreign language is used as an instrument to acquire knowledge. A survey (Cook, 1992) of young Europeans’ attitudes towards learning foreign languages found that 29% wanted to learn an additional language to increase their career possibilities, while 14% wanted to learn in order to live, work, or study in the country. The largest category - 51%, were motivated by “personal interest”.

The main characteristic features of CLIL are as follows (Pavesi et al., 2001):

1. An innovative approach to learning
CLIL is a method supporting European linguistic diversity. In some schools, it has holistic features and represents a shift towards curricular integration. CLIL constitutes motivating force, relying on the students’ intrinsic and instrumental motivation. Learners are involved in interesting and meaningful activities while using the language.

2. Experimentation with different content subjects, languages, methodological approaches and with learners of different ages
“CLIL encompasses many different forms of teaching. It can refer to the whole year instruction of one or more subjects or the teaching of a module on a specific topic, or as a part of a regular course. CLIL mostly applies to the teaching of non-community language such as French in Austria or Spanish in Italy but it can also be used for the teaching of a second language in a bilingual context, e.g. Italian in South-Tyrolo.” (Pavesi et al., 2001, p. 78)

“Although different, types of CLIL have much in common because the reasons for doing CLIL, what we call the dimensions, are inter-linked in CLIL practice.” (Marsh et al., 2001).

3. Dual-focused education
All bilingual programmes including CLIL follow the same aim suggesting equilibrium between content and language learning. Both the subject matter and the foreign language (L2) are developed simultaneously and gradually, depending on the age of students and other variables. The third, long-term goal is to promote an additive form of bilingualism through the development of thinking skills. “The hypothesis is that mutual interference between the bilingual child’s two languages forces the child to develop particular coping strategies which in some ways accelerate cognitive development.” (Ben-Zeev, 1977). It can be said that bilingual learners show higher levels of verbal and non-verbal ability, they do better on concept formation, on rule discovery tasks, etc.

2. COMMUNICATION IN THE TEACHING/LEARNING PROCESS

Teaching/learning in general is primarily communication, regardless of the subject. There are four major communication modes (Mareš and Křivohlavý, 1995): spoken
language (speaking and listening), written language (reading and writing), graphic representation (diagrams, tables, pictures, graphs), active demonstration (performance and physical involvement). In the following text English language is used as L2 and mathematics as the non-language area. However, the presented ideas are valid also for other languages and non-language areas.

Theoretically speaking, classroom communication seems to be a simple process covering both receptive and productive skills of learners. In practice, however, the teacher and the class have to make a number of decisions in order to successfully understand the content matter and to make themselves understood. The task is not easy even in the learners’ mother tongue. In a foreign language which the pupils have been learning for several years only, it is even more complicated. Our initial hypothesis was that communicative strategies that might work well in a language class will probably be an insufficient means in a mathematics lesson conducted in English - in a Czech, i.e. monolingual classroom. The learners will need to learn how to combine thinking strategies so far used in mathematics taught in Czech with the foreign language communicative strategies, and possibly add other strategies in order to quickly and successfully complete the task, i.e. to solve the assigned problem.

3. LEARNING STRATEGIES

Even though learning strategies have been researched for about 20 years, it is not completely clear whether learning and communication strategies overlap or are independent (Ellis, 1999). It seems obvious that for a CLIL learner a large number of learning strategies must encompass both the area of content learning and second language acquisition. That is why in our study we decided to work with strategies originally worked out with regard to the latter area of research. Examples of strategies, which we find most important for the teaching of mathematics in a foreign language, will be presented in the following part of the text.

Learning strategies are defined and classified by a number of researchers. In the 1980s learning strategies were seen as the behaviours and thoughts that a learner engages in during learning that are intended to influence the learner's encoding' process. For O'Malley and Chamot (1990) learning strategies are techniques, approaches or deliberate actions that students take in order to facilitate the learning and recall of both linguistic and content area information. Moreover, there appears a close link between the concept of learner strategies and that of learner autonomy. For Oxford (1990, p. 8), direct and indirect learning strategies are specific actions taken by the learner to make learning easier, faster, more enjoyable, more self-directed, more effective and more transferable to new situations. Basic classification of learning strategies was provided by O’Malley and Chamot (1990):
Cognitive strategies
Metacognitive strategies
Social strategies
Affective strategies

3. OUR RESEARCH

What is common in the way students learn? What are the individual differences? To answer these questions, investigations must be done through real situations. In order to illustrate the use of learning strategies in a CLIL classroom, we prepared a series of experimental lessons. In one of them, mathematics was taught in English to 32 students of an upper secondary school, who have never encountered this type of education. The lesson content is stated below. As the lesson was to be video recorded, the school selected the best students of the final year. All of them expressed interest in the new method and wished to participate. In one group, learners were more successful in mathematics, whereas the second group consisted of students whose competences in English were better than their performance in mathematics. In the follow-up discussions, the teachers expressed their beliefs that students from the second group would make more active, more successful participants of the CLIL lesson. The course of the lesson, however, showed that good knowledge of English and communicative strategies themselves do not suffice for success in a CLIL classroom. During the lesson the students combined a variety of strategies to be able to understand the tasks, to solve them mathematically and what is more, to communicate the results and the solving procedures to the others in English. Furthermore, from the social perspective, through the use of a variety of learning strategies, they were able to exert some control over the learning process.

The learners were to solve the following assignment:

| A refreshment stall is offering three kinds of meals – hamburgers, pizzetas and fried chicken legs. In one day the assistants sold 288 pieces of hamburgers and chicken legs. They sold four times as many hamburgers than pizzetas and seven times as many chicken legs than hamburgers. How many hamburgers, how many chicken legs and how many pizzetas have they sold? |

N.B.:
1. The assignment of this type is not an example of an authentic teaching material. In fact it is a quasi-real situation. In real life, the described problem cannot occur – each sold items is registered automatically.
2. This experiment launched a new stage of research in CLIL focusing on the learners.
Analysis – the use of learner strategies

The choice of learning strategies is not independent from class management. Continually, the teacher must decide whether s/he wants - or needs - to control what the students are doing to be able to accommodate individual learning styles to their general approach. On the one hand, if the whole class works together, it is easier to control everything. On the other hand, some control can be provided through the type of activity. It should be one of the basic teaching skills to make the right decision which part of the lesson is to be teacher controlled and which can be learner directed.

We first carried out a simple analysis of possible learners’ mathematical solving strategies. In order to solve the assignment mathematically, learners are free to choose one or to combine some of the three kinds of strategies: algebraic, arithmetical, or mixed. Those learners, who decide to solve the problem algebraically, can work out the relations from the assignment in random order (this is the advantage of the strategy). The learners who prefer arithmetical strategies, having grasped the first two relations in the text do not have enough information to count H or L or P (two relations for three unknowns). The analysis of possible solving strategies is presented in the attachment.

There might be differences in the decision making between the above solving strategies. Talented students in mathematics are supposed to use algebraic strategies whereas slower/younger students usually refer to arithmetical strategies. From the methodological perspective, there are two basic ways how to conduct the lesson: in a silent way (individual written work), or as pair work, group work or whole class activity (communicative work). In the first case there is no classroom interaction. The decision making depends on each student only and the choice of strategies is restricted to the mathematical solving processes. In the second case, the classroom interaction is varied. The teacher and the students share the control of the class activities. All the statements must be formulated in a comprehensible way.

The students are expected to be active, selecting appropriately various strategies in order to persuade and are given the chance to display their mathematical and linguistic knowledge and skills. Unfortunately, this does not often happen since the students’ lack the necessary communicative skills in the foreign language.

Therefore, as the next step we decided to apply communicative learning strategies as described by O’Malley and Chamot (1990) to the solving strategies used by learners in the mathematics classroom to see whether they equak, overlap or are

\[ H = \text{hamburgers}, \ L = \text{legs}, \ P = \text{pizzetas} \]
entirely different. After careful consideration, we decided not to work with affective strategies. For easier orientation, a brief definition of each strategy is provided in the brackets.

**Cognitive strategies**
- *Contextualization* (Placing the task into a meaningful mathematical/real world context)
- *Resourcing* (Using available reference sources of information about the content area)
- *Elaboration* (Relating new information to prior knowledge, relating different of new information to each other, making meaningful personal association to information presented, in the following ways)
- *Transfer* (Using previously acquired knowledge to facilitate a new task)
- *Substitution* (Selecting alternative approaches and revised plans to accomplish a task; e.g. deduction, induction, recombination)

**Metacognitive strategies**
- *Problem identification* (Explicitly identifying the central points needing resolution in a task or identifying an aspect of the task that hinders the successful completion)
- *Self-management* (Understanding and arranging for the conditions that help successfully accomplish the task)
- *Self-monitoring* (Checking, verifying or correcting one’s comprehension or performance in the course of problem solving)

**Social strategies**
- *Cooperation* (Working with others to facilitate problem solving)
- *Mediation* (Asking questions for clarification)

### 4. CONCLUDING REMARKS

According to some educationalists, intrinsic motivation resides in the learner and not in educational methods and procedures. It is the learner who must, at any given point in time, choose the method of learning and the materials that are reinforcing to him/her. Without the opportunity of the choice, the values of intrinsic motivation will not be realized. The choice of learning strategies depends on a number of variables, one of them being the so called talent.³ It is not surprising that the characteristics of talented students are also manifest in mathematical classrooms.

During the experiment, the learners were alternatively using all of the above mentioned strategies to a) solve the problem mathematically, b) communicate the solving procedure to their peers and the teacher. The significant differences in the

³ For the characteristics of talented students see e.g. www.eddept.wa.edu.au/gifttal/giftiche.htm.
number of strategies used were noted in those students who were identified as talents.

Talented students constantly want to know the reasons why. Therefore some of them might refuse to solve the above problem because they know that it does not correspond with the reality. Some talented learners prefer the silent way of solving problems. They see the solving strategy quickly and take the initiative. The more successful learners are able to make effective decisions about the amount of planning needed. They attack the text as a holistic problem to be solved by coming at it from different angles. Basically, they try to deal with whole blocks of information because they can cope with more than one idea at a time.

These are the reasons why for talented students, communication of the solving strategy in a foreign language might seem an easy task. In reality however, such students often need encouragement to join in whole classroom discussions, they are reluctant to work in pairs or groups. Even though talented students have advanced understanding and use of language, they sometimes hesitate as they look for the correct word. However, even when they are not directly involved in the interaction they seem to use strategies to help them stay focused in the classroom.

Some talented learners use strategies to encourage the teacher to direct their attention on them. This is not always welcome by the teacher. Sometimes a talented student might constitute a burden for the teacher because s/he can ask unusual, even provocative questions or make unusual contributions to discussions.

We can conclude by saying that talented learners use a combination of top-down processing (thinking about the context and their own “knowledge” of world) and bottom-up processing (individual words or digits analyzed for meaning) to decode input. It is in the number, frequency, deployment and combination of strategies that talent and success is to be found.

REFERENCES

**APPENDIX**

1. **Mathematical description of the problem structure:**
   \[ H + L = 288, \quad H = 4P, \quad L = 7H. \]

2. **Solving strategies**

   Non-global: The problem is solved as two subtasks H-L and P-H:
   
   H-L: \[ H + L = 288, \quad L = 7H. \]
   P-H: \[ H = 4P, \]

   Global: All three relationships are elaborated simultaneously.

1. **Arithmetical strategies**

   1a) **Non-global arithmetical strategies**

   - **Direct use of relationships between \( H \) and \( L \)**
     
     288 contains 8L, \( 288 : 8 = 36 \Rightarrow H = 36, \quad L = 252 \)

   - **Systematic exhaustion**
     
     |   |    |    |   | 
     |---|---|---|---|
     | 1 | 7 | 8 | No | Too small |
     | 2 | 14| 16| No | Too small |
     | 3 | 21| 24| No | Too small |

     etc. until the solution is reached

   - **Simple approximation**

     Example

     | \( H \) | \( L \) | \( H + L \) | Yes-No | Reason |
     |---|---|---|---|---|
     | 1  | 7 | 8  | No | Too small |
     | 10 | 70| 80 | No | Too small |
     | 100| 700| 800| No | Too much |
     | 50 | 350| 400| No | Too much (but closer) |
     | 30 | 210| 240| No | Too small (but not too far) |
     | 32 | 224| 256| No | Too small (approaching) |
     | 37 | 259| 296| No | Too much (but not too far) |
     | 35 | 245| 280| No | Too small (really close) |
     | **36** | **252** | **288** | **Yes** |
Induction and systematization

Example

<table>
<thead>
<tr>
<th>H</th>
<th>L</th>
<th>H + L</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>7</td>
<td>8</td>
</tr>
<tr>
<td>2</td>
<td>14</td>
<td>16</td>
</tr>
<tr>
<td>20</td>
<td>140</td>
<td>160</td>
</tr>
<tr>
<td>21</td>
<td>147</td>
<td>168</td>
</tr>
</tbody>
</table>

The difference between $H + L$ in two neighbouring cases

When $H = 21$, $H + L = 168$, I need 288 … Difference 120
$120 : 8 = 15$, $H = 21 + 15 = 36$

Systematic elaboration – false item

If I add 1 hamburger, the number of legs increases by 7, therefore $H + L$ increases by 8
For $H + L$ I need 288,
$288 : 8 = 36$

Ib) Global arithmetical strategies

Analogy to non-global strategies for H-L, all three relationships are elaborated simultaneously.

II. Mixed strategies

The subtask H-L is solved algebraically, P-H arithmetically or vice versa (the inverse use of strategies is only hypothetical).

IIa) 1 equation with 1 unknown

IIa)-1

<table>
<thead>
<tr>
<th>H</th>
<th>L</th>
<th>H + L</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>x</td>
<td></td>
</tr>
<tr>
<td></td>
<td>7x</td>
<td></td>
</tr>
</tbody>
</table>

$x + 7x = 288$
$8x = 288$
$x = 36$

$H = 36$, $L = 7.36 = 252$

IIa)-2

<table>
<thead>
<tr>
<th>H</th>
<th>L</th>
<th>H + L</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>x</td>
<td></td>
</tr>
<tr>
<td></td>
<td>7</td>
<td></td>
</tr>
</tbody>
</table>

$x + x/7 = 288$
$8x/7 = 288$
$x = 252$
$H = 288 : 7 = 36$

IIb) 2 equations with 2 unknowns

IIb)-1

<table>
<thead>
<tr>
<th>H</th>
<th>L</th>
<th>H + L</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>x</td>
<td></td>
</tr>
<tr>
<td></td>
<td>y</td>
<td></td>
</tr>
</tbody>
</table>

$y = 7x$
$x + y = 288$
III. Algebraic strategies

IIIa) 1 equation with 1 unknown

- IIIa)-1
  - P \( \ldots \) \( x \)
  - H \( \ldots \) 4\( x \)
  - L \( \ldots \) 7.(4\( x \))
  
  \( 4x + 7 \cdot (4x) = 288 \)
  \( 32x = 288 \)
  \( x = 9 \)

- IIIa)-2
  - H \( \ldots \) \( x \)
  
  \( x + 7x = 288 \)

- IIIa)-3
  - L \( \ldots \) \( x \)
  
  \( x + x/7 = 288 \)

IIIb) 2 equations with 2 unknowns

- IIIb)-1
  - P \( \ldots \) \( x \)
  - H \( \ldots \) \( y \)
  - L \( \ldots \) \( 7y \)

- IIIb)-2
  - H \( \ldots \) \( x \)
  
  \( x/4 + y = 288 \)

- IIIb)-3
  - P \( \ldots \) \( x \)
  
  \( y + y/7 = 288 \)

- IIIb)-4
  - H \( \ldots \) \( x \)
  
  \( x + y = 288 \)

IIIc) 3 equations with 3 unknowns

- IIIc)-1
  - P \( \ldots \) \( x \)
  - H \( \ldots \) \( y \)
  - L \( \ldots \) \( z \)

  \( y = 4x \)
  \( z = 7y \)
  \( v + z = 288 \)