RF01: TEACHERS RESEARCHING WITH UNIVERSITY ACADEMICS

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A PME working group, *Teachers as Researchers*, first met in 1988, and then met annually for nine years. This working group was based on the belief that classroom teachers could and should carry out research concerned with the practice of teaching mathematics. This theme, based on contributions from members of the group, led to the publication of a book (Zack, Mousley & Breen, 1997). What was the role of the university academic in supporting or challenging teacher-researchers in the chapters in this book? Was there an academic acting as leader or facilitator? Do teacher-researchers aim to become independent of their mentors?

What is meant by teacher research? In Anderson & Herr (1999) the following characterisation is given:

> By practitioner research we refer to a broad-based movement among school professionals to legitimate knowledge produced out of their own lived realities as professionals. This includes an ongoing struggle to articulate an epistemology of practice that includes experiences with reflective practice, action research, teacher study groups, and teacher narratives. (Note 1, p. 20)

The role and status of 'knowledge' in teacher research is an object of sharp and vivid debate not only in the field of mathematics education (Metz & Page, 2002). Breen (2003) comments,

> On the one hand, there is a growing movement for more teachers to become involved in a critical exploration of their practice through such methods as critical reflection, action research, and lesson studies. The contrasting position makes the claim that these activities have done little to add to the body of knowledge on mathematics education. (Abstract, p. 253)

Jaworski (2005) believes that one way to add to the body of knowledge is through 'co-learning partnerships',

> The action research movement has demonstrated that practitioners doing research into their own practice […] learn *in* practice through inquiry and reflection. There is a growing body of research which provides evidence that *outsider* researchers, researching the practice of other practitioners in co-learning partnerships, contribute to knowledge *of* and *in* practice within the communities of which they are a part. (p. 2)

Is academic research useful to practising teachers of mathematics or is it generally inappropriate? What happens when teacher-researchers seek to validate their work through studying for a post-graduate degree? What forms of research collaboration between university academics and teachers of mathematics exist? What are their advantages and limitations? The contributors to this research forum will focus our

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explorations on the theme of teachers researching with university academics through addressing some or all of the following questions:

Question 1: Who are we? What are our connections with teacher-researcher collaboration? How did we start our work in this area? How do we work?

Question 2: Why do we engage in teacher-researcher collaboration? What is it for? Who is it for? *e.g.*, developing theory about teaching and learning; personal transformation; making a difference in classrooms?

Question 3: Who speaks for whom, to whom and for what purposes (balance of the roles for issues of voice, power, reciprocity and identity)?

Question 4: What can we do in such a cooperation that could not be done only by teachers or only by researchers?

We believe that the research done in collaboration between teachers and university academics is a powerful tool for improving both theory and its implementation in practice under the condition that respect is given to the roles of all participants. The diversity of research theories and experiences of the contributors to this forum range through academics approaching teachers and working with them in formal projects, seeking to be as equal as possible; an academic approaching a teacher and working together in a mutually negotiated way; a teacher approaching an academic and working in a way driven by the teacher; an autonomous teacher calling on academics when necessary; a group of teacher-researchers studying for a higher degree becoming independent from the university academic.

What are the lessons that we can take from these various interactions about the way in which academics and teachers can work together collaboratively and mutually successfully and at the same time allow the teacher's voice to flourish? Do some situations provide a greater possibility for this to happen?

SEEING MORE AND DIFFERENTLY – TELLING STORIES: COLLABORATIVE RESEARCH ON MATHEMATICS TEACHING AND LEARNING

Laurinda Brown and Alf Coles

Question 1: We, Laurinda Brown and Alf Coles, met in 1995. At the time, Alf was beginning his career as a mathematics teacher and Laurinda was working in a university education department with student teachers of mathematics on a one-year postgraduate course. We have therefore worked together for just over ten years.

When we met, Laurinda was particularly interested in how new teachers of mathematics develop their teaching styles and strategies and what her role might be in that process. Having taught mathematics herself for fourteen years she saw herself as a teacher and a researcher with no conflict between these roles: "there is only that which I bring to whatever context I am in – I cannot help but bring those perspectives to the range of activities in which I engage" (Brown (with Coles), 1997, p.103). In working with her student teachers to support them in developing a range of teaching

styles beyond their initial images of how they were taught, Laurinda began theorizing about what she called 'purposes' (Brown and Coles, 2000a). These were not 'tips for teachers' (behaviours to implement), nor philosophical positions (beliefs related to mathematics or teaching mathematics). 'Purposes' were in a middle position, motivations to act, such as ways of finding out what their students know from which student teachers can develop a range of teaching strategies. Laurinda was looking to find a teacher, new to the profession, to work with who had not done her course and was finding teaching challenging. As we began working together, Laurinda was the researcher and Alf the teacher but rapidly our frames merged. We starting looking in the same direction as co-teachers and co-researchers.

Alf: Reflecting back on my first year of teaching had produced in me a feeling of inadequacy akin to despair – looking back over all that time, looking for the lessons which had been 'good' from which to start to build next year they had seemed rare. No lesson really seemed to match up to my ideal image of what seemed possible and there was a strong sense of a gap between where my philosophy lay and the day-to-day practice of what was actually happening in the classroom.

Laurinda: Alf and I discussed the possibility of working together. Alf asked me: what do you want to do? and the only answer was that if the work were to take place the agenda would emerge from conversations. What seemed crucial was that the agenda for the work was Alf's. My investigation would be subordinate to his agenda. (Brown (with Coles), 1997, p.106)

Laurinda asked whether Alf could bring to mind particular moments or times during a part of parts of lessons that had felt closest to his ideal. This provoked two 'brief-but-vivid' (Mason, 1994) anecdotes. Without any prompting from Laurinda, Alf made a connection between the two incidents, saying, energetically: 'It's silence, isn't it? It's silence.' Silence was recognized to be a purpose by Laurinda. This was something that we could work with, exploring strategies for using the silence of the teacher within the mathematics classroom. The work that we have done has supported our personal transformation as teachers.

How do we work together? At most once a week, we spend time together in Alf's classroom. Dependent on our focus we might use videotape for data but mostly Laurinda takes observation notes against the current issue. We stay with the detail of what has happened in discussions after the lesson, 'What did we notice?', allowing patterns and differences over time to emerge that become the foci of what we work with – critical incidents noticed by one or other of us. Foci have been, for example, using the questions 'what's the same' and 'what's different?' as a teaching strategy (Brown and Coles, 2000b). Part of our work is writing together and our first joint paper 'The Story of Silence' appeared as a PME paper (Brown and Coles, 1996).

Question 2: These struck us as being really good questions. Why do we engage in this research collaboration? It has always been clear to us that we are personally transformed by the process and changes are apparent over time within Alf's classroom and in Laurinda's work with student teachers. As we engage in (often the

same) activities, such as researching and teaching in classrooms or viewing videotapes, we are literally aware of seeing more – in the sense of what seems like a finer mesh to look through. As we collaborate with each other and with others interested in the teaching and learning of mathematics, we also see things from more perspectives. We have engaged theoretically; through reading and applying the work of other's (particularly Bateson, 2000, 2002; Varela, 1995, 1999; Maturana, 1994, 2004 - these authors talk about using 'difference' as a natural way of learning (a difference that makes a difference) and with David Reid we ran a discussion group at PME 26 to focus attention on the similarities and differences between these (and other) authors; within an enactivist frame (Varela, 1999); through developing our own theories-in-action.

It is clear that the writing process helps us but why would any of these stories of our developing awarenesses of teaching and learning in one classroom be of interest to anyone else? In 2003, a review was written of the British Society for Research into Learning Mathematic (BSRLM)'s work, through consideration of its day conference proceedings (three each year) over seven years. The author of this review, Marilyn Nickson, commented on our corpus of work presented in that community:

> '… worldwide research projects in the development of teaching in mathematics education tend to encourage models of critically reflective practice leading to the development of communities of enquiry together with critical intelligence in them. This type of research is well illustrated by the work of Coles and Brown […]. [Their] initial paper, relating to their ongoing study, includes a reflection on what it is like for teachers and researchers to work together. […] As well as positive outcomes in terms of classroom learning, the study in its entirety is a very good example of the benefits of collaboration over time between a teacher and a colleague for whom research is part of his or her professional life. The fact that the BSRLM community as a whole gains from it is an added bonus to the profession as a whole.' (Nickson, 2003, pp. 63-4)

So, there is something that the UK community values about the process of us sharing our work over that time and we learn through the process of writing those stories. We have also shared our work through research papers in the PME community (1996, 1999, 2000) and in the work of the PME Teachers as Researcher Working group (in Zack, Mousley & Breen, 1997). In research writing more generally, our use of story, "the pattern which connects" (Bateson, 2002, p. 7) tries to convey that "little knot or complex of that species of connectedness which we call relevance" (Bateson, 2002, p. 12). In the process of writing, we make more connections and tell more (different) stories often about the same sequence of data – this again allows us to see more.

From Alf's perspective early in our collaboration:

> One discipline that has also come out of the work with Laurinda is that of staying with the story. In my notes on teaching in the first year - the observations are in general distant - about whole classes - with observation and analysis all mixed in. What I have been working on this year is forcing myself to hold back the analysis and stay just with stories about individuals or groups of individuals. Analysis (or synthesis) from this data then has the possibility of throwing up something I had not been aware of before…previously

> … mixing …analysis and observations …meant that I was never surprised … There was little chance of my accessing those things I did that I was unaware of - but which yet had profound effects. (Diary, Alf, 10/95)

Question 3: In working with the silence of the teacher, Alf started exploring the effects of offering images to students, in silence, inviting their silent responses or asking them to describe what they see. Alf is not the one to whom students listen for explanations but Alf becomes the listening teacher. The children are working on the mathematics and it is their work on the mathematics that Alf is interested in learning about. This situation is parallel to Laurinda being interested in what Alf is working on, whilst Alf concentrates on his learning about teaching. So, the balance of the roles means that we are always both learning, sometimes about different things. Alf has explored issues of silence, listening and hearing through an MEd in Mathematics Education, presenting his dissertation at PME25 (Coles, 2001).

Question 4: There is a reflexivity built in to this co-operation. Laurinda continues to learn about life as a teacher in a classroom and develop as a teacher, which is important to her role as a teacher-educator. Alf learns about being a researcher and has been a named participant in a successful bid for research funding, completed his Master's degree and built his practice through researching in action. We provide mirrors for each other as co-researchers, sometimes co-teachers that allow us to reflect deeply about the teaching and learning of mathematics, specifically the development of mathematical classroom cultures. Neither a researcher with their own agenda nor a teacher perhaps inarticulate about their practice would be in the position to add the component of collaborative writing – learning through outer speech and responding to each other's questioning – that has allowed the weaving of stories in acts of meaning. What we seem to be dealing with over the years is the cultivation of awarenesses of awarenesses, learning about learning, where the other provokes another meta-layer of awareness in ourselves as we work to provoke second level awarenesses for our various students. And mathematics is the vehicle in which we both work – doing the mathematics together.

JOINT REFLECTION AS A WAY TO COOPERATION BETWEEN RESEARCHERS AND TEACHERS

Alena Hošpesová, Jana Macháčková and Marie Tichá

Question 1: We started to give attention to reflection through studying the basic competences of the teacher (Hošpesová & Tichá, 2004). Many authors (e.g., Jaworski, 2003; Schön, 1983) cite a competence of qualified pedagogical reflection (the teacher's analysis of their own thinking and ways of dealing with students suitable for planning their own lifelong education) and consider it to be a determining feature of each teacher's professionalism. Other authors (e.g., Svec, 1996; Steinbring, 2002) assume that reflection creates space for the transition from intuitive to conscious and justified action. We agree with Czech educationist Slavik that: "It is possible to treat reflection connected with interpretation of teaching/ learning

situations as the best way to develop the teachers' professional way of thinking and to present practical didactical theory". (2004, our translation)

On the intuitive level, reflection is present in all human activities and thus in teaching, too. However, if we want to speak about a qualified pedagogical reflection (which includes observation, contemplation and consideration) then we also take into account description and analysis of key elements, evaluation or revaluation, ways of explanation, accepting decisions and determining a new strategy (Slavik & Sinor, 1993). We must consider conscious reflection on our own teaching from the point of view of goals and content of the teaching, and methods of work and their realisation. Knowledge of content is assumed as a given.

We understand reflection not only as a retrospective act but also as part of the whole process of teaching, penetrating preparation, realisation and evaluation. "Joint reflection" seems to be a contradiction. Reflection is often seen as something personal or individual. But if we observe a teaching episode within a group of other people expressing their views freely, our reflection is influenced and changed (cf, Cobb et al., 1997). Joint reflection in a group of teachers and researchers influences positively the improvement of teacher's competences (Hošpesová & Tichá, 2004) and can be seen as a form of cooperation between teachers and researchers.

Our cooperation with a group of primary school teachers (all fully engaged only in teaching) started during a four-year project within the international Comenius project "Understanding of mathematics classroom culture in different countries". The general aim of the project was to contribute to the search for ways to improve the quality of continuous in-service education of primary school teachers and so to support the development of teachers' competence (Hospesova & Ticha, 2003). We initially assumed that we would examine the different approaches coming from different countries. The co-operation within the project itself led to the amendment of our initial intentions, and we focused our attention more deeply towards the preparation of teacher training courses promoting qualitative changes in classroom culture; the development of a more sensitive approach by teachers to pupils' ways of thinking; the ability to use this in lessons; and an awareness of situations that could be valuable from the point of view of the pupils' learning processes. We started to aim at the cultivation of teachers' competences through self-reflection and joint reflection (Scherer & Steinbring, 2003).

How do we work? What are the forms of efficient cooperation in our case? The key feature of cooperation was the equal status of all members of the team in all areas of work, i.e., when preparing, carrying out and analysing instructional experiments. Usually a more active role for researchers and a more or less passive role for teachers is expected. Researchers are supposed to determine and evaluate teachers' work, the teachers are in the role of people putting into practice the ideas of someone else. We gradually persuaded the teachers that we all have the same level of responsibility, although our roles and interests are different. During our cooperation we prepared several teaching experiments realized by the teachers. The cooperation gradually established itself in the following form:

- At the beginning of our work on the mathematical topic of the teaching experiment, we discussed (and when necessary the researchers summarised for the teachers) its mathematical background and its possible didactic elaboration.

- After discussions amongst the whole team, the teachers independently prepared experimental lessons for their classes to be part of the usual school teaching.

- The experimental lessons were video-taped (25 recordings in all) by the researchers. The teaching was as close to "ordinary" as possible.

- The teacher who taught the lesson chose, according to her opinion, the most interesting teaching episodes, usually discussing the selection with a researcher. The members of the team then reflected on the video-recordings individually (including the performer or observer of the action). This meant that each member of the team had at his/her disposal video-recordings of a chosen episode or episodes to analyse and assess, aiming to be prepared for subsequent joint reflection.

- Chosen episodes became the core of joint reflection in the meeting of the whole team. These discussions were usually audio- or video-taped so that it was possible to study the level of reflections of all participants. The level of reflections developed over time, growing in quality. We perceived several mutually-connected levels; a simple dialogue with conversations aimed at intuitively-understood observations such as "I liked/disliked this" in which teachers generally spoke about their feelings; looking for effective methods of teaching for specific mathematical content which aimed to improve teaching; a deep analysis of teaching from the point of view of goals, methods and content, which led in turn to the preparation and realisation of the teacher's own instructional experiments.

It is obvious, that all teachers in our group did not reach the last level. For some teachers (regardless of their age), it is very difficult to take part in discussions and to express their opinion. Apparently, they need more time to think the situation over and, say, study literature. Their low self-evaluation of their teaching and uncertainty in their own mathematical understanding may be impeding their progress.

Question 4: The teachers from early on in the relationship realised that joint reflection facilitates their personal motivation.

> Diana: For me, the self-reflection and help of other colleagues are important. In some situations I would not be able to change by myself even if I wanted to.
>
> Betty: The opportunity to communicate about problems in teaching is a huge 'driving engine' for me.

The teachers also gradually grasped the need to videotape the lessons, because it enabled them to balance their involvedness in education and acquire the critical distance, *i.e.*, to follow (their own) education from "outside" and to fall into the role of reflective teacher and teacher-researcher.

> Ann: The video recordings, which are authentic, are excellent and allow me to observe my work from a different standpoint, from the position of an observer of the efficiency and quality of my teaching – verbalization, correctness and

> accuracy of formulation of the tasks, and the quality of my communication with pupils.

After the third year of cooperation, the teachers themselves formulated the benefits from the work on the project as follows:

> The work on the project brings me a lot of new things. Some topics (of school mathematics) I do not understand quite precisely and the discussion of the background and didactic elaboration, *i.e.*, about theory and practice, helps me a lot. ... My responsibility has grown. I think more about what I do in education, what the children do, whether all the children understand...Even in the project I realized the importance of the personality of the teacher.

We perceived the development of more sensitive teachers' approaches to pupils' ways of thinking and of the ability to use knowledge in this area in teaching. Teachers became more interested in the pupils' understanding. *i.e.*, what does it mean to understand certain school mathematics topic?

> Cecily: Thanks to the project, I have an opportunity to see the teaching of mathematics more deeply. As soon as I realised what we had learned thanks to the joint reflection and self-reflection, how our approach towards the teaching of mathematics had changed, I began to realise that the changes did not affect only my mathematics lessons. I started to ponder on the idea that the method of self-reflection and joint reflection can be useful in other lessons, too. I asked myself the question: If self-reflection and joint reflection lead to the improvement of mathematics lessons in terms of both mathematical and didactic aspects and force a teacher to work on him/herself and educate him/herself, why could it not work in the Czech language or in geography or in any other lesson?

Joint reflection brought a shift (an improvement) in the researchers' knowledge in various ways. It deepened their understanding of various aspects of mathematics education, opening ways to recognise causes of failure for some teachers; why problems appear in teaching and how to remove them; understanding of processes going on during mathematics teaching in the social context of the classroom and helping to show the possibilities of using this knowledge for person- centred education and the strengthening of the constructivist approach.

Analysis of reflection could be used as a diagnostic instrument by the researchers, allowing them to recognise shifts and improvements in teachers' professional competence, opinions and approaches on the basis of external representations (Duval, 1995), seeing which aspects (subject, didactic and pedagogical) should be emphasised in the education of future teachers and practising teachers. During the meetings and joint reflection of chosen episodes from teaching mathematics lessons, the researchers had an opportunity of influencing practising teachers indirectly, informing them about actual research results in mathematics education.

Pupils indirectly profit from the fact that their teachers pay them more attention and discover both misunderstandings and problems, and abilities that are not necessarily apparent in everyday teaching. (We have not done the analysis of the video recordings with pupils so far.)

We consider that joint reflection can bring about an improvement in the mathematics classroom culture, *e.g.*, studying key phenomena for assessment of processes going on during mathematics lessons; following changes in approaches and behaviour of participating teachers; studying the impact on students' knowledge. We need to do a longitudinal study to study these areas.

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OPENING THE SPACE OF POSSIBILITIES: TALES FROM THREE TEACHERS

Agatha Lebethe, Neil Eddy and Kendal Bennie

We are three teachers who were brought together because we shared similar strong feelings about our work as teachers. All three of us were drawn to an advertised Masters programme in Teaching which foregrounded the practice of teachers (see Breen, 2002). All three of us decided that we wanted to take the themes of a particular module, which introduced us to the ideas of complexity science and enactivism (Maturana and Varela, 1986; Davis, 1996) as well as those of the Discipline of Noticing (Mason, 2002), into our research dissertation. This proved to be an extremely complex experience as our first hurdle was to convince the traditionally-minded academics who were presenting a generic Research Methods course that our planned research was legitimate and acceptable. Despite the difficulties that each of us experienced along the way, we were pleased when each of us received recognition from the academy that our work was worth the award of distinction.

Our presence as part of this Research Forum stems from our growing belief that our experience is generalisable. Not enough is being done to ensure that the authentic and embodied voices of teachers are being heard and respected to a sufficient degree in an undomesticated form.

Agatha: I spent 10 years working as a mathematics in-service field worker at the University of Cape Town. I took a very long time to begin my Masters Studies. I was waiting for a Masters programme that would respect and acknowledge my Practice and would allow for the voice of the teacher, the teacher educator and the researcher to be heard. My earlier experience with the way most Academics supported teachers in Research made me determined to engage in a form of research that would allow me to open up and reflect upon my experiences as a teacher educator; tell of my discomfort, my turmoil, and confusion and also explore the complexity in trying to make sense of my teaching. By weaving my voice into the research I wanted to understand my practice and describe my journey through the research as I unravelled the layers of habitual actions of my teaching. This meant that I could not disembody my multiple voices from the text and this necessitated writing in the "I". I needed to interact with the text, with the research as a living medium and therefore my intention was not to seek answers but rather to understand the journey through research and to appreciate my interaction with my environment and so embrace possibilities.

I wanted the research to be a text that was honest and authentic. However, crafting such a text meant challenging the existing genres of dissertations at my University. One of the ways I did this was by insisting that my existence in the research process and in the dissertation was that of the hermeneutic inquirer. As an hermeneutic inquirer I was not seeking a truth and I was not a detached observer in the inquiry and therefore the research was grounded in interpretation. This was evident in the methods I employed for both the data collection and analysis.

I believe that there is a space for teacher researchers like myself to make the process of inquiry systematic and public. We need to uncover our own beliefs, assumptions and our biases. We need to make explicit our own theorizing both in our everyday practice and in formal research. I would like to see us examining how these theories have influenced the nature and structure of our work.

I would also like to see University postgraduate courses for teachers place less of a heavy emphasis on being pre-packaged, pre-determined and linear. Teachers experience needs to be validated and this can be done by researching how we live within worlds (our teaching world) of interpreted meaning.

For van Manen (2001):

> … to do research is always to question the way we experience the world, to want to know the world in which we live as human beings. And since to know the world is profoundly to be in the world in a certain way, the act of researching – questioning– theorising is the intentional act of attaching ourself to the world, to become more fully a part of it, or better, to become the world. (2001, p. 5)

Neil: I am a teacher. Initially I flourished in the world of the academy – the world of immutable truth obtained from data analysed objectively. I then began teaching in my classroom where the faces in the desks staring at me were neither numbers nor objects.

It was my practice here that forced me back to the academy, with the hope of finding a means to understand my practice, from within my practice. I was not comfortable with research that required me to isolate – I needed a method that embraced wholeness. I heard of a Masters course offering a different philosophy. In the first year, I became aware of people using methodologies that promoted the improvement of one's practice and privileged teacher experience.

In the second year I endured a compulsory course on research methodology. Suddenly the exclusionary walls of the academy, so effectively deconstructed, were now bulldozed back into place – twice as oppressive now that I had a view beyond. "Choose a small question, for which data is easily obtained, write it up and submit". "Get the cloth on your back, prove that you can research, then you can start asking the questions that truly matter to you." I put together a proposal, but somewhere lost the track and disappeared from the Academy. I never did get my proposal passed, but, after a while, my practice forced me back to continue the research.

I chose to expose my research through the metaphor of a science fiction story by Clarke (1956) and the story's hero, Alvin. In the story, the city of Diaspar reproduces itself in an unchanging cycle. The comfort of the citizens is ensured by removing the pain that is associated with change.

Through this story I was able to tell my story of change from a deeply scientific and disembodied paradigm to one more deeply rooted in the "I". It led through emerging deep ecology works onto a set of statistical techniques that I used to open questions.

In a parallel story, I am concerned with the voice of the teacher and the lack of acceptance of this voice. I am convinced that the teacher has a story to tell – one rich in knowledge and a truth – but we lack a language with which to converse. The university academic has a language, a voice, a firm grasp of the status quo – and therefore owns the truth. The researcher can, in the status of "doctor", diagnose problems in practitioners, operate on them to correct these, and extract the truth, thus building their power. I feel the teacher is often left with the numbed feeling of the anaesthetic and a distrust of the academy of which he or she is an integral, but voiceless part.

My supervisor allowed me the space to develop a voice. Very little came from him on how things should be done and this, although difficult, allowed me to sit with the data and find my way, while he kept the academy away. I ended up not only with a dissertation, but also with a more mindful practice.

Kendal: My aim was not to contribute to a body of knowledge outside of myself but rather to contribute to my own (en)active knowledge, to improve my teaching practice on a daily basis and through this have a greater impact on those that come into contact with me, especially learners. It started and continues with me wanting to be(come) a better teacher. It has to do with a belief about the need for personal responsibility. If each of us could make a commitment to improving ourselves, the change in the world would be phenomenal.

> The universe changes when something as miniscule as a thought changes - because that thought is not merely in the universe, it is part of the universe. (Davis, 1996, p.14).

The nature of this research meant that I was not starting with a question that I would answer. The questions were to emerge from the research (process) and the answers were unlikely to be simple solutions but more likely to create more questions. It was to be a step out of a universe of binary questions and answers and into a multiverse of awareness of the uncertainty with which we live. I would not be following the well-worn path of illusory objectivity but rather riding in a subjective ocean where the ground beneath me was not solid and the path in front nonexistent.

Education is "about sensitivity to and transformation in others. The only certain place to stand is in the most unlikely place: ourselves." (Mason, 1994, p. 5). And yet when I did find the sea, a part of me cried out for the security of land and a path to follow.

I proposed to research and write about my learning while using the analogy of surfing to distance myself and help me understand and analyse my experiences. My research

proposal presenting this approach was rejected by the academy because my research method came before the questions I was to answer. Views I invoked were called heretical and in the minority.

While most research has a frame that researchers work towards, the vagueness of my frame (plan) was the most exciting part. My frame (or lack thereof) didn't formalise proceedings, it opened space for (r)evolution. To learn required me being aware, (ob)serving, listening and (re)visiting writing.

Capra (1991, p. 51) explains how Eastern religions use mythology, metaphor and paradoxes to explain reality better than language, in its linear fashion, is able to.

All through the writing I found myself tempering a flair knowing the sharks didn't like me being in the water. Fortunately I was not scared out of the water and the rewards are still reverberating.

DIVERSE ROLES, SHARED RESPONSIBILITY

Jarmila Novotná and Alena Pelantová

The scientific aim of the cooperation of teachers and university academics is to accomplish the research necessary for advancement of knowledge of the mathematics education phenomena. In this contribution, a model involving a limited number of staff in school, university researchers and teacher trainers is presented. The focus is on the different types of participants' involvement and responsibilities as well as on the scientific and practical results of such cooperation.

Our cooperation developed from being significantly unbalanced with most responsibility put in the hands of the university academic, towards real cooperation with a clear division of responsibilities. As the basis for the characterization of participants' involvement and responsibilities, the organization of the COREM (le Centre d'Observation et de Recherche sur l'Enseignement des Mathématiques) school (Salin & Greslard-Nédélec, 1999; Novotná, Lebethe, Rosen & Zack, 2003) is used. The benefits and limits of cooperation as well as the differences and similarities with COREM as a representative of a whole institution working on the basis of cooperation form the framework for our contribution.

Questions 1, 2 and 3: Alena is qualified for teaching mathematics and geography to pupils aged 11-15. At present she is the head of a school in Prague. Jarmila is a University teacher training future teachers of mathematics. She is involved in research in the domain of mathematics education cooperating intensively with researchers from abroad.

We met for the first time in 1992 when Jarmila was the coordinator of the project *Integration* between basic school and general upper school. The project represented something new at that time, breaking the uniformity of the educational system. It represented a challenge for the participating teachers, an attempt to improve the organisation of education offering more responsibility and professional freedom to the school and its teachers. Alena was the teacher of mathematics and geography in

the school and actively participated in the project. In the first period of our cooperation, proposals of what to do and plans were elaborated by researchers from the Faculty of Education of Charles University. Teachers implemented these ideas.

The cooperation with Alena continued after the end of the project and after she became head. Our roles and responsibilities in the cooperation have gradually changed. Our roles are different but our present position in the cooperation is balanced with clear division of responsibilities.

We will illustrate this division of our roles and contributions by one episode from our cooperative research. We can identify three different roles of Alena and Jarmila: Alena in the role of a teacher (we will label it as Alena-teacher) and a researcher (Alena-researcher). Jarmila acts here as a researcher only (including the role of an observer) - labelled simply Jarmila.

We dealt with solving word problems, dealing with the division of a whole into unequal parts with 12-year old students. The long-term practical experience of Alena-teacher confirmed by Jarmila's (and not only hers) investigations and research (*e.*g., Novotná, 2003) and their discussions with other teachers signalled the didactical demands of the topic for students before and after being taught school algebra.

We decided to focus in our cooperation on this type of word problems at pre-algebraic level. Our experience confirms that word problems dealing with the division of a whole into unequal parts belong to those school mathematics domains where we can clearly see that the arithmetical and algebraic processing of the problems impose different solving strategies (Bednarz & Janvier, 1994). In school mathematics, algebra is often presented to students as a new and more efficient tool for solving problems. But students at the elementary (pre-algebraic) level have already experiences with arithmetic solving strategies and they can profit from them when starting to use algebraic procedures.

Jarmila proposed the framework of the theory of didactical situations in mathematics (Brousseau, 1997) in which we started to look for teaching strategies that could help students to overcome the difficulties that they face when solving this type of word problem. Jarmila and Alena-researcher performed the *a priori* analysis of the type of word problems including possible solving strategies (correct and incorrect) including the level of mathematical thinking required, necessary knowledge and possible obstacles. In this period, Alena-teacher was not too active; her role of researcher was much stronger. Nevertheless, her experience as a teacher was irreplaceable. She helped to keep a "realistic" platform in our plans and proposals. Based on this analysis, Jarmila, Alena-researcher and Alena-teacher prepared the didactical unit to be realized in the classroom (Pelantová & Novotná, 2004). The didactical unit was designed as a sequence of adidactical situations of action, formulation and validation and the following institutionalization. The lesson was taught by Alena-teacher and video-recorded and observed by Jarmila. We used the same division of roles for the lesson as was used in COREM – Jarmila did not intervene, the whole responsibility and all decisions during the lesson were left to Alena-teacher. The next step, the *a*

posteriori analysis of the realized didactical unit was done by Jarmila and Alena-researcher with Alena-teacher's explanations and other ideas mainly focusing on the reasons for her decisions to modify the prepared course of the lesson.

All "three" of us, Jarmila, Alena-researcher and Alena-teacher prepared the test which was assigned to students three months later, the aim of which was to see the stability of knowledge built in the didactical unit. The results were surprisingly good. Even children who behaved rather passively during the adidactical situations showed good command of applying the acquired knowledge.

The final part of the described activity, done mainly by Jarmila, was the integration of the results into the broader research framework. Jarmila and Alena-researcher disseminated the results at various scientific events.

Question 4: In Novotná, Lebethe, Rosen & Zack (2003), the following questions are posed: "Does the teacher need the direct presence of a researcher during his/her teaching?" Answer, "No"; "Does the researcher in education need the direct cooperation with one or more teachers?" Answer, "Yes". The reasons for the answers are given there, together with the benefits of such cooperation for a teacher.

For Alena (Alena-teacher), the cooperation offers access to theoretical frameworks and research that she would not acquire without our cooperation. The career of Alena-researcher was born in our cooperation. Alena's position is that described by Brousseau (2002):

> When I am acting as a researcher, the interpretation of each step of teaching begins with a systematic questioning of everything, a complex work of a priori analyses, of comparisons of various aspects of the contingencies, of observations first envisaged and then rejected, etc. … When I am a teacher, I have to take a number of instantaneous decisions in every moment based on the real information received in the same moment. I can use only very few of the subtle conclusions of my work as researcher and I have to fight with starting to pose myself questions which are not compatible with the time that I have, and that finally have the chance to be inappropriate for the given moment. I react with my experience, with my knowledge of my pupils, with my knowledge of a teacher of mathematics which I am treating. All these things are not to be known by the researcher.

At the theoretical level, Jarmila's research questions can be treated independently from the school reality. However, to find answers to her research questions she needs the direct contact with Alena and the access to the teaching reality. Thus, she can avoid the danger of producing superficial answers to research questions, results not having "real roots" and with a doubtful applicability in the school reality.

Our cooperation influences significantly not only the teaching and learning of mathematics in the school but several other domains of the life of the school. The following list is not exhaustive but illustrates this impact:

- involvement in international cooperation (Socrates Comenius 1) began with advice and contacts gained from Jarmila's institution;

- other possibilities of learning about other good practice in teaching various subjects and the space for presenting good practice of Alena's school during visits of foreign colleagues from institutions training future teachers and teacher students;

- access to student teachers, facilitating the recruitment of young qualified teachers for the school with new, fresh ideas about the teaching/learning process;

- involvement of the school in various surveys (not only about teaching mathematics) with the outputs enlarging the horizons of staff knowledge about new trends in education;

- access to information about conferences, seminars and summer schools focusing on education and the possibility for active or passive participation at these events. The result is not only the increasingly good reputation of the school but also the staff's increasing knowledge of new educational trends.

And, finally and importantly, the key to successful cooperation is the harmonious cooperation of all partners involved, which is true for us, Jarmila and Alena.

RESEARCH WITH TEACHERS: THE MODEL OF COLLABORATIVE RESEARCH: STUDY OF JOINT DEVELOPMENT MECHANISMS FOR AN APPROACH TO THE TEACHING OF MATHEMATICS TO INUIT CHILDREN IN KINDERGARTEN AND PRIMARY GRADES 1 AND 2

Louise Poirier

In the spring of 2000, the Inuit community and the Kativik School Board were pondering over the difficulties encountered by students in mathematics and the measures that could be taken to help them. One significant fact that could help explain these difficulties is that Inuit students learn Inuit mathematics in their own language in the first three years of their schooling and then go on to study in either French or English. Having heard of the work I was doing at that time in Montreal with immigrant children learning mathematics in French as a second language, members of the Kativik School Board of Nunavik (Northern part of the province of Québec) asked for my help.

In the Fall of 2000, I visited several Inuit villages in order to observe classrooms, to meet teachers and their students. Those visits prompted several remarks:

Mathematics and language: The Inuit children start school (kindergarten, first and second grade) in Inuktitut. The first concepts they learn in math are learned in Inuktitut. Then, at that time in third grade (the situation has changed this year) they would switch either to French or to English and they would pursue their learning of math in that second language.

Mathematics and culture: Until recently, mathematics was seen as a universal language but this view is now questioned. Inuit children learn mathematics in Inuktitut but they also learn Inuit mathematics. And Inuit mathematics is quite different from the "southern maths", the mathematics taught in Montreal, which they will learn in third grade and up in the second language. For example, when children

learn how to count, they are using a base-20 system and not the base-10 system they will use when they change to French or English. It would seem that for these students two separate and distinct universes are cohabiting: the world of day-to-day life and the "southern" mathematical world. Furthermore, the first world, the world of every day life, has nothing to do with the second one, the one of mathematics done in school. Mathematics is not perceived as helpful in day-to-day life.

Spatial capacities: The students I met are very good at spatial representation and geometry. Unfortunately, the present curriculum does not put enough emphasis on these strengths.

Mathematics and teaching methods: Teaching methods used by some teachers up North (paper/pencil exercises) are not "natural" methods of learning for these Inuit children. Traditional teaching and learning are done through observation and listening to stories or enigmas.

Faced with this dual phenomenon of first learning mathematics in Inuktitut and then in French or English, the instructional situation becomes highly complex: how can these two cultures be combined and accommodated in mathematics teaching situations? The main purpose of our project is to study the joint development process of mathematics teaching situations adapted to Inuit classrooms.

For this project, we have two theoretical frameworks: the studies done in ethnomathematics to help us better understand the impact culture has on the learning of mathematics and collaborative research that guides us in our work with teachers (Bednarz, Poirier, Desgagné et Couture, 2001; Desgagné, Bednarz, Couture, Poirier et Lebuis, 2001).

The social dimension of mathematics has grown in importance in the teaching of mathematics (*e.g.,* Lakatos, 1976; Ernest, 1991). If mathematical knowledge is a social construction, the community and the culture of the learners will play an important role in their learning. According to Bishop (1988), we are more and more concerned by what he calls the cultural interfaces in the teaching of mathematics:

> In other countries, like Papua New Guinea, there is criticism of the 'colonial' or 'Western' educational experience, and a desire to create … an education which is in tune with the 'home' culture of the society. The same concern emerges in other debates about … Lapps and of Eskimos. In all of these cases, a culture-conflict situation is recognized and curriculum are being re-examined. (Bishop, 1988, p. 179)

The Inuit community of Québec is no exception. If we want to re-examine the Inuit curriculum and develop learning activities adapted to the Inuit culture, the researcher who is not a member of that community can not do that alone. The risk of developing activities that will not be suitable, or well-adapted, is too great.

When a researcher develops such teaching situations the question of the validity of those situations for the school rises (Artigue, 1990; Arsac, Balacheff and Mante, 1992; Desgagné, Bednarz, Couture, Poirier et Lebuis, 2001). The teachers will use these situations according to their environment and their conceptions about teaching

and learning. This process can have an impact on the learning situations and the researchers sometimes do not recognize the situations they have created. On the other hand, teachers sometimes have great difficulties reproducing what the researchers have put on paper: the environment and the context are not the same. How can we bridge the difference between these two worlds?

The development of learning situations, in our view, has to go through the understanding that the teacher has of the environment and his conception of teaching and learning. This seemed particularly important in the context of teaching mathematics in the Great North to Inuit children. It was then necessary to integrate people of the Inuit community in the development of the learning situations. Our team included 4 Inuit teachers, 3 Inuit teacher trainers and myself. This group helped us get the triple point of view that we felt necessary: Inuit culture, the teachers' experiential knowledge and didactics.

The collaborative research framework seemed an interesting path to follow since it implies that the teacher's actions and the rational behind these actions are part of the data of the research). The aim is not only to develop interesting didactical situations that will help students acquire certain knowledge (what didactical analysis would help) but those situations must be viable in context, in the classroom for which they are meant. This can only be achieved, in my point of view, with the help of the teachers' experience and in this particular project, with their knowledge of the Inuk culture. Cooperation between the researcher and teachers in creating adapted teaching situations is given concrete expression in reflective practice (Schön, 1987). It involves a planned alternation of situation development, classroom experimentation, and feedback. This planned alternation looks like this: Team meeting to elaborate learning situations – experiment with these situations in the classrooms – discussion of those experiments and development of new situations – experiment with these new situations and so on.

In order to start the discussion, we used Bishop's framework. Thus Bishop (1988) recognizes that mathematics is a cultural product and as such has been developed in several different ways depending on the culture. However, he has recognized 6 domains that are present in the different cultures. These domains seam to be necessary to the development of mathematical knowledge (number, localization, measurement, design, games and explanation). It is interesting to note that these domains constitute the mathematical content of primary school. One way, according to Bishop, to diminish the gap between the phenomena of enculturation and acculturation would be to develop a bi-cultural strategy:

> One possible way is to use as a structural framework the six activites… If these activities are universal and if they are both necessary and sufficient for mathematical development, then a curriculum which is structured around those activities would allow the mathematical ideas from different cultural groups to be introduced sensibly. It is indeed possible by this means to create a culturally-fair maths curriculum. (Bishop, 1988, p. 189)

During our meetings we discuss one or several of these domains: how were they dealt with in the Inuk tradition? How are they taught in the Kativik curriculum? How are the teachers teaching them to the students? These discussions give us the opportunity to refresh the teachers' memory about these mathematical concepts. Some of these teachers are not trained. Experimentation and analysis thus take place in two phases: analysis of the meetings between the teachers and the researcher, and analysis of classroom experimentation. This project is about half way through. But already we can see some benefits of that type of research but also some drawbacks.

In September 2002, after having read the project, Betsy Annahatak who is in charge of the Curriculum Development Department of the Kativik School Board wrote this about the collaborative research:

> The collaborative research project that you described have some important elements that will help Kativik on the process of developing a Math Curriculum for Kativik School Board. Although the subject being addressed is on Math, as a curriculum developer I am very interested in this research because I expect to see elements and factors extracted form this research that will help us structure other programs and help us develop a culturally responsive curriculum for Kativik School Board. This research proposal is also a unique project in the history of KSB research specifically addressing curriculum questions in a minority, bicultural, and bilingual situation. As described in your paper, the dual phenomena with two cultures in contact in a learning environment, and in a school setting using the subject of math, is like an unexplored expedition to a foreign area of the universe of learning. (Betsy Annahatak, Curriculum development department, Kativik School Board, September, 2002).

The Inuit People, one of the most studied in the world, unfortunately do not get much back from research. Collaborative research, involving people from the community, gives a better assurance that it will bring something back to the community. Being part of a research group such as this one is a great learning experience for all participants. But it raises several challenges that will be discussed further in our presentation: difficulties of bringing people from different cultures together (teaching culture and research culture; Inuk culture and Quallanat culture), the language issue (several mathematical words and expressions simply do not exist in Inuktitut and when they do, some may induce erroneous conceptions, for example, a rhombus in Inuktitut is said as the " square from the playing card"), the teacher training…

DEVELOPING A VOICE

Gershon Rosen

Question 1: I am a full time teacher in a secondary comprehensive school in Israel, teaching mathematics as well as other subjects. Being a full time teacher, I see my role in this RF as representing the practitioner in the school situation, trying to make our voices heard. I am in the privileged position of being on the front line on a day-to-day basis, coping with all the frustrations as well as enjoying the highlights of educating our youth, not just teaching them mathematics. I am also a link between the practitioner and the researcher as I am in regular contact with those in Israel who

research mathematics education and produce texts and materials for the classroom. My role for many years has been that of disseminating, interpreting and adapting current research for the teachers through workshops, in-service courses and working with the teacher in the classroom as well as bringing back to the researcher the comments of the teachers and adaptations made in order to "make things work".

Results from my own research as a practitioner have been published mainly, and on a regular basis, in professional journals in the UK and in Israel. When I started teaching mathematics all that was required was a Bachelors Degree in Mathematics. No teaching certificate was needed. My first teaching post? - an all boys comprehensive in inner city London. I was the new boy who was given the classes that no one in the department wanted to teach, mostly non-English speakers, in a lecture theatre - the boys sitting up there looking down on me. This was my first realization that not everyone should be taught maths in the "traditional way". Without being aware of it I had already started to research my practice. Forty years of accumulating practical knowledge: what works in what situations; when to give pupils an answer to their questions; when to help them discover the answer for themselves; when is a pupil ready for a mathematical proof? and when to leave a proof to a later date in order not to interrupt the thought process driven by intuition. This kind of research is not driven by a specific question but varies from lesson to lesson, class to class and from year to year and is influenced by so many external factors. There is no possibility of a clinical or quasi-clinical investigation, and in any case, such a "laboratory" investigation has very little relevance to what is going on in the classroom. The priorities are different.

Over the years I learned to try out different approaches, adapt them, re-write them. As I look back and forward, I see that what I have done in my own classroom and in my work with other teachers has emerged from questioning established wisdom, in both curriculum practices and research practices. I found conventional ways of doing mathematics as prescribed in official textbooks were not working for me in my classroom, and I was driven by the need to search for new ways. Thus I have looked closely at what I am doing while teaching and learning, studying it, seeing what works and what doesn't and trying to find out 'why.' I have felt the need to share with other teachers, especially with those who are living with challenging school situations in order to share with them what has worked for me, and to help them explore their own ways of doing mathematics both for themselves and with the children. In my work with them, I encourage them to build upon their own life experience as a learner as a model for ways into mathematics. I will illustrate with the following vignette:

> I was recently asked by my colleagues, a group of experienced and successful teachers but very traditional in their approach, to give a workshop on teaching probability. They implied that they had never learnt it for their degree and were hesitant of teaching the topic. One member of staff said that she had once solved a very simple question with her class. It had to do with drawing two coloured balls from a bag with replacement. She was not sure that she handled it correctly. I encouraged her to describe how she proceeded

> and we would work from there. What was of particular interest to me was that, for example when teaching geometry, this teacher endeavoured to ensure that the students could quote definitions of the various geometrical figures and could set out formal proofs even if it meant rote-learning. When it came to solving the probability problem she drew pictures and tree diagrams and said forcibly: "I don't use any formal words. I just draw pictures and use elementary procedures like counting. I know that there exist formulae but I don't know which ones to use - I need to see the full picture - That's how I understand it and that's how my students will understand it."

There are two main elements in this teacher's response which are key features of my research and which have guided me in my work with my students in my classroom, as well as in my work with other teachers. These elements are: "I need to see the full picture" and "I don't use any formal words. I just draw pictures and use elementary procedures like counting. I know that there exist formulae but I don't know which ones to use." The idea of "using your own words" is a crucial one: keep close to your own way of doing. Seeing the full picture is another vital idea. I take issue, as I will state below, with research that breaks things down into small entities, with the result that the whole picture is lost. My theory about teaching is that with *less* we can do *more*, and I have expanded upon my theory elsewhere (see Rosen, 2003, pp. 91-96). Put very briefly, I submit that we can often achieve an understanding of a task using more primitive methods than the textbooks prescribe. Globally we consider the world we are about to explore mathematically. *With less* we find an elementary technique with which to explore and with *do more* we explore as much of that world as possible with that elementary technique . . .

For the first ten years of my teaching career, including the years in which I attained both a teaching certificate and a master's degree in mathematics, I took little account of research in mathematics. The only personal contact I had with researchers in the UK was with the late Edith Biggs who was a practitioner–researcher and in many ways has been a role model for me. Since coming to Israel and taking a course at the Weitzmann Institute, I have regularly collaborated with many of the researchers both there and at other academic institutes in Israel. Through recent encounters with educational researchers in Israel and the UK, I have been introduced to some forward-looking possibilities.

However, it has seemed to me that generally there is too much research for research sake with little connection to the realities of the classroom situation; looking at pupils' mistakes and misunderstandings and concluding with the feeling that teachers should "do something about it". Many maintain that mathematics is hierarchical and that a mastery of the basics is required before moving on to higher levels. I have read learned papers that break down a topic, such as word problems, into levels of difficulty and formats concluding that these formats should be worked on by the teacher. I have argued (as you will see in regard to my theory *with less do more*) that this type of breakdown leads to the writers of material and the teachers of the mathematically less gifted, taking ever decreasing steps until pupils loses interest because they feel they are not making any progress, or more importantly, lose sight of the whole because the little pieces have become discrete and thus meaningless.

Researchers could do more to connect their work with life in classrooms, adapting their research papers to appear in journals that have a teacher audience, and show how their research work applies in practice. They might suggest other articles, which would, for example, point out theoretical frameworks that ground their study and provide links to further reading, thus enabling readers to extend their understanding of the article.

Question 2: I am driven by the need to make a difference in classrooms, for the non-academic students I teach. In addition, I would like the teachers with whom I work to see that they can make a difference in their classrooms. I have developed my own theory about teaching and learning (*with less do more*). The work I have done with my children and with other teachers has transformed me as a person and as a teacher. Thus my answer to the question is yes to all three points. My aim is to empower the people with whom I work, the children in my classrooms, and the teachers with whom I engage in workshop sessions. I endeavor to elicit from them/show them how they can succeed. Dilemmas regarding how to go about teaching curriculum topics designated by the Ministry are a key focus in my discussions with the teachers. I will present one example. It is one of a number of dilemmas that have arisen in discussion with the teachers. In this case, the teacher remembered that when she herself had learned arithmetic series in school she substituted in a formula and solved equations, but her pupils couldn't handle even simple algebraic manipulations. How was she to proceed?

> I opened the book at random and pointed to the following question:
>
> Given the arithmetic progression 11, 14, 17, . . . how many terms must be added together to reach the sum 861?
>
> She said that she couldn't remember the formula. I said that she didn't need to, just use any knowledge she had as this would be the way she would have to work with her pupils. I produced a calculator, paper and a pencil and told her to start writing. She used the calculator to continue the series down the page. We didn't even define arithmetic series. She started to add the column of figures until she reached the required sum. She then counted how many numbers and wrote down the answer. Here was a case in point of the two basic principles, that of *with less do more* and never losing sight of the generality or globality of the problem involved. To get to a particular sum was a blip in the generality. The sum could have been any number reached before or after the designated sum. At the same time it was also clear which totals could not be achieved by summing this series. The control of the question was in her and hopefully in her pupils' hands. I said that now she and her pupil should see how many of the questions they could solve using the calculator as a tool and being in control of the problem.

I contend that the strategies I share with/elicit from these teachers are ones useful not only in their classroom work with non-academic students, they can inform the teaching and learning of mathematics by all learners.

Question 3: An essential focus of this paper is that of developing voice, power and identity in regards to working with mathematics. My attention is threefold: to help the less-mathematically-gifted non-academic student develop his or her mathematical

voice and identity by helping the non-mathematics graduate develop his or her voice as a mathematics teacher. As a bonus, develop my voice as a researcher. My belief in my students is paramount. My work with them enhances their self-esteem. Helping them develop their mathematical voice, even if their mathematical vocabulary is limited, is a vital goal. In my interaction with the teachers, I am encouraging them to question, to re-shape and re-invent their practice, and to try to determine what works for them, and why. The strategies I emphasize with the teachers – strategies which I elicit from them, such as saying things in one's own words, looking at the big picture, drawing pictures and using elementary procedures - foster the emergence of the teachers' voices, and through them, their students' voices. The students, and at times the teachers, are re-shaping their identities as doers of mathematics, and as people who can engage with it in strength, or if not with strong positive feelings, as least not with avoidance or fear. I continue to develop my voice as researcher by questioning, by studying, by learning with and from others in a reciprocal way.

LEARNING ABOUT MATHEMATICS AND ABOUT MATHEMATICS LEARNING THROUGH AND IN COLLABORATION

Vicki Zack and David Reid

Question 1: We are Vicki and David. Vicki is an elementary school teacher and a researcher of her own practice for the past twelve years. David is a university educator and researcher interested in teaching. We first met in 1995 during the PME conference in Portugal, a surprise given that we had both lived and worked in Montreal for many years, but had somehow never run into each other. Our collaboration began eight years ago when Vicki invited David to help with an inquiry that had stumped her and her students that year (1996-1997). Since then our work together has evolved as we have explored, individually and together, ways of stimulating and studying children's learning and our own learning.

Our collaboration has taken several forms. Vicki's research has generated a corpus of video and written data recording her student's interactions in solving mathematics problems in small groups. We sometimes view videos together and discuss what we see through the filter of our own research interests. At other times we watch separately, and discuss by email or phone. Sometimes our research focus arises from an interesting episode, and at other times we wish to explore a general phenomenon in more detail and choose specific episodes to study that are suitable. At times David has taken on the role as a guest teacher in Vicki's classroom and this provides us with additional video and written data from a different context. Quite often, as we will describe below, we see something that puzzles us and having a second person to view, analyse and discuss the data helps us to move our understanding forward. At other times we are theorising together and our work with the data grounds our discussions.

Question 2: In this section we will use our individual voices to address the question of why we engage in teacher-researcher collaboration. Vicki speaks first.

Vicki: Through close study (research) and at times with crucial input from David, I have learned more about the children's ways of thinking and more about the mathematics, and this in turn has affected my practice, in a continuing cycle. I will reflect below on the diverse and vital roles David has played in my learning: David as resource, as catalyst, and as collaborative partner as we explored questions about mathematics and about how one comes to understand mathematics. For me, engaging in teacher research work alone and in collaboration with David has resulted in personal transformation, in making a difference in classrooms, and in developing theory about teaching and learning.

David as resource person: I will begin by discussing David's role as an invaluable resource and support. I have enlisted David's help on a number of occasions when aspects in the mathematics have puzzled and intrigued me. My background in mathematics is weak. At times I feel vulnerable when I do not understand, and I will only seek help if I feel I can trust the other person to not make me feel inept.

In one instance, about which I have written and spoken previously at PME (*e.g.*, 1997), I was startled to discover that the children and I could not construct an algebraic formula for the Count the Squares task (a variant of the chessboard problem), which I had assigned to them (Zack, 1997). I was stuck. The 'non-obvious' algebraic expression which was available in a mathematics journal and which I showed the students, $n(n+1)(2n+1) \div 6$, was of interest to many of the students in my class, but they wanted to know why it worked as it did. During the 1997 PME conference in Finland, I appealed to members of PME to see if any could suggest a way to make this formula -- $n(n+1)(2n+1) \div 6$ -- meaningful to 10-11year olds. A number of people with whom I spoke shared their individual understanding of the proof but were perplexed in regard to how they would make it meaningful to fifth graders, and one wondered why I would even pursue this endeavour. David took on the challenge, and worked for a number of years, trialling a number of approaches with various cohorts of my students (1995-1996, 1998-2001), with the goal of showing the children how the non-obvious formula works (see Zack & Reid 2003, 2004 for an example of one of the visual proofs David constructed).

In another instance, again in regard to a component of the Count the Squares task, in response to an idea proposed by two of their team members (Ted and Ross), three students in the five-member team offered counterarguments embedded in everyday language, but which upon closer analysis revealed a complex mathematical structure. In considering the children's arguments, I asked David to use a mathematician's phrasing to express the children's ideas; as a result I and others were better able to appreciate the complexity inherent in the children's ideas (Zack, 1999). In yet another instance, in a situation in which I had asked all the children in the class to consider the Ted-Ross idea heard a number of years before and to see if they agree/disagree, and to state why, one child, Jake, offered a counterargument which was startling and clever. In a follow-up interview I asked Jake to explain his thinking. I, however, could not understand what Jake was saying, and appealed to David to explain Jake's thinking to me (Zack, 2002). Only then could I appreciate the power of Jake's

pattern, and understand why it worked as it did. Later I was startled to realize that the pattern Jake constructed was the same pattern which Mason, Burton and Stacey (1982) present in the book Thinking Mathematically. Thus, in the above-mentioned examples and other instances, due to close study and essential input from David at critical junctures, I have grown in my understanding of the children's ways of thinking and of the mathematics with which we are engaged.

David and Laurinda (Brown) as catalysts: On the idea of doing the "same problem" again and again: I will share here an instance in which David and Laurinda served as catalysts to me, asking that we all consider the question of what happens when we assign the 'same problem' again and again with different groups. In deliberating upon what I gained by re-visiting the 'same problem', I noted that the first year gives me a feeling for the preliminary framework. In subsequent years, most of the learnings which emerge are common (though never commonplace) to my classes over the years, but it is the unusual pathway(s) and the resultant learnings which have been of particular interest to me.

Laurinda has suggested that the teacher's 'noticing', which becomes more finely tuned with each encounter, "has everything to do with 'what is possible to see and hear' (Brown, Reid, & Zack, 1998, p. 55).

David and I as collaborative partners exploring together the idea of "good-enough understanding": For the past few years David and I have been discussing how one comes to understand complex ideas. Our interest arose as a result of our in-depth study of the thinking of the fifth-graders in my classroom, and as a result as well of our reflections on our own learning. The episodes focal to our discussion of "good-enough understanding" were the ones during which David met with my fifth-graders during one week in May (1995-1996, 1998-2001) to discuss with them the visual proof he had constructed. The discussions led to the two of us theorizing about how one learns complex ideas (Zack & Reid, 2003, 2004). I feel odd to be speaking about theorizing since my feeling had always been that theories were woven by philosopher-academics and handed down to teachers who then tried to understand them. And yet here I am theorizing. We will briefly explain our thinking. Learning mathematics is often portrayed as sequential; complete understandings of underlying concepts is assumed to be necessary before new concepts can be learned. However, we contend that all learners operate with good-enough understanding. When confronted by many complex ideas the first time through, learners (children and adults alike) make many tentative, temporary decisions and keep a number and sometimes contradictory possibilities 'in the air', waiting at times to the end to make sense of what has happened. Opting for a temporary decision which is 'good enough for the time being' is not only a good move, it is one we make all the time when in the midst of learning. In the everyday use of the term, some have equated the ideas of 'good enough' and 'making do' with laziness. However, we submit that good enough is the best we can do when doing our best, that is, when putting in maximum effort. As we show in our two-part article (Zack & Reid, 2003, 2004), the students press to make sense of complex ideas. The untidy and inevitably partial nature of the

students' work is part and parcel of coming to understand. The students' disposition to proceed on the basis of an incomplete grasp is, we contend, an essential component in complex problem solving. The evolution of our thinking about "good-enough understanding" could not have happened without our longstanding work together.

David: Most of the work I do, I do in collaboration. This is an extension of my belief that learning is a social process, so I pursue my own learning through research in social contexts. Because I am interested in learning more about the way students reason in the specific context of school mathematics classes, much of my collaboration occurs with teachers. My collaboration with Vicki is unusual as she had already made her classroom a research site before I met her and began collaborating with her. This means that she brings a rich theoretical background, a commitment to teaching through problem solving, and an unusually rich data set to our work. I benefit from all these. Vicki's expertise in communication and discourse offers an alternative to my more psychological and mathematical perspective. She has taught me a great deal about this way of viewing mathematical activity. Vicki's commitment to teaching through problem solving results in a classroom context that is (unfortunately) unusual in Canadian schools. Not all teachers have this commitment, or the background to create such learning contexts. As I am interested in observing the reasoning that takes place in such contexts my collaboration with Vicki gives me access to data that is otherwise hard to come by. And because Vicki was already researching in her classroom before we met, the data she has gathered stretches back in time and covers a wide range of children with different styles of approaching problems. Vicki also has a phenomenal memory of individuals and events and can usually locate examples of similar or contrasting behaviour by other children in other classes in other years.

Question 3: Vicki: One of my goals has been to show others the power of children's thinking. The children know that what they say matters to me, that I am listening and observing closely because I am genuinely interested in them and in their thinking. In regard to aspects of proving, in particular in regard to counterarguments (refutations), the children have pointed the way. They formulate generalizations about observed regularities in regard to diverse patterns they have detected (NCTM, 2000, p. 262) and use this reasoning in situations which are real and meaningful to them, to prove or disprove mathematical claims. My role is to study provocative instances, work to understand them, go at times to David for help which further deepens my appreciation of the children's thought processes and their relationship to the history of mathematical thought, and work with the children to make explicit to them the power of their reasoning. I have shown them how singular their work is, and that at times they have engaged in the problem-solving process with ideas which reflect original thought. I want to be sure that my voice is heard and that through me the children's voices are heard, and so I write. Knowing that I share their ideas with teachers and researchers through conference sessions and publications is powerful for the children. Our identities as mathematics learners with important ideas to share and pursue are established.

David and Vicki: In our collaboration we sometimes speak together, to the community of mathematics educators (*e.g.*, Zack & Reid, 2003, 2004; Brown, Reid & Zack, 1998). But we also speak separately at times, writing papers independently but reading and commenting on each other's writing throughout the process. These papers are also directed to the mathematics education research community (Reid, 2001), as well as to mathematics teachers and other educators (*e.g.*, Zack, 1999).

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