WHAT IS THE PRICE OF TOPAZE?

Jarmila Novotná, Alena Hošpesová

Charles University in Prague, University of South Bohemia České Budějovice

Abstract: In this paper we study the influence of Topaze effect on 14-15 year old students' learning in a sequence of mathematics. We use transcripts of interaction between the teacher and her students and statements of the students and the teachers from post-lesson interviews to document both the teacher's pedagogical beliefs leading to the effect and the consequences of overuse of Topaze effect on quality of students' understanding of mathematics.

> "If we consider the intimate sphere of everyday discourse in mathematics classroom, we can discover patterns and routines in the lessons' micro-structures which constitute the "smooth" functioning of the classroom discourse, while nevertheless having undesirable consequences for the pupils' learning behaviour."
>
> Voigt (1985, p. 71)

RATIONALE OF THE STUDY

Learning mathematics is viewed as a discursive activity (Forman, 1996). It is broadly accepted that the social dimension of learning influences individual's ways of acquiring and using knowledge of mathematics. Environmental effects on learning mathematics are e.g. explained using Brousseau's concept of *didactical contract* presented in the 1980s, i.e. the set of the teacher's behaviours (specific to the taught knowledge) expected by the student and the set of the student's behaviour expected by the teacher. It equally concerns subjects of all didactical situations (students and teachers). This contract is not a real contract; in fact it has never been 'contracted' either explicitly or implicitly between the teacher and students and its regulation and criteria of satisfaction can never be really expressed precisely by either of them. (Brousseau, 1997; Sarrazy, 2002)

The interplay of relationships and constraints between the teacher and students may also produce certain unwanted effects and developments that can be observed (e.g. the Topaze effect, the Jourdain effect, metacognitive shift, the improper use of analogy). (Brousseau, 1997) They are inappropriate for the learning (especially from the metacognitive point of view) but often inevitable (Binterova et al., 2006). It is more their systematic use that is detrimental. In our analysis of videotaped lessons we will focus on the Topaze effect. Its occurrence in a teaching unit influences significantly the quality of the envisaged learning process.

Topaze effect

The Topaze effect can be described as follows: When the teacher wants the students to be active (find themselves an answer) and they cannot, then the teacher disguises the expected answer or performance by different behaviours or attitudes without

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providing it directly. In order to help the student give the expected answer, the teacher 'suggests' the answer, hiding it behind progressively more transparent didactical coding. During this process, the knowledge, necessary to produce the answer, changes. (Brousseau, 1997)

In most cases, the use of Topaze effect is accompanied by lowering of intellectual demands on students (lowering of intellectual demandingness of the given tasks). It is a reaction, an action or an answer that is expected from students. Understanding is not checked. The teacher replaces explanation by a hint.[1]

Let us illustrate what we perceive as manifestations of Topaze effect using a teaching episode from teaching of linear equations in the 8th grade (students aged 14, 15). The students' task was to solve the following word problem: In a laboratory, 2l of 30-percent solution of sulphuric acid is mixed with 4.5 l of 50-percent solution of sulphuric acid. What percent solution is created? The transcript of the episode comes from the initial phase of the solving process. The teacher, guided by in the Czech Republic deep-rooted methodology of solving word problems led the students through analysis of the problem and its brief record. A brief written record of the word problem as the first step is, with very few exceptions, used by all teachers in the Czech Republic. This method imitates word problem solving carried out by experts (Odvárko et al., 1999; Novotná, 2000) and is widespread in Czech schools.

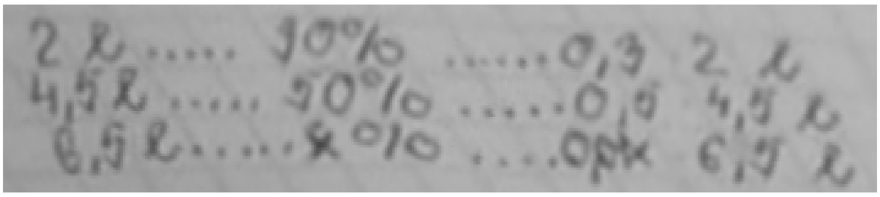
In our teaching episode, the teacher wrote the record of the given data on the blackboard on her own, without any effort to activate her students. Only then did she try to involve them in the activity. However, the effect of her questions was not that her students would look for the data needed for the problem solving and their coherence, which was crucial in this phase of the solving process. Answers to her questions only needed retrieval and reproduction of data from the wording of the problem. The teacher wanted to activate the students but was not patient enough. Application of Topaze effect secured smooth progress of the solving process. The teacher formulated the questions in such a way that the students had no chance to influence its direction. Her aim seems to have been to secure that each step of mathematization of the word problem is understood by the students.

That is why we perceive this teaching episode as a manifestation of the Topaze effect. The students did not search for a structure in the assigned data, they only reacted to the teacher's questions. We present here a relatively long extract with the aim to illustrate that what is at stake here in not just one isolated question but the teacher's

[1] In mathematics teaching and learning, explanations play an important role. Traditionally, explanation belongs to monological teaching methods where the information is transmitted in the direction teacher to students. In this perspective, explanation is seen as the task fulfilled by the teacher with students passively receiving what is presented. In (Levenson, Tirosh, Tsamir, 2006), explanation is seen in a much broader sense: An explanation can be given by students and teachers as a means for clarification of their mathematical thinking that they consider not clear to the others.

strategy as a whole. The places which are in our opinion manifestations of the Topaze effect are printed in bold letters.

601	Teacher (from now on T):	So what? 2 l, that is an important piece of data. And with it goes 30 %, see, and thirty percent acid. And 4 and a half and with it goes fifty percent acid. **What percent solution is created? Let's complete the third line.**
602	Lenka:	**That is x.**
603	T:	**And how much of it is there, Denisa?**
604	Denisa:	**6,5**
605	T:	So I can write it down there. *(The teacher finishes her record on the blackboard.)*



606	T:	So. Now we have an unknown and we will have to make an equation. **What will we equate with what?** .... **What does thirty percent acid mean in fact?** Or fifty. Well?
607	Student:	**Concentration.**
608	T:	Pardon? Concentration. To put in other words, what is it **... Dddddd... Di ...** Dilution, isn't it? There is some chemical **and the remainder is ...**
609	Students:	**water**
610	T	Yes. **And when we mix it …**
611	Student:	**New dilution is created**
612	T:	New dilution. And that which was added, that of the different concentration, you see, and that which was added from the first liquid, the chemical substance, and from the other, must in the end be in the result, mustn't it? (*She imitates by gestures pouring of two liquids into one container.)* So what will be equated, what? ... The pure chemical substance, not the concentration, but the chemical substance. **So, how much of it is there in the first liquid, Vojta? Say, how much, Vojta?**
613	Student:	**Thirty percent.**
614	T:	Thirty percent. How much of this is 30 %? (*She points at the item "2l" in the record on the blackboard.*) **How can this be calculated? Lucka.**
615	Lucka:	0.6
616	T:	Yes, but how did you come to it? I would prefer to have it written, you see, to…
617	Lucka:	30 % in one litre is 0.3.
618	T:	Aha. **How do we calculate 30 % of anything? Vojta.**

619	Vojta:	**Divided by 100, times 30.**
620	T:	OK. Now to avoid division, it is always better to avoid it wherever possible, **it will be**
621	Student:	**Times 0.3.**
622	T:	Times 0.3, yes. So it is 0.3 times 2 litres, is it? Written it down so that everybody can see how we got it. Which is exactly what you said, that was correct, 0.6, yes. **So how do we calculate 50% from the other liquid?**

The consequences can be seen in the following sequence which is basically routine solving of the problem arising from mathematization. The teacher well aware of the students' passivity says: "You only copy it. I can see that very few of you are making any calculations." It is due to the Topaze effect that the teacher, instead of asking for mathematization of the wording of the problem (i.e. its translation from common language to the language of mathematics), contented herself with simple expression with the help of decimal numbers.

OUR RESEARCH

Research questions

To be able to study the influence of Topaze effect on students' learning, our decision was to start by attempting to answer the following questions: How does Topaze effect reflect teacher's beliefs? How does Topaze effect influence students' work?

Method

Data for this study were gathered in the 8th grade (students aged 14-15) of a junior secondary grammar school, the alternative to more academic education. The framework was based on the method used in Learner's Perspective Study (LPS) (Clarke, 2001). A significant characteristic of LPS is its documentation of the teaching of sequences of lessons. This feature enables to take into account the teacher's purposeful selection of instructional strategies. Another important feature of LPS is the exploration of learner practices. LPS methodology is based on the use of three video-cameras in the classroom supplemented by post-lesson video-stimulated interviews (Clarke, Keitel, Shimizu, 2006). What is vital here is that all interviews be held immediately after the lesson. They enable revelation of the teacher's beliefs.

Our experiment

In the following text, we will call *experiment* the ten consecutive lessons on linear equations in one eighth grade classroom together with post-lesson interviews. The length of one lesson is 45 minutes. The experiment was video-recorded, transcribed and analysed in order to localise Topaze effect. (The teaching episode commented above was from Lesson 6, 28:00 – 31:50)

The experiment was carried out in a school in a county town with approximately 100 000 inhabitants. The chosen teacher was recommended for the experiment by the headmaster. The observed teaching is rated as 'outstanding' in the school. The teacher is very experienced and respected by parents, colleagues and educators as one of the best teachers in the town. The fact that she agreed with being recorded reveals that she is confident in her professional skills.

Classification of Topaze effect types in our experiments

It is our belief that types of Topaze effect considerably differ in being either explicitly stated or only implicitly suggested. Let us recall here that Topaze effect can only be considered when a previously explained subject matter is discussed. It is not connected to the process of explanation. The following types can be observed in our experiment:

Explicit prompting (overt) can be of the following nature: (a) description of steps which students are expected to follow, (b) questions related to the following solving procedure, (c) warning on possible mistake, (d) pointing out analogy, either with a problem type or with a previously solved problem, (e) recollection of previous experience or knowledge.

Covert, indirect, implicit prompting can be of the following nature: (a) rephrasing, (b) use of signal word, (c) prompting of beginning of words, (d) asking questions that lead to simplification of the solving process, (e) doubting correctness, usually in situations where the student's answer is not correct, (f) comeback to previous knowledge or experience.

Let us now illustrate the specific types by examples from our experiment and let us look for answers why the teacher is doing that.

Description of steps which students are expected to follow was used by the teacher in our experiment whenever she wanted to ensure that each step of the solving process is clear to her students: (Lesson 2, 12:28) "Somebody may have used brackets, so I will wait for your next step in which you will get rid of the brackets and then we will check it together … OK. If you use them they are all right, there is no haste …" It seems that the teacher is trying to prevent occurrence of mistakes in this way. For that matter, she explicitly states it in (Lesson 9, 6:16): "Some of you feel to be experts in how you do it and start using shortcuts by heart. Please, be so kind and don't do it. Be patient, we will get to that and then you will be asked to use a more economic record. But don't do that until you are 100% sure that you can handle it." (Lesson 9, 37:24): "Let's now multiply out the brackets … To make sure that you don't make any mistake, you should multiply out each side of the equation separately." (Lesson 5, 5:50)

T:	What did we always say that was the best way of adjusting the sides of the equation? Vítek … All variables (gesturing) … out loud …
Vítek:	All variables on one side.
T:	... and real numbers on the other side. Then it is easiest to select the method.

Sometimes the students were asked to state **how the solving would proceed**; in some cases she suggested the progression herself. A good illustration of this point is step 612 in the initial episode. Similar stimuli, however, can be found in many other places (Lesson 2, 14:13): "Well, and now comes another equivalent adjustment … which, Aneta …. (Aneta does not react.) To take two steps at once. What do you have to do?"

In some cases the teacher **directly warned on a mistake** that could happen. She was guided here by her long-time teaching experience (Lesson 2, 25:43): "Watch out, watch out, don't forget to multiply everything, yes?…"

Pointing out to analogy was of various nature: recollection of problem type, recollection of previously solved similar problem, comeback to previous knowledge or experience: (Lesson 3, 11:40): "OK, now *you* will do more work, because it is something similar." (Lesson 2, 29:02): "There is one thing that you forgot … How I did it here, see (She points at record on the blackboard). I did something similar here …".

Covert, indirect prompting was in our experiment often of the nature of **appeal to reformulate:** "How could you put this in different words?" This was used by the teacher whenever the students' explanations were essentially correct but inaccurately formulated. It may be that these reformulations were not necessary for the other students because they joined in the dialogue with the teacher easily. This can be observed in the above presented transcript – steps 614 – 622.

The teacher often directed the solving process with the help of **signal words** which she used in her questions and instructions. For example in analysis of a word problem (Lesson 10, 26:15):

Teacher:	So we know what?
Student:	That there were 52 bicycles.
Teacher:	That the sale lasted in total ...
Student:	4 months ...

This is even more striking when the teacher **prompts the beginning of the word** that she wants to hear from the student (see step 608 in the above presented transcript). This type of prompting was used by the teacher whenever she was not able to find a suitable question.

Simplification of the situation by focusing on sub steps often resulted in disintegration of the solving process and students' loss of overview. See e.g. steps 613 – 622 in the above presented transcript.

In other places, the teacher **doubted correctness of answers.** This was usually used when the answer was wrong (Lesson 3, 35:09): "This sounds really strange, doesn't it? … Really? … Are you sure?"

(Lesson 10, 31:21 – 33:30): The teacher records the word problem on her own and uses simplified formulations which make the record much easier. Later this method leads the students directly to the solution. Even if the students suggest an "intellectually higher" answer, the teacher keeps coming back to the procedure:

T:	Why do we use arrows?
Student:	To know whether it is direct or inverse proportion.
T:	Well, to be able to record it.

DISCUSSION AND SOME CONCLUSIONS

The main reason for the frequent use of Topaze effect in our experiment is the teacher's belief that students' success in mathematics can be reached by repeated execution of a series of similar procedures and that her students need this type of support for successful completion of the assigned tasks. For example in post lesson interview of Lesson 5 the teacher, when asked by the experimenter whether her students would not have problems when having to solve similar problems on their own, expressed her belief that her students are not able to work individually without being prompted: "Even if we do it with older students, almost nobody is able to solve it on their own the second time. Well, as long as it is stereotypical, you see, the algorithm is always the same, there is no exception, then perhaps the students would be able to solve it on their own."

Our hypothesis is that frequent use of Topaze effect decreases students' responsibility for successful completion of the assigned mathematical problems. Students do not work on their own, they do not discover, experiment. They wait for directions of the teacher whom they trust and imitate his/her procedure instead of individual activity. (Post-lesson interview 6)

Interviewer:	And here, when solving the task, did you solve it with them, or on your own or …
Student:	Well, the beginning, the multiplication on my own, and then I preferred to wait for the rest and continue with them.
Interviewer:	And now, if you were to solve the problem today, is it better? Or a similar one?
Student:	Well, may be a similar one, but if it were somehow different, I would probably not solve it.

What we proved in our experiment is that the price of Topaze effect is high: At the first glance everything in the lesson seems to be running smoothly. However, students lose self-confidence and are only seemingly active. They rely on the teacher's help, mistake is understood as transgression. Students routinely repeat the learned process, often without understanding. They do not attempt to find their own suitable solving strategies. The learning process fails to work with one of the key elements – mistake, its recognition and elimination.

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