Teachers could promote this ability by facilitating the atmosphere of curiosity in the classroom.

It may be very demanding to be often interrupted by children′ questions during lessons. My proposal is to teach children how to make short notes on their questions, for example in a pictorial form. The other possibility is to create a box of questions. Children can put their questions into the box and once a week one lesson could be devoted to answering these questions. Interests of children change often and if we wait too long the motivation could decrease. Questions asked by children are often quite complicated and therefore it could be useful for the teacher to read them before the lesson starts.

References

Earl, C. *What is Effective Pedagogy when Informally Educating Electively Home-Educated Children? Case Study Research*. http://www.education-otherwise.org/Links/Research%20Papers/Cassie%20Earl.doc accessed on 23. 03. 2007.

Fortune-Wood, J. (2002). *Doing It Their Way: Home-based Education and Autonomous Learning*. Educational Heretics Press.

Hejný, M. & Kuřina, F. (2001). *Dítě, škola a matematika.* Praha: Portál.

Lowe, J. & Thomas, A. (2002). *Educating your children at home*. Continuum.

Meighan, R. (2001). *Learning Unlimited.* Educational Heretics Press.

Nováčková, J. (2001). *Mýty ve vzdělávání*. Kroměříž: Spirála.

Petrie, A.J., Windrass, G. & Thomas, A. (1999). *The Prevalence of Home Education in England: a feasibility* study. London: Department for Education and Employment.

Thomas, A. (1998). *Educating Children at Home*. Cassell.

SOLVING NUMERICAL PROBLEMS IN THE 3RD YEAR OF PRIMARY SCHOOL: TEACHING AND LEARNING WITH CONNECTIONS

***Maryvonne Priolet* ☜ *and Jarmila Novotná* ☜☜**

Abstract

This paper summarizes the results of a study devoted to networking in solving word problems with numerical data. In the experiment, eight French and two Czech 3rd classes of the primary school participated. The study has two aims: to compare teachers' practices to explain, at least partly, the observed differences in pupils' performances, and evaluate to which extent teaching the solving of problems with numerical data based on networking of assignments and their representations could contribute to improving pupils' performance in this domain of mathematics.

☞ University Lyon 2, France; e-mail: maryvonne.priolet@wanadoo.fr.

☞☞ Charles University in Pargue, Czech Republic; e-mail: jarmila.novotna@pedf.cuni.cz.

1. Introduction

The paper summarizes the results of a study devoted to investigating the effect of explicit identification of mathematical and situational connections in solving word problems with numerical data. Ten classes from the 3rd grade of primary school in France and in the Czech Republic participated in the study. The impulse for this study was the relatively poor performance of eight French classes on a test involving word problems. This prompted the desire to try to stimulate teachers to actively support the use of connections among problems and mathematical situations. We will show that, despite the progress in the four experimental classes in France, with the performance higher for these students than in the control group of the other four French classes (also discussed in Priolet, Régnier, 2007), the rate of success was lower than in the two Czech control classes where explicit connection identification was a well-established practice.

This study has two aims: (i) to compare teachers' practices to explain, at least partly, the observed differences in pupils' performances, and (ii) to evaluate the extent to which teaching the solving of problems with numerical data based on identifying connections among problems and their representations could contribute to improving pupils' performance in this domain of mathematics.

In the first section of the paper, the theoretical framework used for the study is presented. The second section describes the methodology. Finally, the first results of the investigation are presented and discussed.

2. Theoretical framework

The study is based on the Theory of Didactical Situations (Brousseau, 1997). We adopt the concepts *didactical contract* (Brousseau, 1984, 1988; Sarrazy, 1995), and *obstacle* (Brousseau, 1997) for explaining the differences in the performance in French and Czech classes. We affirm that the teacher, when initializing a modification to their teaching strategies by introducing approaches focussing on conceptualisation and connections, has effectively taken hold of the didactical responsibility for what takes place in the classroom (Sarrazy, 1995).

Our ***hypotheses*** are:

Following the example of Roditi (2001), working with lower secondary classes on decimal numbers, and Sayac (2006), working with upper secondary classes, we consider the primary level, where there exists a large variety of teachers' practices for solving problems. This variety results in a diversity of pupils' activities. The teaching approach explicitly emphasising connections and representations among problems is little used in France. We hypothesise that such an emphasis can help pupils in the activity of solving problems with numerical data.

However, we do not ignore the difficulties related to the researcher's transmission of a lesson plan to a teacher, since each teacher will use his/her own interpretation (Arsac & Mante, 1989). In our experiment, we are considering the variety of teachers' practices. We respect the "pedagogic culture" of the class that influences the way how pupils experience the didactical contract (Marchive, 1995). After having identified teachers' practices, the problem of learning lies exactly in the perspective of *breaking didactical contract* (Brousseau, 1997), to identify the unspoken assumptions and habits that arise in a classroom and determine if they need to be changed.

On the other hand, we support our experiment by the theoretical framework borrowed largely from cognitive psychology. The works of Duval (1995, 2005), on the conversion of registers of representations, led us to introduce an inter-representational dimension. In other words, our experiment includes pupil's use of "referential boxes", which is a way of identifying and setting out the components and representations that will solve the problem, and a repertoire of referential schemes conducive to conversions of semiotic representations (such as iconic, textual, numerical, and so on). It concerns not only the creation of a break of didactical contract largely oriented towards a quasi-exclusive treatment of numerical data linked with the last studied operations, but more significantly it concerns bringing the pupil to use models. This last item refers to introducing the referential schemes connected with each referential box for categorising problems; here we adopt Vergnaud's classification (1990). Each time the pupil uses a referential box, (s)he has to implement changes of registers (e.g., between iconic representations and numerical). Reading the problem task, presented in textual form, should lead partly to a conversion in the iconic register when the pupil has to represent the proposed problem by a drawing, and then link the schema to a class of problems. It will also lead partly to a conversion in the numerical register, when the pupil has to look for a corresponding operation. Finally the pupil must return back to the textual register to answer the question in the form of a sentence. This approach, involving a change of registers, was used with the aim to help pupils to solve the word problems dealing with the division into unequal parts (see also Novotná, 1997).

Finally, the theory of schema (Riley, Greeno, Heller, 1983) constitutes our third loan from cognitive psychology. It allows us to explain the positive change between the pre- and post-tests in experimental classes.

3. Methodology

Ten classes of students in the 3rd year of primary school participated: four experimental classes and four control classes in France, and two control classes in the Czech Republic. In France the use of a pre-test consisting of twelve problems with numerical data enabled us to measure the pupils' initial performances.

A questionnaire was used to obtain data to characterise the practices used by teachers. In France, video recordings accompanied by simple auto confrontation interviews (Clot & Faïta, 2000) allowed us to refine the data collecting by including the theoretical framework developed by Leplat (2000) in the psychology of labour and *ergonomie* (cost-benefit analysis). In accordance with Rabardel et al. (1998) and Rogalski (2003), we considered the education teaching environment as a working situation.

In the period January to March, the four French experimental classes underwent experimental arrangements aiming to evaluate the effects of teaching problem solving based on identifying problem types, representations and connections. The teachers in the experimental classes were taught about the use of referential boxes and how to implement them in the classroom. The problem solving lessons were video recorded to detect possible changes in intrinsic and extrinsic teaching practices linked to the proposed activities. The other four French classes used as control group continued their usual work, with their teachers' ordinary practices. In June, a post-test identical to the pre-test was given to the eight French classes. The Czech classes were given the same test (translated and with minor contextual changes to reflect different cultural experiences) on a single occasion. No intervention took place for the Czech classes.

The performances of the French and Czech classes on the tests were compared. All problems belonged to the textual register and were close to pupils' everyday experience (Nunes, Schliemann & Carraher, 1993); in all problems, the question was placed at the end of the task in order to neutralise the variable "place of the question" whose influence on the solution process has been pointed out by Fayol and Abdi (1986). Seven problems were solely multiplicative; solving the other five problems required intermediate steps.

In summary, we obtained two types of data: qualitative ones in the form of transcriptions of video recordings and interviews in France allowing us to inventory and characterise teachers' practices, as well as Czech questionnaires referring to some previously video-taped lessons and the teachers' teaching strategies; and quantitative data from the pupils' solutions of the twelve problems, with pre- and post-test data from France, and a single set of test data from the Czech Republic.

4. Results and discussion

4.1. France: Comparison of performances of the four control classes and the four experimental classes

The first results reveal that in the 12 post-test problems, pupils from the experimental group succeeded, on average, on one problem more than those from the control group. In the control group, the difference between the two means in the pre- and post-tests (4.68 vs. 5.60) is 0.92, while for the experimental group it is 1.97 (4.38 vs. 6.35).

After the verification that the constitution of the groups was independent of variables like gender, age, performances in national evaluations, we attempted to detect the changes in the practises used by teachers in the experimental group, focusing on the use of connection identification, and changes in referential schemes in teachers' teaching of problem solving. Analyses are based on the following sources of information about teachers' practices: transcriptions of video recordings and the auto confrontation interviews; we distinguish between Lesson Series 1 recorded before the experiment and Lesson Series 2 recorded during the experiment; Lesson Series 3 are videos of isolated lessons in the last two years before the experiment from the two Czech classrooms to illustrate the practices of the Czech teachers involved in the study.

In this paper, we concentrate on three variables: explicit reference to previous knowledge, the place of the inter-representational approach (the use of different referential schemes), and the place of modelling.

4.1.1. Explicit reference to previous knowledge

We identified a difference between the Lessons Series observed before and after the experiment (Fig. 1). Teachers moved from a teaching free of all connections to previous learning to a teaching approach provoking the search for links to other problems, in our case done by the referential boxes. Used repeatedly during the lessons, the use of such boxes appears to have contributed to helping pupils to implement networking among the problem assignments, associated schemas and the operations necessary for the solving process. The progress of the experimental group could be, at least partially, explained by the changes in teachers' practices positively influencing the creation of suitable mental schema for a family of problems.

Example:
Class of Mrs. L. (Lesson Series 1)
No return to previous knowledge or work

Class of Mrs. L. (Lesson Series 2)

Item	Times	Speaker	
39	02.43	T.→Cl.	Who wants to remind me to the small picture that we did in order to represent the problem?

Figure 1: Request for an explicit return to the previous knowledge or work

4.1.2. Place of the inter-representational approach

Before the experiment, the eight teachers participating in the study used instructions that were nearly exclusively in the numerical register, often asking for the answers of the type "solution, operation". The experiment of introducing referential boxes in the four experimental classes and provoking a change in the four teachers' practices induced pupils' use of several registers of representations and the transitions from one register to another when performing the conversions among referential schemes. After the experimental phase we followed the modifications in pupils' written records in the experimental classes, mainly those integrating iconic representations when translating data from the problems.

Example:

Class of Mrs C. (Lesson Series 1)

Item	Times	Speaker	
108	08.42	T.→Cl.	Solution. A capital S. And O, Operations. (The teacher writes on the blackboard simultaneously with pupils.)

Class of Mrs C. (Lesson Series 2)

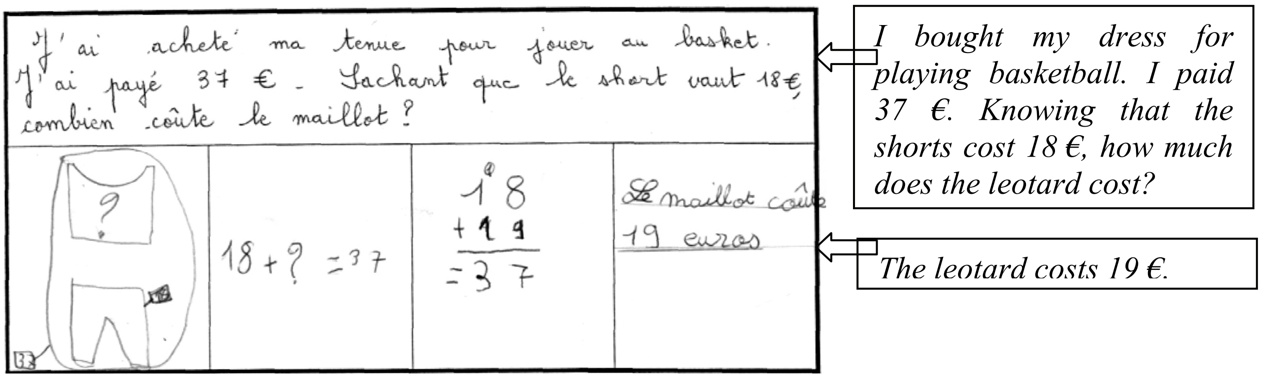

Figure 2: Example of pupils' conversion of resisters

4.1.3. Place of modelling

Neither the construction nor the use of diagrams or iconic schemes to solve a family of problems was found in the first data collected. On the contrary, the class of Mrs. L., Lesson Series 2, from the experimental group includes (Fig. 3 – carried out collectively on the blackboard) an introduction of a modelling process.

1st volume: 403 pages Two volumes: 758 pages 2nd volume: ?	403 ? 785	403 + … = 785	Known number ? Whole

Figure 3: Phases of modelling of the problem - Class of Mrs. L. Lesson Series 2

4.2. Czech Republic

At the beginning of the 3rd school year (September 2006), pupils from the control group in the Czech Republic solved the 12 problems from the test used in France translated into Czech. Instructions and conditions of testing were strictly identical with France. The average for the group of 43 Czech pupils is 7.67. The difference between the two means between the French control group in pre-test and the Czech group is 2.99 (4.68 vs. 7.67), i.e. pupils from the two Czech classes were successful at least in three more problems than the control group in France. After the experiment in France, the experimental group that moved from the average 4.38 in the pre-test to 6.35 in the post-test reduced the difference in the performance in comparison with the Czech Republic. Nevertheless, despite the experiment and the time between the two tests, the results of the French experimental group remained inferior to those of the Czech Republic (6.35 vs. 7.67).

Although the progress was seen in France after the experiment, the comparison with the Czech control group led us to investigate the teachers' practices in the two Czech classes. Three sources of information were used: video recordings of individual lessons taught by the two teachers during the last two school years and questionnaires were analysed and followed by interviews with the teachers focused on getting more details about their practices.

The data collected reveal that both teachers apply the system of "small hints" (*I will offer just a little help but nothing more*). They often assign "cascade problems": the task consists of a series of sub-problems; the solution of the next sub-problem is based on the solution of the previous one. They encourage pupils' independent reasoning and justifications, discussions, formulation of ideas from the very beginning of school attendance. Frequent individual and group work activities are accompanied by the teachers' pointed help for those having difficulties with the use of connection identification, and changes in referential schemes (see Fig. 4 for an example). The questions *Why*? and *What does it mean*? are ordinary components of lessons. Pupils are encouraged to decompose problems into several simpler sub-problems (if possible). Using modifications of problems with the same mathematical background, the "mathematics" in the problem is emphasized (*What am I interested in? Is it that it is a courtyard, a garden – that is not what interests me. What is important are the side lengths …*) Verifications of answers are requested.

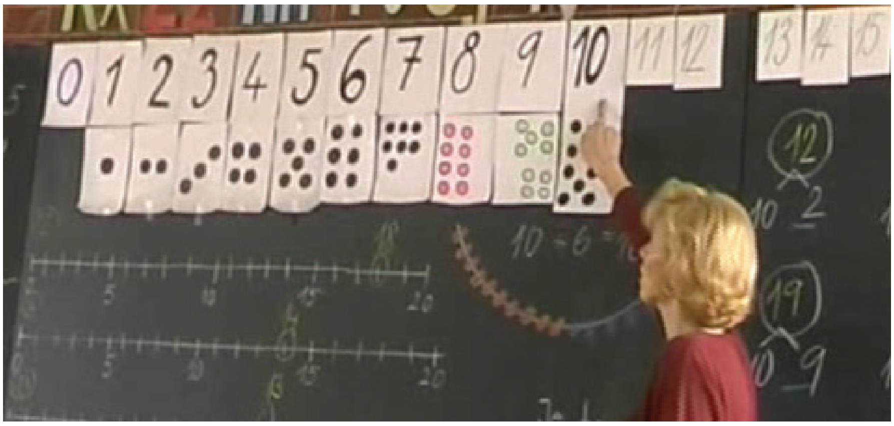

Figure 4: Different representations of numbers

The analysis of these data confirms that both teachers use practices supporting the use of pupils' previous knowledge, connection identification, and changes in referential schemes as well as changes of registers of representation.

5. Concluding remarks

There are three important issues arising from our results: The direct improvement of pupils' success in problem solving when the teacher's practices support pupils' connection-building (i.e. use of previous knowledge, changes of registers, modelling) was detected. The French control group showed a significant improvement when these practices were applied for one topic for three months. We attribute the success of pupils from the Czech control group to the long-term and systematic work with pupils from the beginning of the school attendance supporting the above mentioned activities. Pupils in these classrooms regarded it as natural to search for similar problems, to try to use the register of representation mostly suitable for their solving problem strategy, to decompose a new problem into several simpler sub-problems for which they already knew the solving algorithms, etc. Replacing the systematic and extended application of these practices by a "one-shot" (even lasting a certain period) change of practices for one topic cannot substitute the systematic work with pupils to anywhere near the same degree of effectiveness.

References

Arsac, G. & Mante, M. (1989). Le rôle du professeur : aspects pratiques et théoriques, reproductibilité. *Cahiers du Séminaire de Didactique des mathématiques et de l´informatique*. Grenoble: LSD-IMAG, 79-105.

Brousseau, G. (1997). *Theory of Didactical situations in mathematics 1970-1990*. Kluwer Academic Publishers, 304 p.

Clot, Y., Faïta, D., Fernandez, G. & Scheller, L. (2000). Entretiens en autoconfrontation croisée: Une méthode en clinique de l´activité. *Pistes.* Vol. 2, n°1, 7 p.

Duval, R. (1995). *Sémiosis et pensée humaine*. Berne: Peter Lang, 395 p.

Duval, R. (2005). Langage, symboles, images, schémas… De quelle manière interviennent-ils dans la compréhension, en mathématiques et en dehors des mathématiques. *Bollettino dei Docenti di Matematica*. N°50, 20 p.

Fayol, M. & Abdi, H. (1986), Impact des formulations sur la résolution de problèmes additifs chez l'enfant de 6 à 10 ans. *European Journal of Psychology of Education, 1*, *(1),* 41-58.

Leplat, J. (2000). L'environnement de l'action en situation de travail. *Séminaire du centre de recherche sur la formation : L'analyse de la singularité de l'action*. CNAM, Paris, 107-132.

Marchive, A. (1995). L'entraide entre élèves à l'école élémentaire: relations d'aide et interactions pédagogiques entre pairs dans six classes de cycle trois. Université de Bordeaux 2, 573 p.

Novotná, J. (1997). Geometrical Models in Solving Word Problems That Include the Division of the Whole Into Parts (Theory and Practice). In Tocki, J. (Ed.) P*roceedings Interakcja teorii i praktyki nauczania matematyki w szkole podstawowej i średniej* (pp. 109-119). Rzeszów: VSP.

Nunes, T., Schliemann, A.S. & Carraher, D.W. (1993). *Street mathematics and school mathematics.* Cambridge : Cambridge University Press.
Priolet, M. & Régnier, J.C. (2007). Résolution de problèmes numériques au cycle 3 de l´école primaire. Etude des relations entre pratiques d´enseignants et performances des apprenants. In : *Actes du Colloque international des Iufm du Pôle Nord-Est.* In print.
Rabardel, P., Carlin, N., Chesnais, M., Lang, N. Le Joliff, G. & Pascal, M. (1998). *Ergonomie, concepts et méthodes*. Octarès – Collection Formation, 175 p.
Riley, M.S., Greeno, J.G. & Heller, J.I. (1983), Development of children's problem-solving ability in arithmetic. In: H.P. Ginsburg (Ed.) *The development of mathematical thinking*. New-York: Academic Press.
Roditi, E. (2005). *Les pratiques enseignantes en mathématiques. Entre contraintes et liberté pédagogique*. Paris: L'Harmattan, 191 p.
Rogalski, J. (2003). Y a-t-il un pilote dans la classe ? Une analyse de l'activité de l'enseignant comme gestion d'un environnement dynamique ouvert. *Recherches en Didactique des Mathématiques*. Vol. 23, n°3, 343-388.
Sayac, N. (2006). Etude à grande échelle sur les pratiques des professurs de mathématiques de lycée : résultats liés à des variables spécifiques et essai de typologie. *Recherches en Didactique des Mathématiques*, Vol.26, n°2, 231-278.
Sarrazy, B. (1995). Le contrat didactique. *Revue Française de Pédagogie*, 112, 85-118.
Vergnaud, G. (1990). La théorie des champs conceptuels. *Recherches en Didactique des Mathématiques*, Vol.10, n°2-3, 133-170.

SOME ASPECTS OF COMMUNICATION IN THE TEACHING OF MATHEMATICS – THE SEMIOTIC VIEWPOINT

Filip Roubíček☜

Abstract

Communication in the teaching of mathematics is accomplished using various languages, semiotic representation systems. Their proper acquirement plays an important role not only in communication but also in cognitive processes. Some problems in communication between the teacher and the pupil can be subjected to semiotic analysis. The problems tend to be caused by different mental representations of the teacher and the pupil, by not distinguishing between of the communication contexts, non-accepting of conventional writing forms of problem solutions by the pupils, insufficient acquaintance of the pupils with rules of usage of semiotic systems and alternative forms of mathematical writing.

☞ Mathematical Institute of the Academy of Sciences of CZ, Czech Republic; e-mail: roubicek@math.cas.cz.