Should I remember more than you?

– On the best response to factor-based strategies –

René Levínský[1]

Abraham Neyman[2] Miroslav Zelený[3]

[1]Max Planck Institute of Economics, Jena

[2]Hebrew University, Jerusalem

[3]Charles University, Praha

June 23, 2015

Repeated games

Consider that a general game

G

Repeated games

Consider that a general game

$G = (N,$

Repeated games

Consider that a general game

$G = (N, (A_i)_{i \in N},$

Repeated games

Consider that a general game

$G = (N, (A_i)_{i \in N}, (u_i)_{i \in N})$.

is (infinitely) repeated. Suppose that the stage game is deterministic with a finite set of actions.

Repeated games

Consider that a general game

$G = (N, (A_i)_{i \in N}, (u_i)_{i \in N})$.

is (infinitely) repeated. Suppose that the stage game is deterministic with a finite set of actions.

Formally, by a *supergame of G* (in notation G^∞) we mean an infinite sequence of repetitions of G.

Repeated games

Consider that a general game

$G = (N, (A_i)_{i \in N}, (u_i)_{i \in N})$.

is (infinitely) repeated. Suppose that the stage game is deterministic with a finite set of actions.

Formally, by a *supergame of G* (in notation G^{∞}) we mean an infinite sequence of repetitions of G.

At each period $t = 1, 2, 3, \ldots$ players 1, 2,... make simultaneous and independent moves $a_t^i \in A_i$, $i = 1, 2, ...$

History

Initial history $h_0 = \emptyset$.

History

Initial history $h_0 = \emptyset$.

The symbol $A^{<\mathbf{N}}$ denotes all finite sequences of elements of A including the empty one.

History

Initial history $h_0 = \emptyset$.

The symbol $A^{<\mathbf{N}}$ denotes all finite sequences of elements of A including the empty one.

The set of histories $H := A^{<\mathbf{N}}$.

History

Initial history $h_0 = \emptyset$.

The symbol $A^{<\mathbf{N}}$ denotes all finite sequences of elements of A including the empty one.

The set of histories $H := A^{<\mathbf{N}}$.

A *strategy for player i in* G^∞ is a mapping $\sigma : H \to A_i$.

History

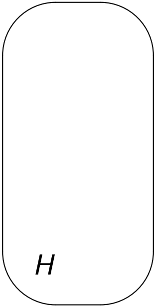
Initial history $h_0 = \emptyset$.

The symbol $A^{<\mathbf{N}}$ denotes all finite sequences of elements of A including the empty one.

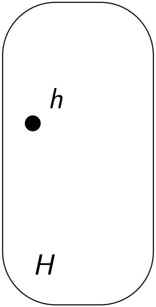
The set of histories $H := A^{<\mathbf{N}}$.

A *strategy for player i in G^∞* is a mapping $\sigma : H \to A_i$.

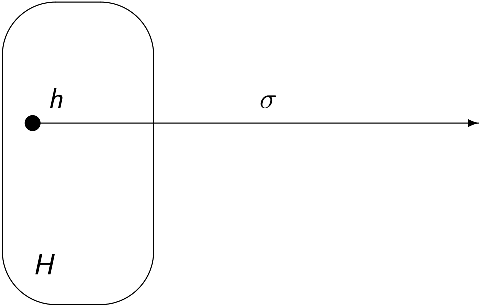
General strategy of player i



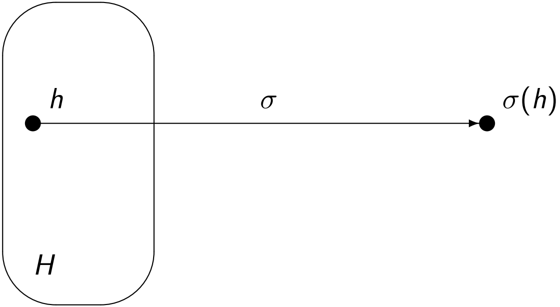
General strategy of player i



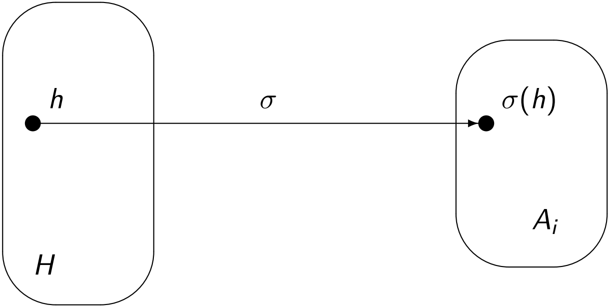
General strategy of player i



General strategy of player i



General strategy of player i



Bounded strategies of player 1

Suppose that player 1 is not able to recall the full history of the game played.

Bounded strategies of player 1

Suppose that player 1 is not able to recall the full history of the game played.

Her action in the current stage game relies only on k previous signals she observed.

Example: SBR strategies

Suppose that player 1 is capable to "remember" only last k action profiles in the repeated game and this "depth of recall" k as well as the strategy itself, are time independent.

Example: SBR strategies

Suppose that player 1 is capable to “remember” only last k action profiles in the repeated game and this “depth of recall” k as well as the strategy itself, are time independent. Formally: Let $k \in \mathbf{N}$.

Example: SBR strategies

Suppose that player 1 is capable to "remember" only last k action profiles in the repeated game and this "depth of recall" k as well as the strategy itself, are time independent. Formally: Let $k \in \mathbf{N}$.

By a *k-SBR strategy for player i* in G^∞ we mean a pair (e, ω), where

Example: SBR strategies

Suppose that player 1 is capable to "remember" only last k action profiles in the repeated game and this "depth of recall" k as well as the strategy itself, are time independent. Formally: Let $k \in \mathbf{N}$.

By a *k-SBR strategy for player i* in G^∞ we mean a pair (e, ω), where

- $e = (e_1, e_2, \ldots, e_k) \in A^k$ and

Example: SBR strategies

Suppose that player 1 is capable to "remember" only last k action profiles in the repeated game and this "depth of recall" k as well as the strategy itself, are time independent. Formally: Let $k \in \mathbf{N}$.

By a *k-SBR strategy for player i* in G^∞ we mean a pair (e, ω), where

- $e = (e_1, e_2, \ldots, e_k) \in A^k$ and
- $\omega : A^k \to A_i$ is a mapping.

Example: SBR strategies

Suppose that player 1 is capable to "remember" only last k action profiles in the repeated game and this "depth of recall" k as well as the strategy itself, are time independent. Formally: Let $k \in \mathbf{N}$.

By a *k-SBR strategy for player i* in G^∞ we mean a pair (e, ω), where

- $e = (e_1, e_2, \ldots, e_k) \in A^k$ and
- $\omega : A^k \to A_i$ is a mapping.

Player i following the strategy (e, ω) plays as follows.

Example: SBR strategies

Suppose that player 1 is capable to "remember" only last k action profiles in the repeated game and this "depth of recall" k as well as the strategy itself, are time independent. Formally: Let $k \in \mathbf{N}$.

By a *k-SBR strategy for player i* in G^∞ we mean a pair (e, ω), where

- $e = (e_1, e_2, \ldots, e_k) \in A^k$ and
- $\omega : A^k \to A_i$ is a mapping.

Player i following the strategy (e, ω) plays as follows. If moves $a_1, \ldots, a_\ell \in A$ have been played,

MAX-PLANCK-GESELLSCHAFT

Example: SBR strategies

Suppose that player 1 is capable to "remember" only last k action profiles in the repeated game and this "depth of recall" k as well as the strategy itself, are time independent. Formally: Let $k \in \mathbf{N}$.

By a *k-SBR strategy for player i* in G^∞ we mean a pair (e, ω), where

- $e = (e_1, e_2, \ldots, e_k) \in A^k$ and
- $\omega : A^k \to A_i$ is a mapping.

Player i following the strategy (e, ω) plays as follows. If moves $a_1, \ldots, a_\ell \in A$ have been played, then player i takes the sequence s, which is formed by the last k elements of the sequence $(e_1, .., e_k, a_1, .., a_\ell)$,

Example: SBR strategies

Suppose that player 1 is capable to "remember" only last k action profiles in the repeated game and this "depth of recall" k as well as the strategy itself, are time independent. Formally: Let $k \in \mathbf{N}$.

By a *k-SBR strategy for player i* in G^∞ we mean a pair (e, ω), where

- $e = (e_1, e_2, \ldots, e_k) \in A^k$ and
- $\omega : A^k \to A_i$ is a mapping.

Player i following the strategy (e, ω) plays as follows. If moves $a_1, \ldots, a_\ell \in A$ have been played, then player i takes the sequence s, which is formed by the last k elements of the sequence $(e_1, .., e_k, a_1, .., a_\ell)$, and her $(\ell + 1)$-th move is $\omega(s)$.

MAX-PLANCK-GESELLSCHAFT

Example

Prisoners' Dilemma

Example

Prisoners' Dilemma

		Player 2	
		cooperate	defect
Player 1	cooperate		
	defect		

Example

Prisoners' Dilemma

Player 2

Player 1		cooperate	defect
	cooperate	$(3,3)$	
	defect		

Example

Prisoners' Dilemma

		Player 2	
		cooperate	defect
Player 1	cooperate	$(3,3)$	$(0,5)$
	defect		

Example

Prisoners' Dilemma

		Player 2	
		cooperate	defect
Player 1	cooperate	$(3, 3)$	$(0, 5)$
	defect	$(5, 0)$	

Example

Prisoners' Dilemma

Player 2

Player 1		cooperate	defect
	cooperate	$(3, 3)$	$(0, 5)$
	defect	$(5, 0)$	$(1, 1)$

Tit for tat as SBR strategy

Tit-for-tat is 1-SBR strategy, with

Tit for tat as SBR strategy

Tit-for-tat is 1-SBR strategy, with

- $\omega(c, c) = \omega(d, c) = c$

Tit for tat as SBR strategy

Tit-for-tat is 1-SBR strategy, with

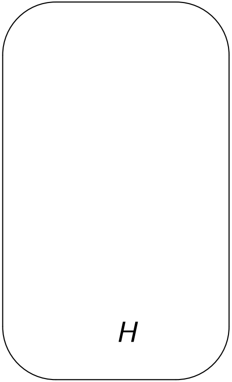
- $\omega(c, c) = \omega(d, c) = c$
- $\omega(d, d) = \omega(c, d) = d$

Tit for tat as SBR strategy

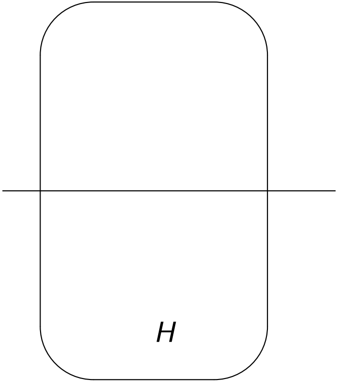
Tit-for-tat is 1-SBR strategy, with

- $\omega(c, c) = \omega(d, c) = c$
- $\omega(d, d) = \omega(c, d) = d$
- $e = (c, c)$

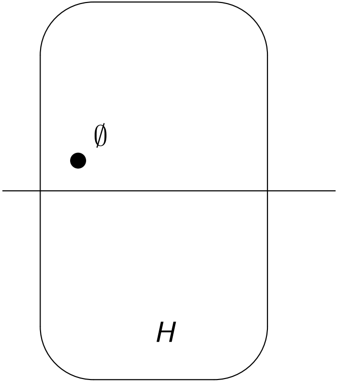
Tit-for-tat of player 1



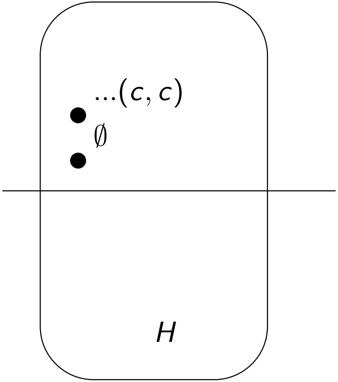
Tit-for-tat of player 1



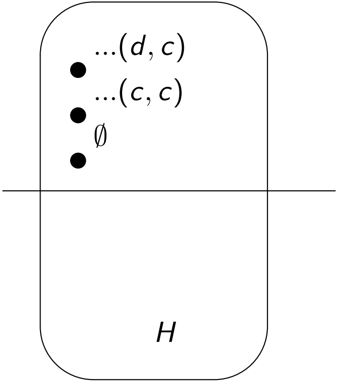
Tit-for-tat of player 1



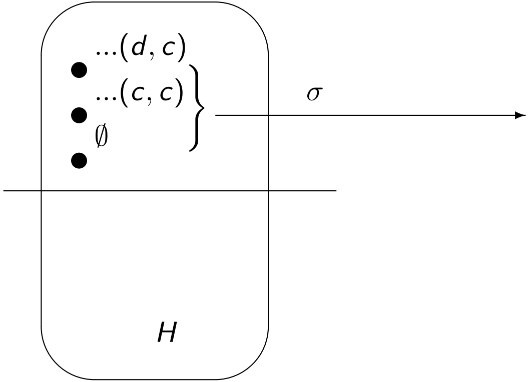
Tit-for-tat of player 1



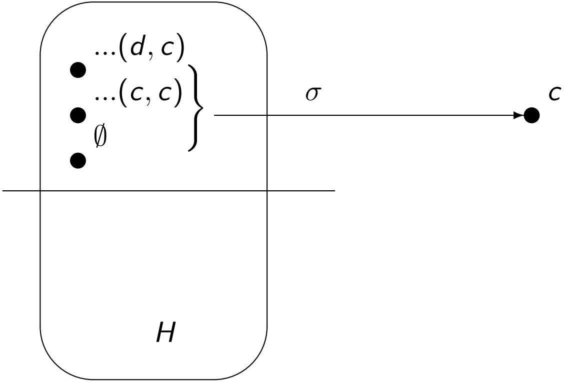
Tit-for-tat of player 1



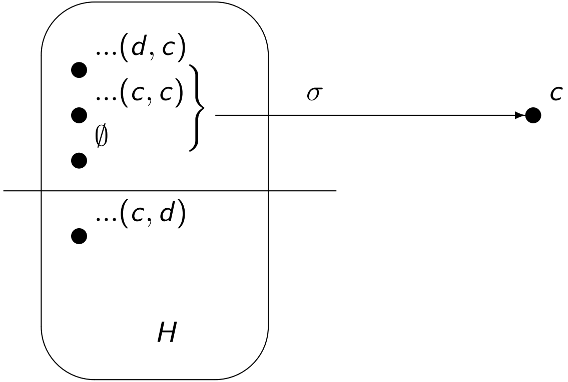
Tit-for-tat of player 1



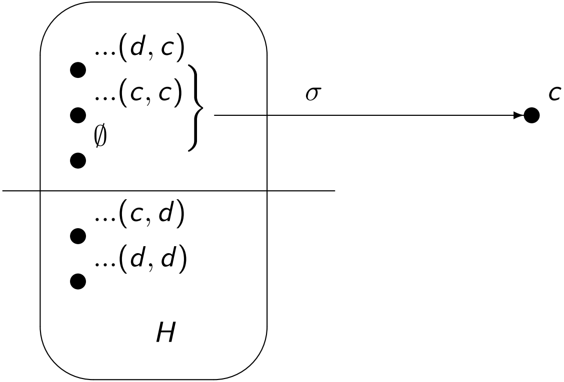
Tit-for-tat of player 1



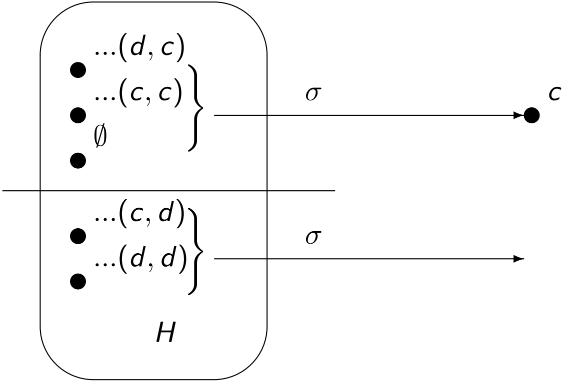
Tit-for-tat of player 1



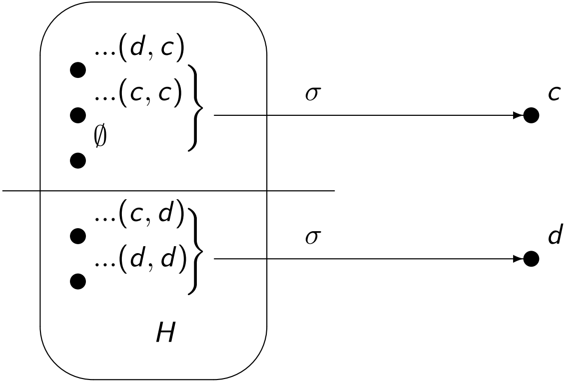
Tit-for-tat of player 1



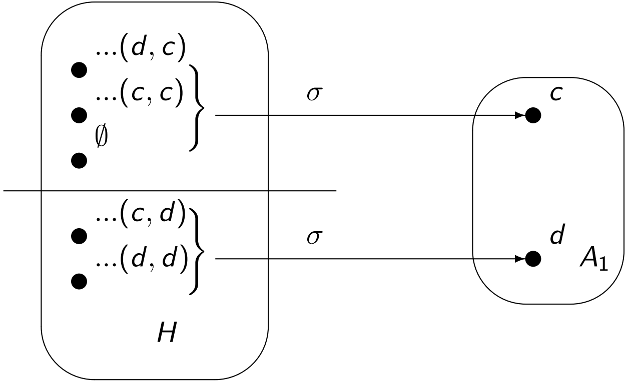
Tit-for-tat of player 1



Tit-for-tat of player 1



Tit-for-tat of player 1



Example: Automata

An *automaton* (for player 1 in the supergame G^∞) is a quadruple $\langle M, m^*, \alpha, \tau \rangle$, where

Example: Automata

An *automaton* (for player 1 in the supergame G^{∞}) is a quadruple $\langle M, m^*, \alpha, \tau \rangle$, where

- M is a nonempty set (the state space),

Example: Automata

An *automaton* (for player 1 in the supergame G^∞) is a quadruple $\langle M, m^*, \alpha, \tau \rangle$, where

- M is a nonempty set (the state space),
- $m^* \in M$ is the initial state,

Example: Automata

An *automaton* (for player 1 in the supergame G^∞) is a quadruple $\langle M, m^*, \alpha, \tau \rangle$, where

- M is a nonempty set (the state space),
- $m^* \in M$ is the initial state,
- $\alpha : M \to A_1$ is an action function, and

Example: Automata

An *automaton* (for player 1 in the supergame G^∞) is a quadruple $\langle M, m^*, \alpha, \tau \rangle$, where

- M is a nonempty set (the state space),
- $m^* \in M$ is the initial state,
- $\alpha : M \to A_1$ is an action function, and
- $\tau : M \times A \to M$ is a transition function.

MAX-PLANCK-GESELLSCHAFT

Example: Automata

An *automaton* (for player 1 in the supergame G^∞) is a quadruple $\langle M, m^*, \alpha, \tau \rangle$, where

- M is a nonempty set (the state space),
- $m^* \in M$ is the initial state,
- $\alpha : M \to A_1$ is an action function, and
- $\tau : M \times A \to M$ is a transition function.

A *k-state automaton* is an automaton where the set M has k elements.

Tit-for-tat automaton

- $M = \{m^*, m\}$

Tit-for-tat automaton

- $M = \{m^*, m\}$
- $\alpha(m^*) = c, \alpha(m) = d$

Tit-for-tat automaton

- $M = \{m^*, m\}$
- $\alpha(m^*) = c, \alpha(m) = d$
- $\tau(m^*, c) = \tau(m, c) = m^*$

Tit-for-tat automaton

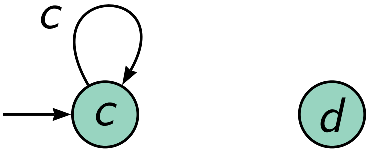
- $M = \{m^*, m\}$
- $\alpha(m^*) = c, \alpha(m) = d$
- $\tau(m^*, c) = \tau(m, c) = m^*$
- $\tau(m^*, d) = \tau(m, d) = m.$

MAX-PLANCK-GESELLSCHAFT

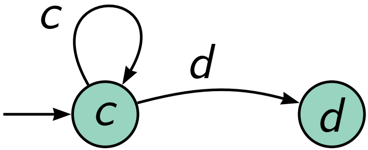
Tit-for-tat

Tit-for-tat

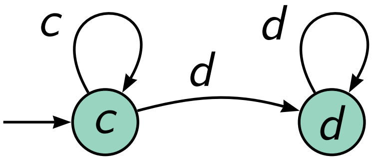
Tit-for-tat



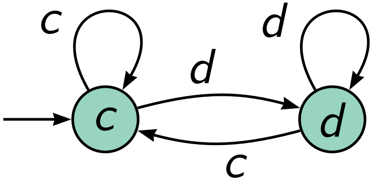
Tit-for-tat



Tit-for-tat



Tit-for-tat



Axelrod's tournaments

1. Tournament: TFT, Tideman and Chieruzzi, Nydegger, Grofman, Shubik, Stein and Rapoport, Friedman, Davis, Graaskamp, Downing, Feld, Joss, Tullock, Random

Axelrod's tournaments

1. Tournament: TFT, Tideman and Chieruzzi, Nydegger, Grofman, Shubik, Stein and Rapoport, Friedman, Davis, Graaskamp, Downing, Feld, Joss, Tullock, Random

Each strategy was paired with each other strategy for 200 iterations of a Prisoner's Dilemma game, and scored on the total points accumulated through the tournament.

Axelrod's tournaments

1. Tournament: TFT, Tideman and Chieruzzi, Nydegger, Grofman, Shubik, Stein and Rapoport, Friedman, Davis, Graaskamp, Downing, Feld, Joss, Tullock, Random

Each strategy was paired with each other strategy for 200 iterations of a Prisoner's Dilemma game, and scored on the total points accumulated through the tournament. The winner was a tit-for-tat (TFT) strategy submitted by Anatol Rapoport.

Axelrod's tournaments revisited

Which strategy we will submit?

Axelrod's tournaments revisited

Which strategy we will submit?

A 2-SBR strategy!

Axelrod's tournaments revisited

Which strategy we will submit?

A 2-SBR strategy!

t-2	∅	∅	∅	∅	∅	cc	dc	cd	dd	cc	dc	cd	dd	cc	dc	cd	dd	cc	dc	cd	dd
t-1	∅	cc	dc	cd	dd	cc				dc				cd				dd			
	?	?	?	?	?	?	?	?	?	?	?	?	?	?	?	?	?	?	?	?	?

Axelrod's tournaments revisited

Which strategy we will submit?

A 2-SBR strategy!

t-2	∅	∅	∅	∅	∅	cc	dc	cd	dd	cc	dc	cd	dd	cc	dc	cd	dd	cc	dc	cd	dd
t-1	∅	cc	dc	cd	dd	cc				dc				cd				dd			
	?	?	?	?	?	?	?	?	?	?	?	?	?	?	?	?	?	?	?	?	?

Which one?

Axelrod's tournaments revisited

Which strategy we will submit?

A 2-SBR strategy!

t-2	∅	∅	∅	∅	∅	cc	dc	cd	dd	cc	dc	cd	dd	cc	dc	cd	dd	cc	dc	cd	dd
t-1	∅	cc	dc	cd	dd	cc				dc				cd				dd			
	?	?	?	?	?	?	?	?	?	?	?	?	?	?	?	?	?	?	?	?	?

Which one?

We have consequently submited all 2-SBR strategies.

Axelrod's tournaments revisited

Which strategy we will submit?

A 2-SBR strategy!

t-2	∅	∅	∅	∅	∅	cc	dc	cd	dd	cc	dc	cd	dd	cc	dc	cd	dd	cc	dc	cd	dd
t-1	∅	cc	dc	cd	dd	cc				dc				cd				dd			
	?	?	?	?	?	?	?	?	?	?	?	?	?	?	?	?	?	?	?	?	?

Which one?

We have consequently submited all 2-SBR strategies. So, we have played $2 \times 2^4 \times 2^{16} = 2\,097.152$ tournaments.

Axelrod's tournaments revisited

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	∅
1 Davis	300	231	300	299	300	111	300	288	300	300	17	300	300	297	300	263
2 Feld	346	111	113	175	330	109	346	228	114	169	111	205	346	114	245	204
3 Friedman	300	113	300	154	300	108	300	296	300	300	111	300	300	298	300	252
4 Graaskamp	301	170	151	294	301	109	301	276	153	299	111	300	301	157	301	235
5 Grofman	300	223	300	299	300	276	300	165	300	300	38	300	300	297	300	266
6 Joss	111	111	108	111	306	106	312	227	109	111	112	197	312	111	312	177
7 Nydegger	300	231	300	299	300	282	300	149	300	300	17	300	300	297	300	265
8 Random	68	208	53	99	360	212	399	198	83	223	121	59	58	69	64	151
9 Shubik	300	114	300	155	300	109	300	283	300	300	111	300	300	298	300	251
10 T-f-T	300	166	300	299	300	109	300	223	300	300	111	300	300	298	300	260
11 Tullock	489	111	113	113	405	110	489	266	113	113	111	173	169	113	115	200
12 T-CH	300	182	300	298	300	187	300	294	300	300	96	300	300	298	300	270
13 Downing	300	231	300	299	300	282	300	293	300	300	97	300	300	297	300	280
14 Stein Rap	302	114	300	160	302	109	302	289	300	300	111	300	302	298	302	253
15 s517572	300	205	300	299	300	282	300	276	300	300	110	300	300	297	300	278

Axelrod's tournaments revisited

The results are not robust w.r.t. realisation of the random variables. TFT is not winning all the tournaments...

TFT is too harsh

The Joss strategy from the 1. Tournament is a five-line program by Johann JOSS of the TH Zurich. This rule cooperates 90% of the time after a cooperation by the other.

TFT is too harsh

The Joss strategy from the 1. Tournament is a five-line program by Johann JOSS of the TH Zurich. This rule cooperates 90% of the time after a cooperation by the other. It always defects after a defection by the other.

TFT is too harsh

The Joss strategy from the 1. Tournament is a five-line program by Johann JOSS of the TH Zurich. This rule cooperates 90% of the time after a cooperation by the other. It always defects after a defection by the other. So, what is the run of the game?

TFT is too harsh

The Joss strategy from the 1. Tournament is a five-line program by Johann JOSS of the TH Zurich. This rule cooperates 90% of the time after a cooperation by the other. It always defects after a defection by the other. So, what is the run of the game? It starts with $(C, C), (C, C) \ldots (C, C)$

TFT is too harsh

The Joss strategy from the 1. Tournament is a five-line program by Johann JOSS of the TH Zurich. This rule cooperates 90% of the time after a cooperation by the other. It always defects after a defection by the other. So, what is the run of the game? It starts with $(C, C), (C, C) \dots (C, C)$ and after first (random) defection of Joss switch to $(C, D), (D, C), (C, D) \dots$

TFT is too harsh

The Joss strategy from the 1. Tournament is a five-line program by Johann JOSS of the TH Zurich. This rule cooperates 90% of the time after a cooperation by the other. It always defects after a defection by the other. So, what is the run of the game? It starts with $(C, C), (C, C) \dots (C, C)$ and after first (random) defection of Joss switch to $(C, D), (D, C), (C, D) \dots$ Already after second (random) defection it will result in neverending mutual defection!

Axelrod's tournaments revisited

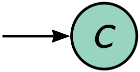
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	∅
1 Davis	300	231	300	299	300	111	300	288	300	300	17	300	300	297	300	263
2 Feld	346	111	113	175	330	109	346	228	114	169	111	205	346	114	245	204
3 Friedman	300	113	300	154	300	108	300	296	300	300	111	300	300	298	300	252
4 Graaskamp	301	170	151	294	301	109	301	276	153	299	111	300	301	157	301	235
5 Grofman	300	223	300	299	300	276	300	165	300	300	38	300	300	297	300	266
6 Joss	111	111	108	111	306	106	312	227	109	111	112	197	312	111	312	177
7 Nydegger	300	231	300	299	300	282	300	149	300	300	17	300	300	297	300	265
8 Random	68	208	53	99	360	212	399	198	83	223	121	59	58	69	64	151
9 Shubik	300	114	300	155	300	109	300	283	300	300	111	300	300	298	300	251
10 T-f-T	300	166	300	299	300	109	300	223	300	300	111	300	300	298	300	260
11 Tullock	489	111	113	113	405	110	489	266	113	113	111	173	169	113	115	200
12 T-CH	300	182	300	298	300	187	300	294	300	300	96	300	300	298	300	270
13 Downing	300	231	300	299	300	282	300	293	300	300	97	300	300	297	300	280
14 Stein Rap	302	114	300	160	302	109	302	289	300	300	111	300	302	298	302	253
15 s517572	300	205	300	299	300	282	300	276	300	300	110	300	300	297	300	278

MAX-PLANCK-GESELLSCHAFT

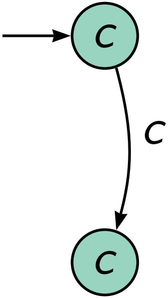
TF2T and hell

t-2	∅	∅	∅	cc	dc	cd	dd	cc	dc	cd	dd	cc	dc	cd	dd	cc	dc	cd	dd
t-1	∅	cc	cd	cc				dc				cd				dd			
s517572	c	c	d	c	c	c	d	d	d	c	d	c	c	d	d	d	d	d	d

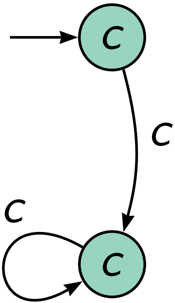
TF2T and hell



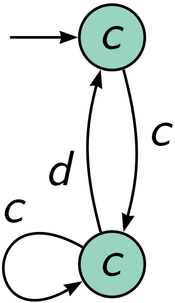
TF2T and hell



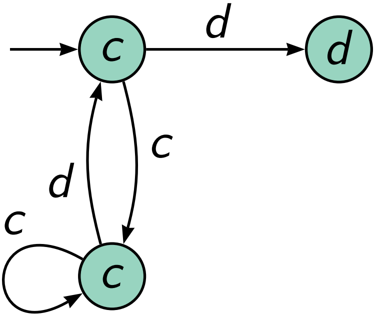
TF2T and hell



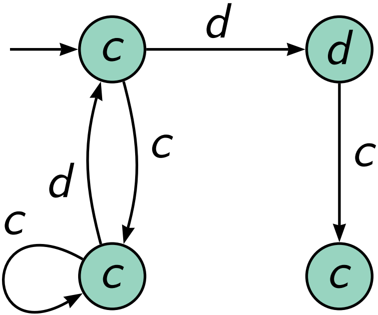
TF2T and hell



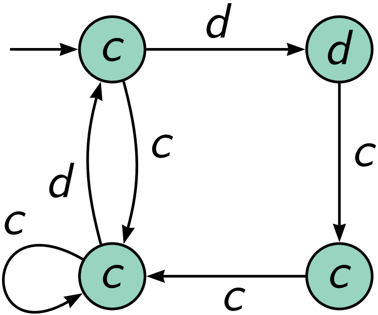
TF2T and hell



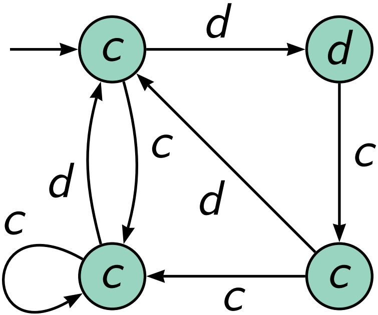
TF2T and hell



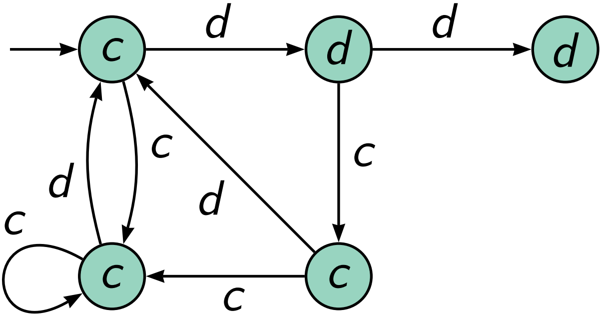
TF2T and hell



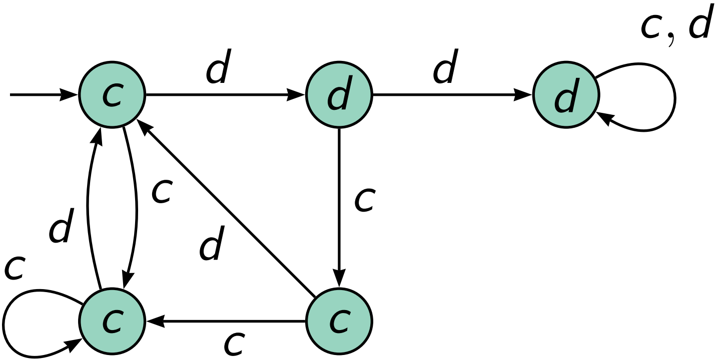
TF2T and hell



TF2T and hell



TF2T and hell



Tournament 2

The results of the first tournament were analyzed and published, and a second tournament held to see if anyone could find a better strategy.

Tournament 2

The results of the first tournament were analyzed and published, and a second tournament held to see if anyone could find a better strategy. TFT won again.

2. Tournament (representatives) Adams R., Pinkley, Gladstein, Feathers, Graaskamp

Gladstein

Defects on the very first move in order to test the other's response.

Gladstein

Defects on the very first move in order to test the other's response.

If the other player ever defects, it apologizes by cooperating and playing tit-for-tat for the rest of the game.

Gladstein

Defects on the very first move in order to test the other's response.

If the other player ever defects, it apologizes by cooperating and playing tit-for-tat for the rest of the game.

Otherwise, it defects as much as possible subject to the constraint that the ratio of its defections to moves remains under .5, not counting the first defection.

Gladstein

Defects on the very first move in order to test the other's response.

If the other player ever defects, it apologizes by cooperating and playing tit-for-tat for the rest of the game.

Otherwise, it defects as much as possible subject to the constraint that the ratio of its defections to moves remains under .5, not counting the first defection.

This means that until the other player defects, Gladstein defects on the first move, the fourth move, and every second move after that.

Gladstein vs. TF2T

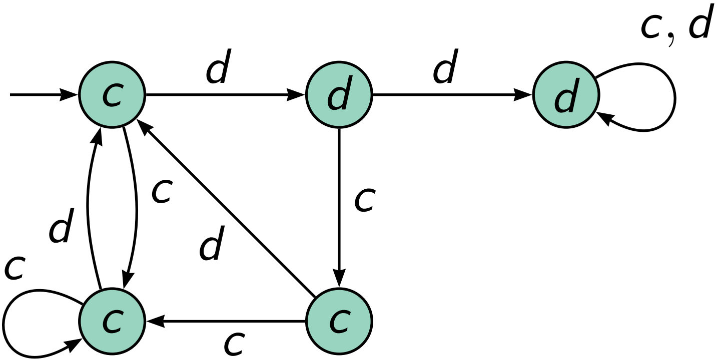
Gladstein never does defect twice in a row.

Gladstein vs. TF2T

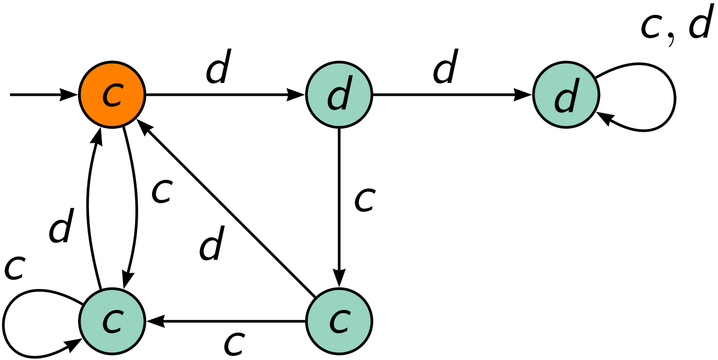
Gladstein never does defect twice in a row.

So TF2T always cooperates with Gladstein, and gets badly exploited for its generosity.

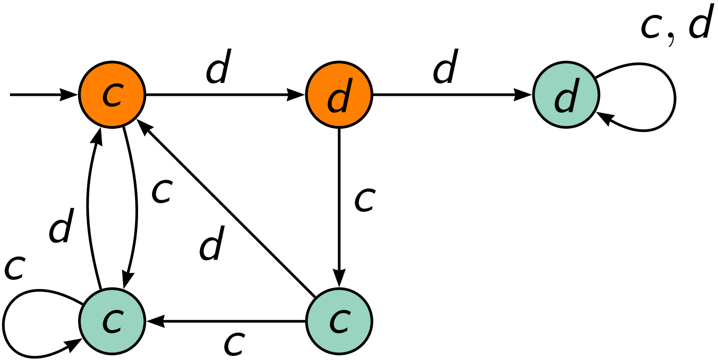
TF2T and hell vs. Gladstein



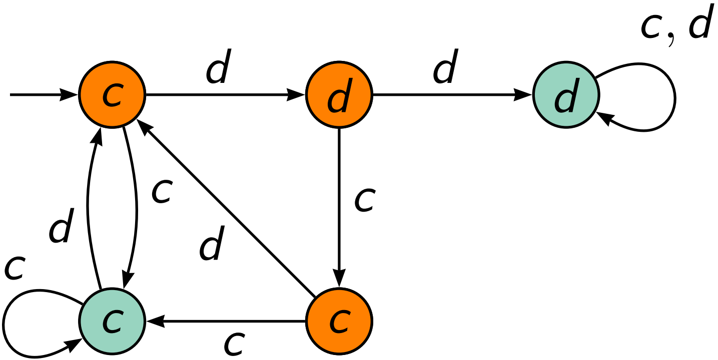
TF2T and hell vs. Gladstein



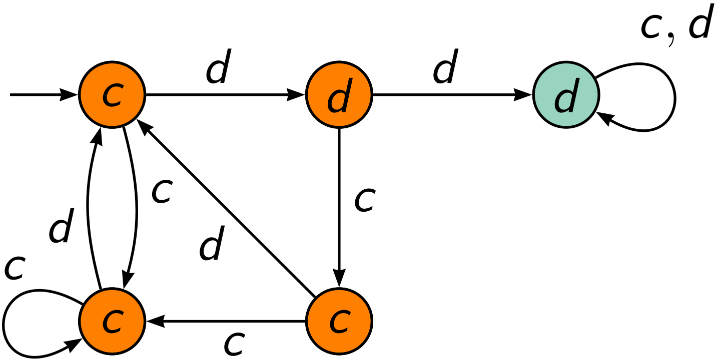
TF2T and hell vs. Gladstein



TF2T and hell vs. Gladstein



TF2T and hell vs. Gladstein



Tournament 2

	1	2	3	4	5	6	∅
1 Pinkley	300	252	263	300	300	300	286
2 Gladstein	249	299	296	300	105	300	258
3 Feathers	228	296	298	297	173	334	271
4 Graaskamp and Katzen	300	300	297	300	300	300	299
5 Adams, R.	300	105	238	300	300	300	257
6 Tf2T&hell	300	300	249	300	300	300	291

Tournament 2

	1	2	3	4	5	6	∅
1 Pinkley	300	252	263	300	300	300	286
2 Gladstein	249	299	296	300	105	300	258
3 Feathers	228	296	298	297	173	334	271
4 Graaskamp and Katzen	300	300	297	300	300	300	299
5 Adams, R.	300	105	238	300	300	300	257
6 Tf2T&hell	300	300	249	300	300	300	291
6a T-F-T	300	300	297	300	300	300	299

Tournament 2

	1	2	3	4	5	6	∅
1 Pinkley	300	252	263	300	300	300	286
2 Gladstein	249	299	296	300	105	300	258
3 Feathers	228	296	298	297	173	334	271
4 Graaskamp and Katzen	300	300	297	300	300	300	299
5 Adams, R.	300	105	238	300	300	300	257
6 Tf2T&hell	300	300	249	300	300	300	291
6a T-F-T	300	300	297	300	300	300	299
6b Magic circles	300	298	319	300	300	300	303

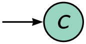
Tournament 2

	1	2	3	4	5	6	∅
1 Pinkley	300	252	263	300	300	300	286
2 Gladstein	249	299	296	300	105	300	258
3 Feathers	228	296	298	297	173	334	271
4 Graaskamp and Katzen	300	300	297	300	300	300	299
5 Adams, R.	300	105	238	300	300	300	257
6 Tf2T&hell	300	300	249	300	300	300	291
6a T-F-T	300	300	297	300	300	300	299
6b Magic circles	300	298	319	300	300	300	303

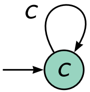
Magic circles in T1 – 270 (T-F-T 260, TF2T&hell 278)

MAX-PLANCK-GESELLSCHAFT

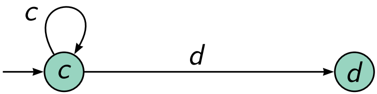
Magic circles



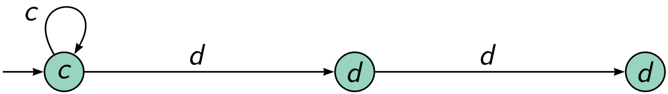
Magic circles



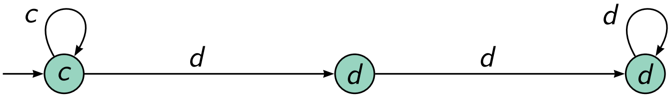
Magic circles



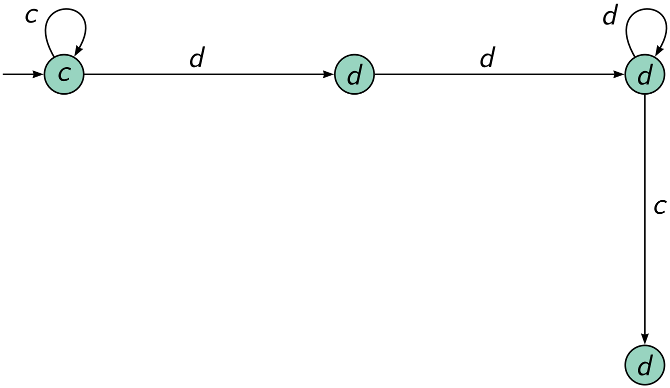
Magic circles



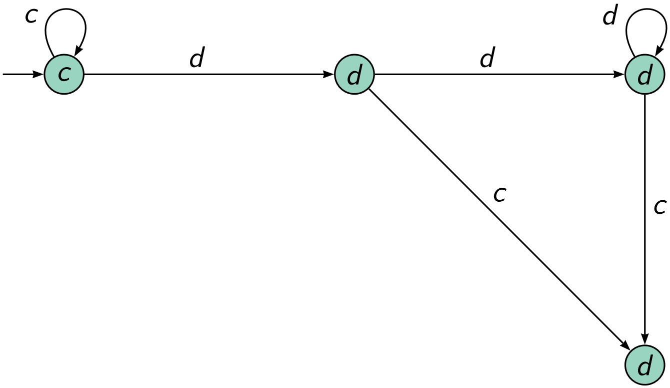
Magic circles



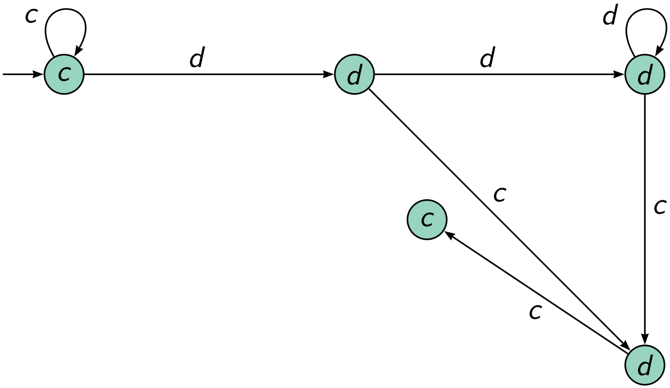
Magic circles



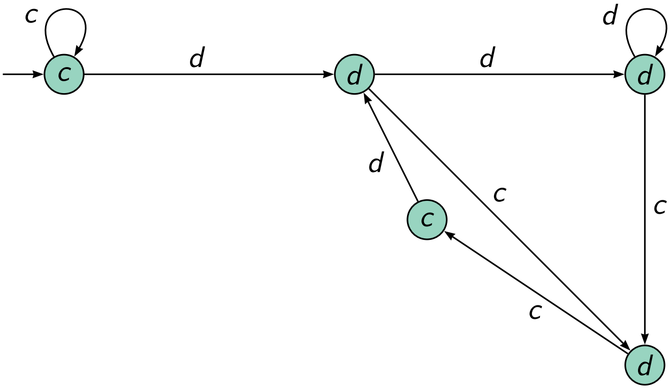
Magic circles



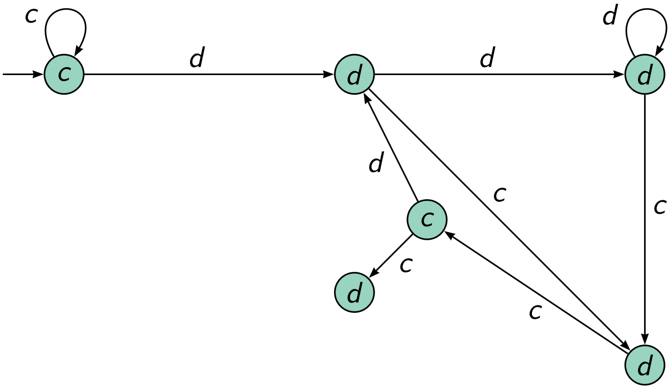
Magic circles



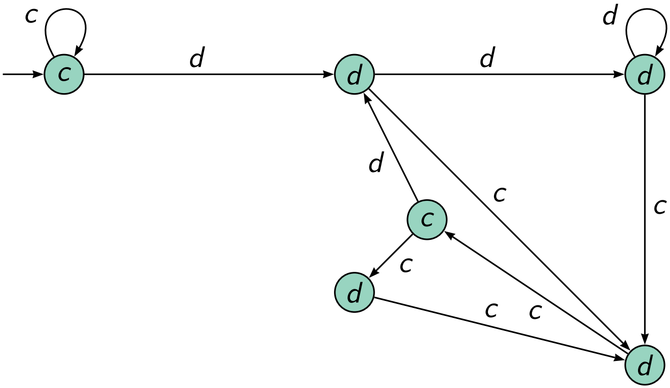
Magic circles



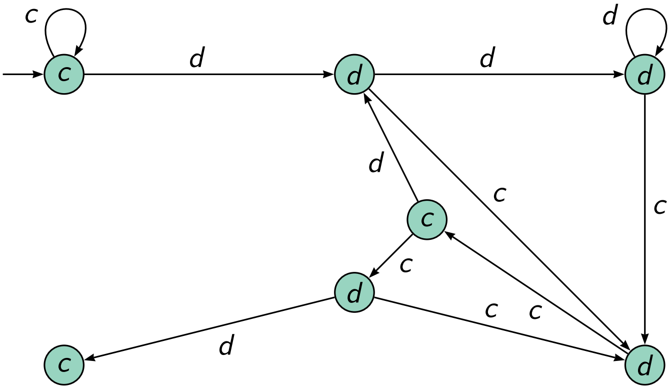
Magic circles



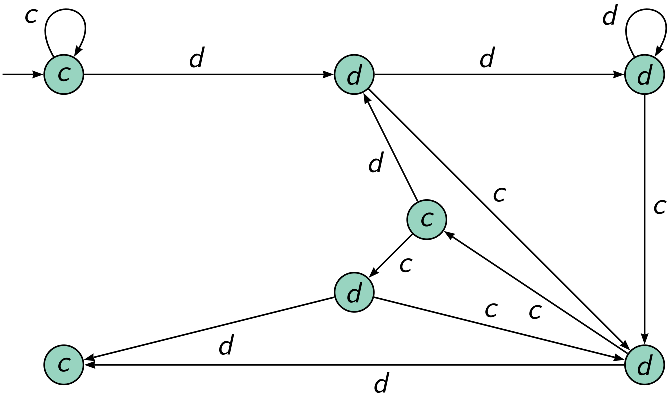
Magic circles



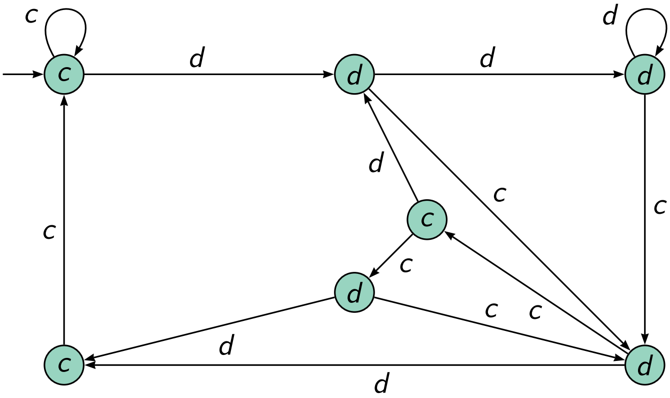
Magic circles



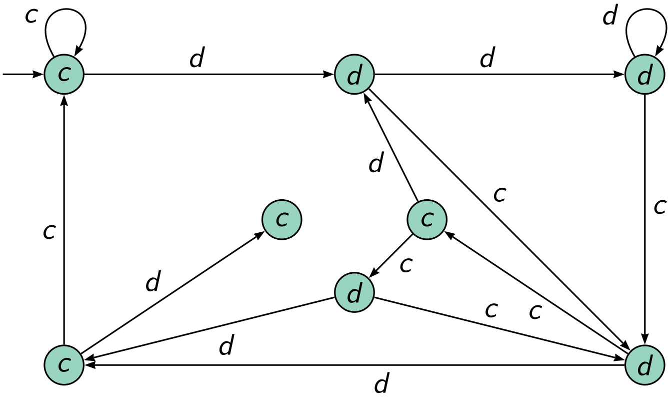
Magic circles



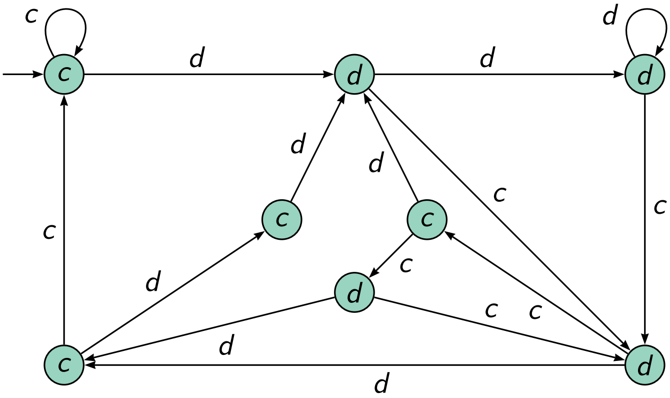
Magic circles



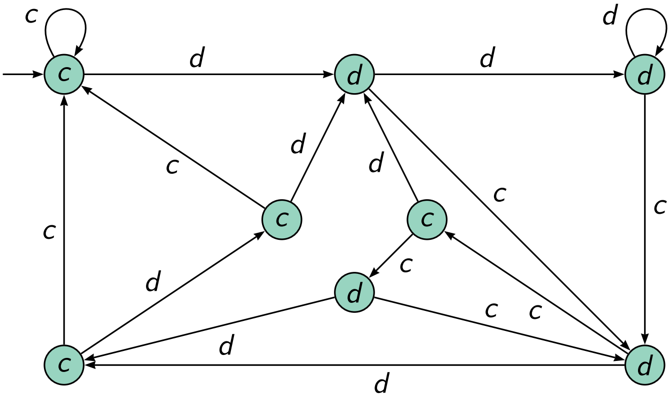
Magic circles



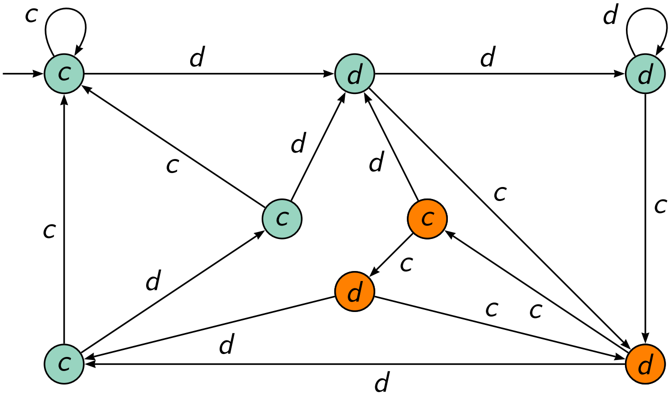
Magic circles



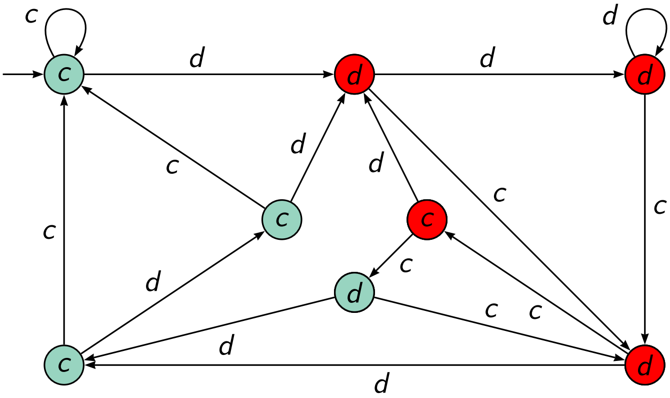
Magic circles



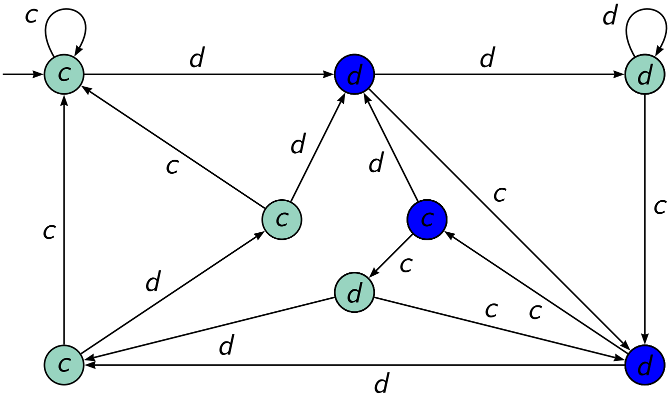
Magic circles



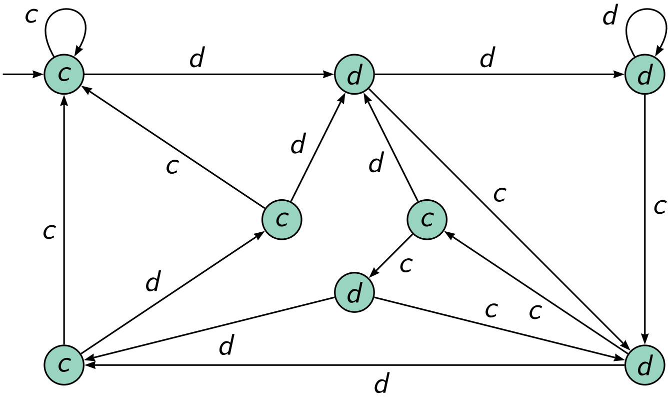
Magic circles



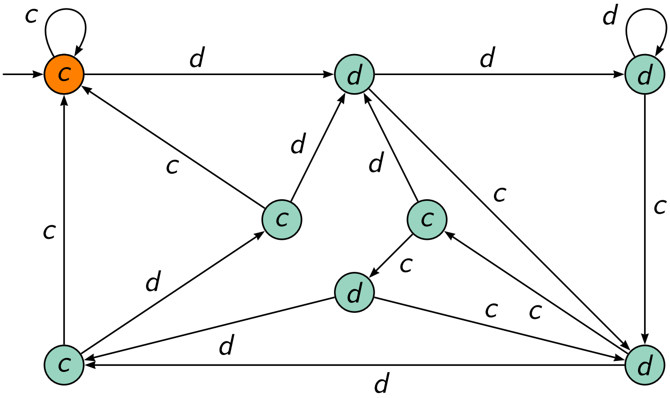
Magic circles



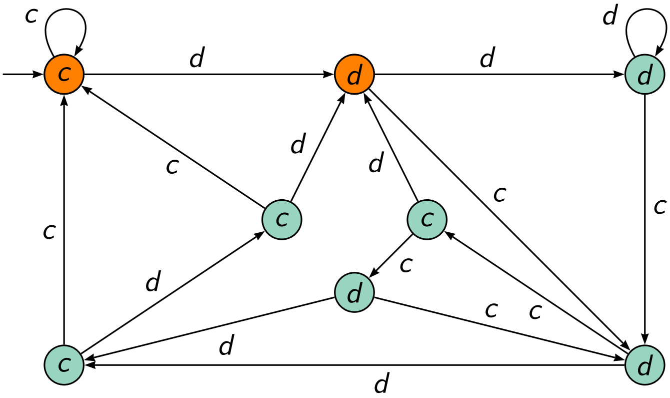
Magic circles and Gladstein



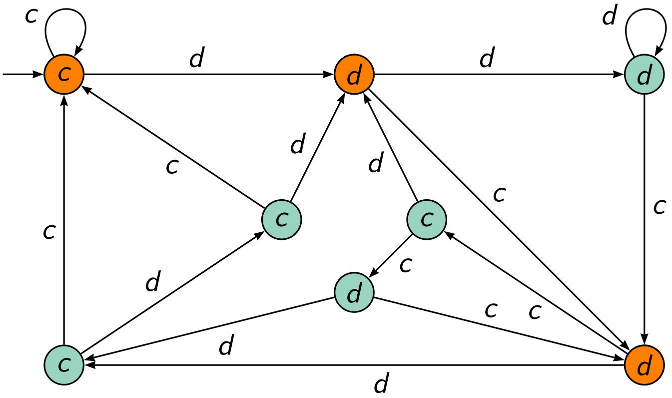
Magic circles and Gladstein



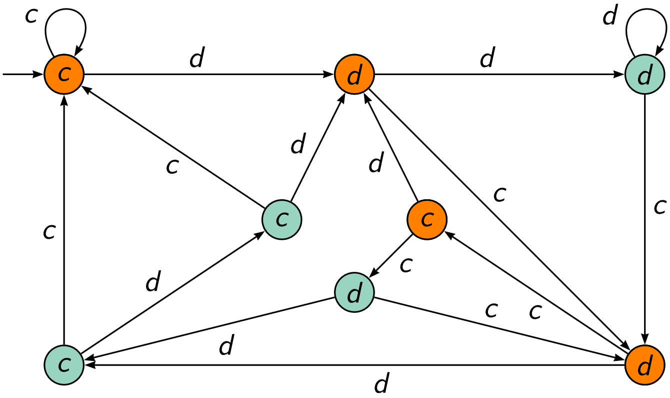
Magic circles and Gladstein



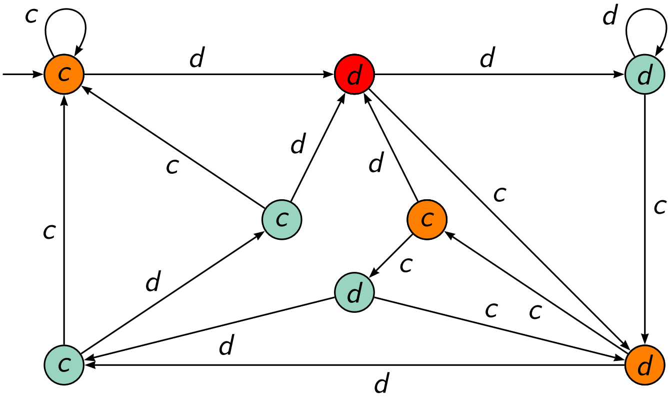
Magic circles and Gladstein



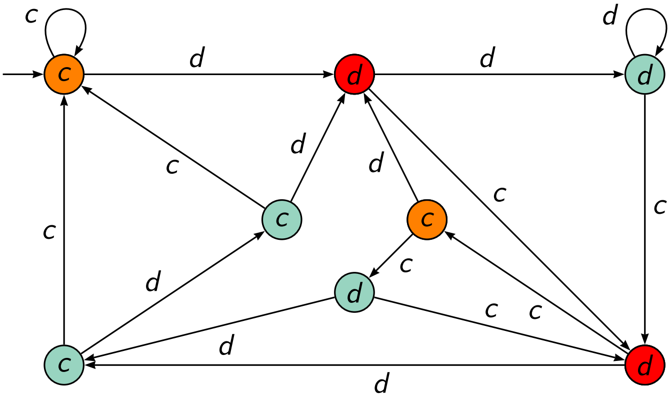
Magic circles and Gladstein



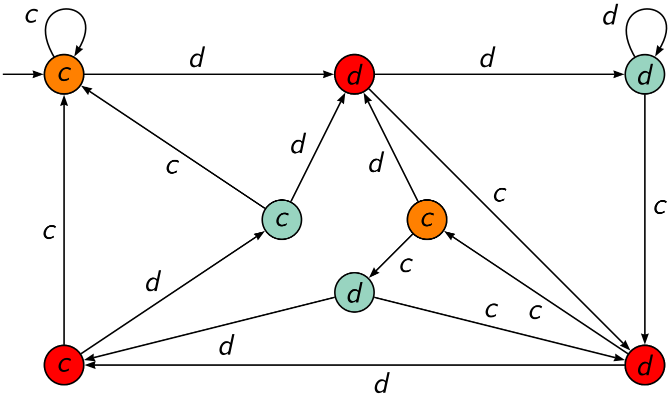
Magic circles and Gladstein



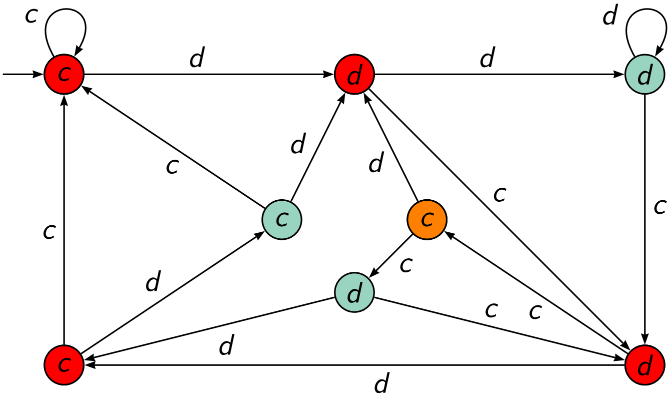
Magic circles and Gladstein



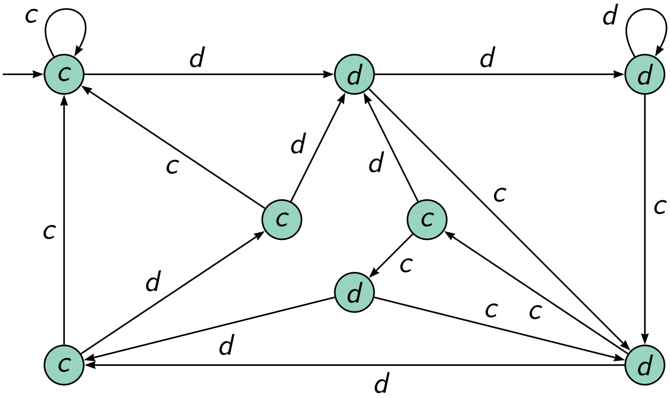
Magic circles and Gladstein



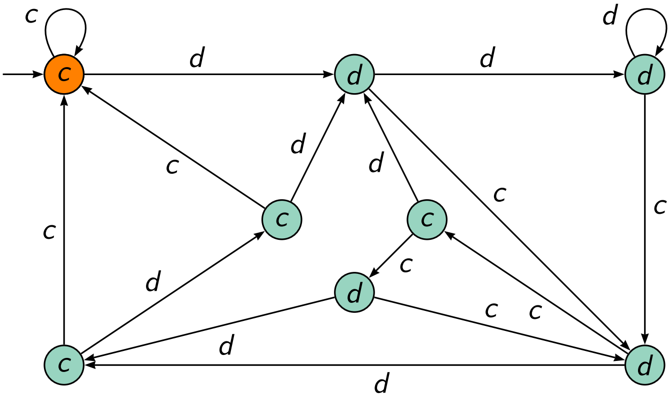
Magic circles and Gladstein



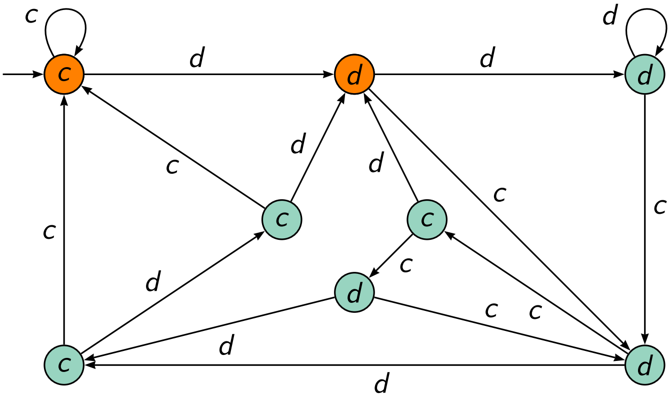
Magic circles and Joss



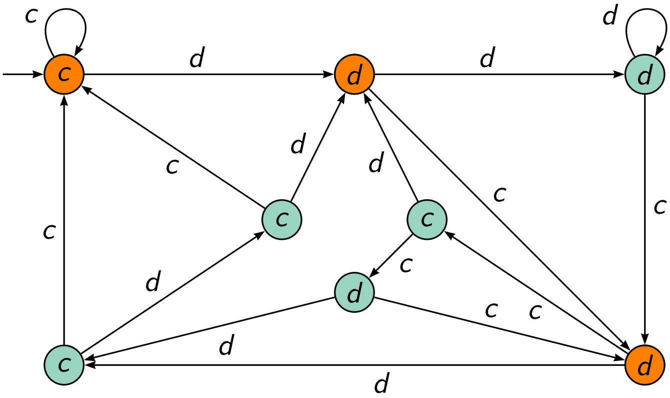
Magic circles and Joss



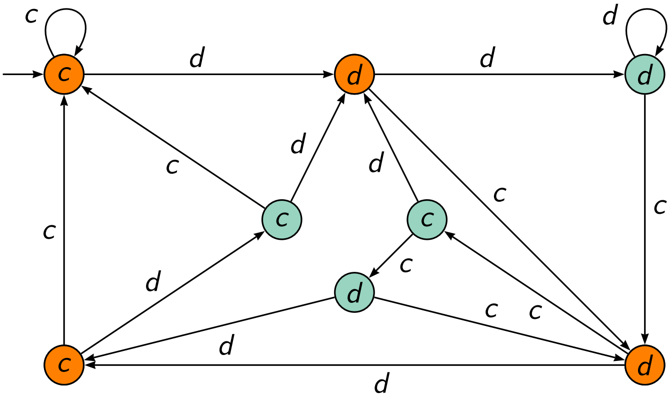
Magic circles and Joss



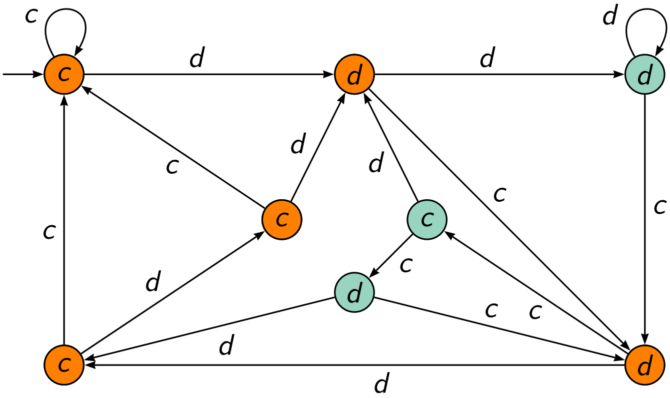
Magic circles and Joss



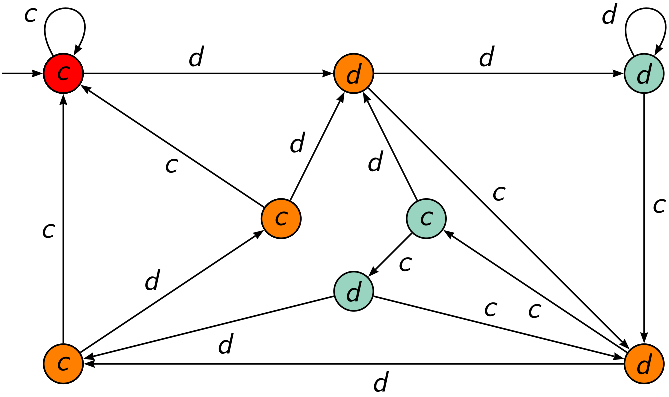
Magic circles and Joss



Magic circles and Joss



Magic circles and Joss



Exploit and excuse

$(d, c)(d, c) \to d$

Exploit and excuse

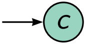
$(d, c)(d, c) \to d$

$(d, c)(c, c) \to d$
$(c, c)(d, c) \to c$

Exploit and excuse

$(d, c)(d, c) \to d$

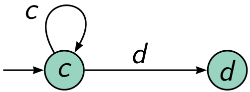
$(d, c)(c, c) \to d$
$(c, c)(d, c) \to c$



Exploit and excuse

$(d, c)(d, c) \to d$

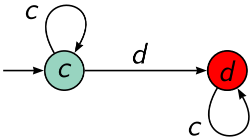
$(d, c)(c, c) \to d$
$(c, c)(d, c) \to c$



Exploit and excuse

$(d, c)(d, c) \to d$

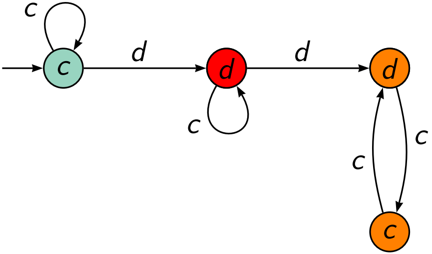
$(d, c)(c, c) \to d$
$(c, c)(d, c) \to c$



Exploit and excuse

$(d, c)(d, c) \to d$

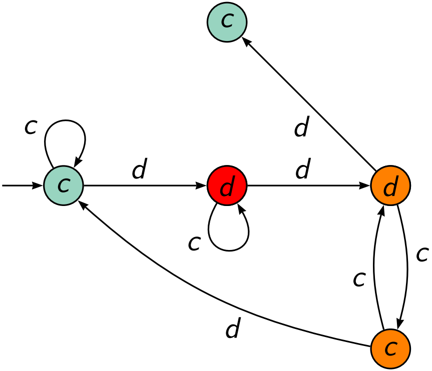
$(d, c)(c, c) \to d$
$(c, c)(d, c) \to c$



Exploit and excuse

$(d,c)(d,c) \rightarrow d$

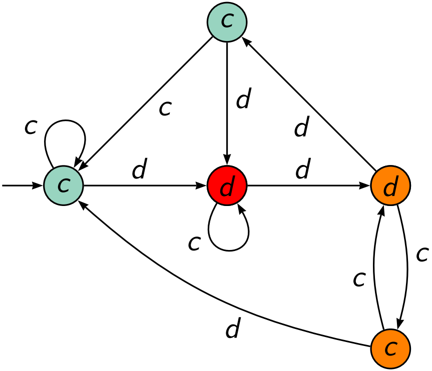
$(d,c)(c,c) \rightarrow d$
$(c,c)(d,c) \rightarrow c$



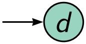
Exploit and excuse

$(d, c)(d, c) \to d$

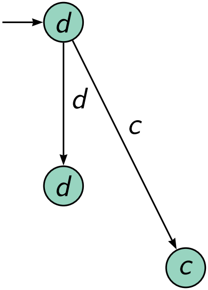
$(d, c)(c, c) \to d$
$(c, c)(d, c) \to c$



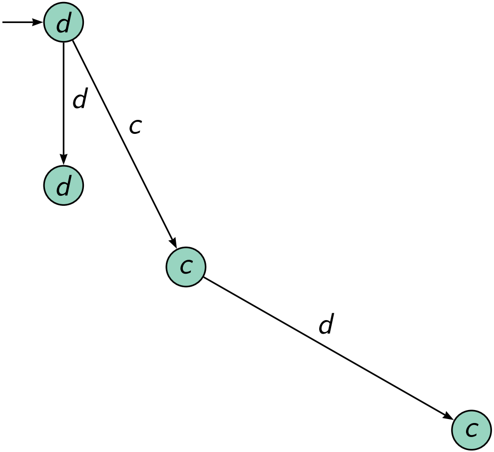
Provocateur



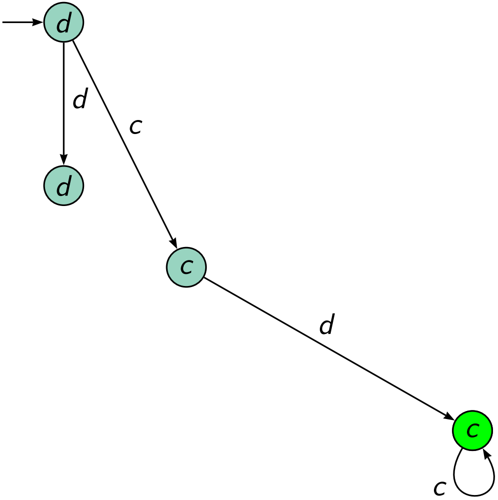
Provocateur



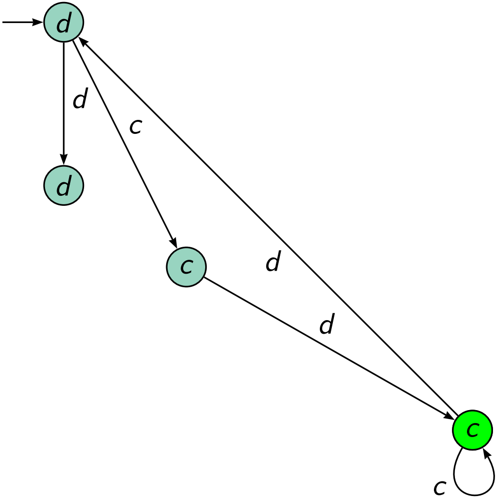
Provocateur



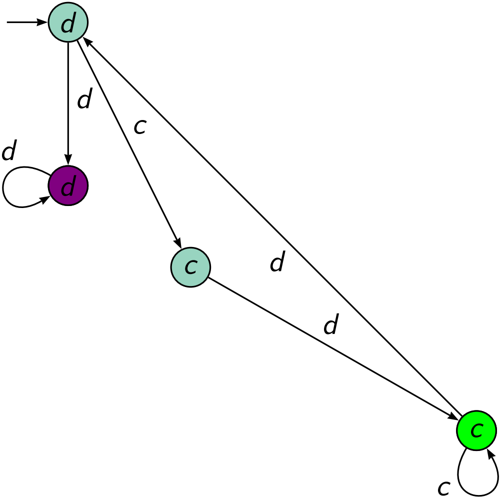
Provocateur



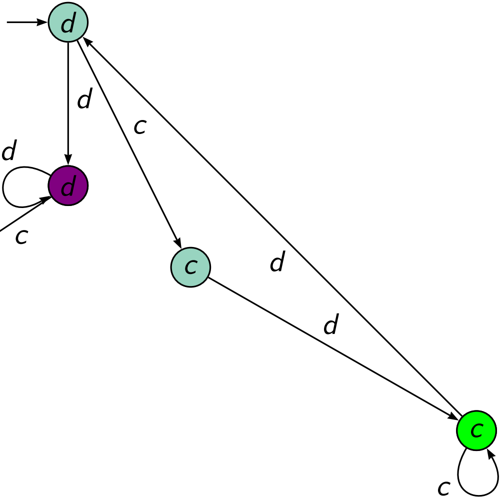
Provocateur



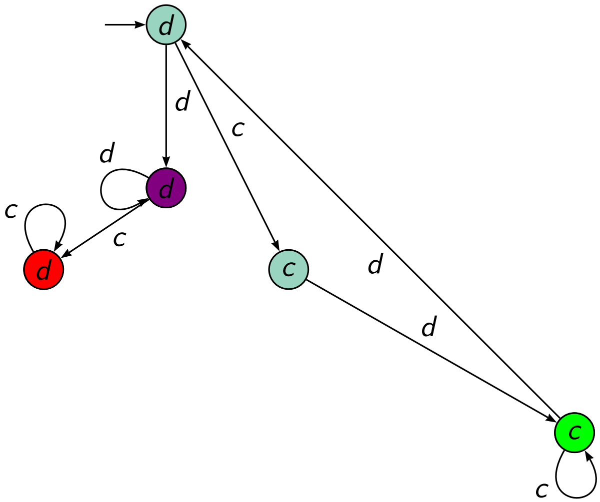
Provocateur



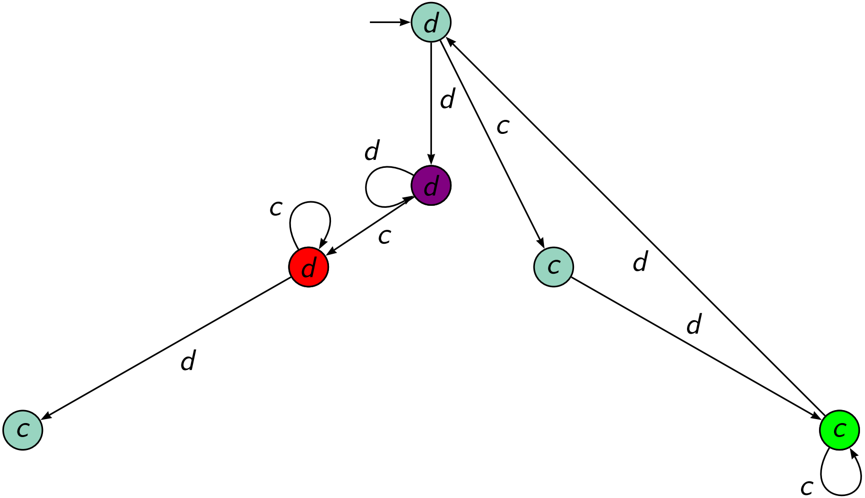
Provocateur



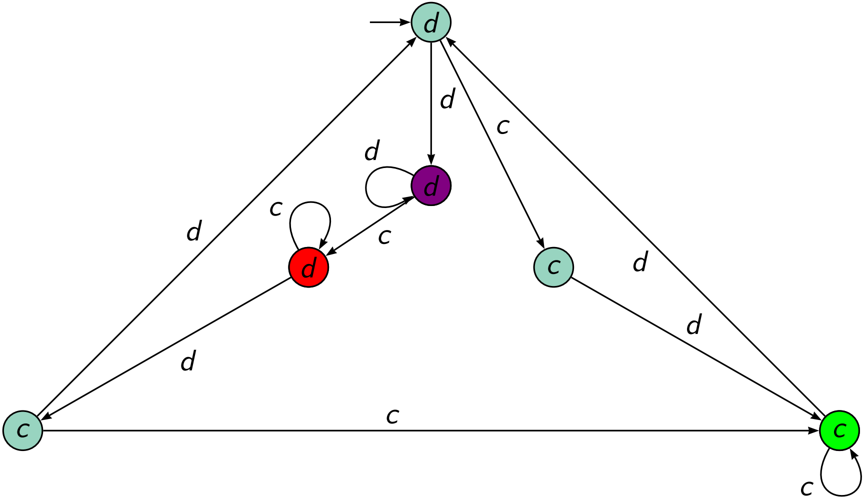
Provocateur



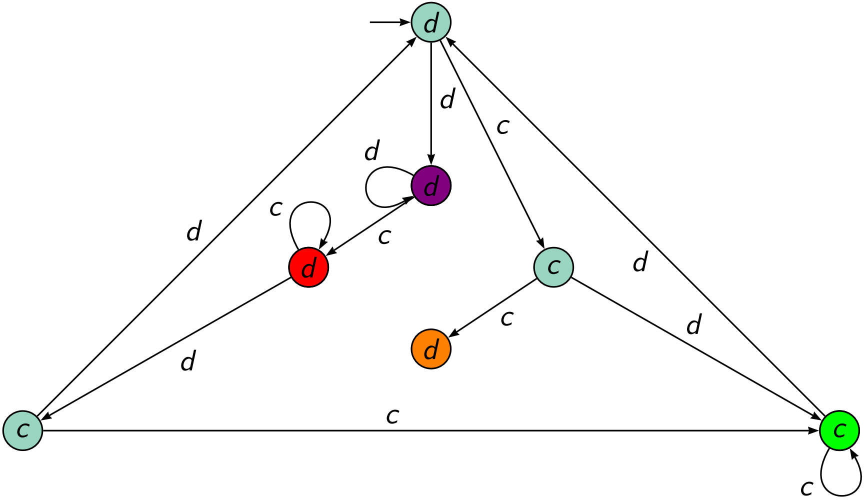
Provocateur



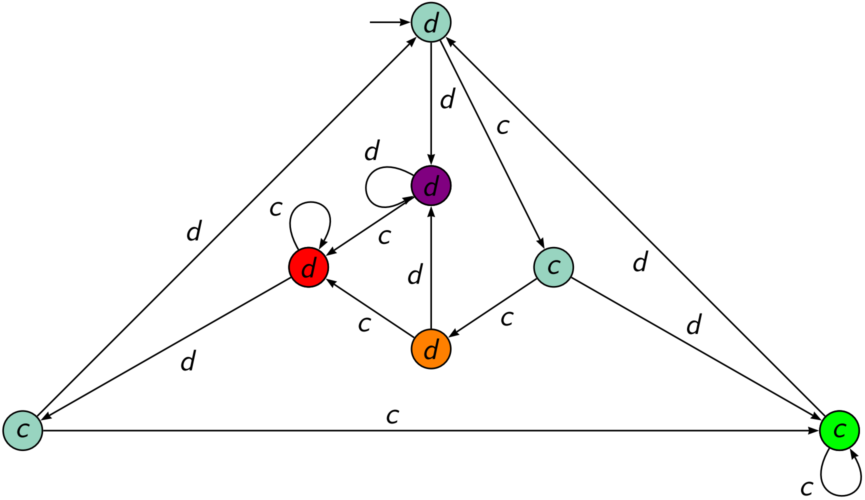
Provocateur



Provocateur



Provocateur



Research question

Kalai (1990): “What information system (size and structure) should a player maintain when playing a strategic game?”

Research question

Kalai (1990): “What information system (size and structure) should a player maintain when playing a strategic game?”

Here, we try to answer the question of Kalai in the context of strategies of bounded complexity.

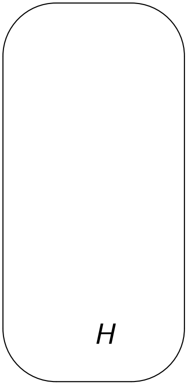
Research question

Kalai (1990): "What information system (size and structure) should a player maintain when playing a strategic game?"

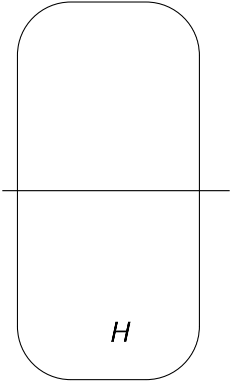
Here, we try to answer the question of Kalai in the context of strategies of bounded complexity.

In detail, we study the complexity of the strategy that is the best response to a strategy with a given complexity.

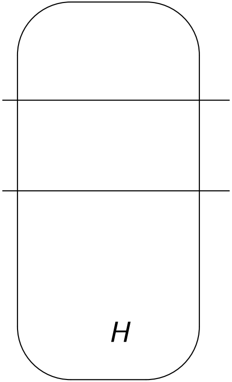
Bounded strategy



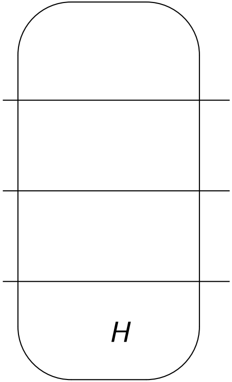
Bounded strategy



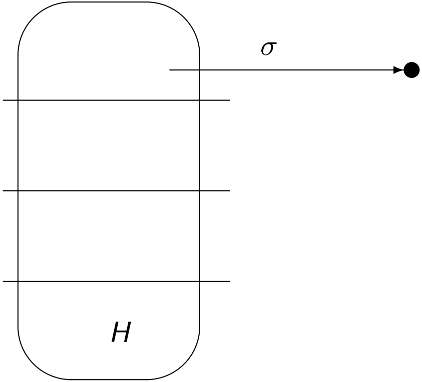
Bounded strategy



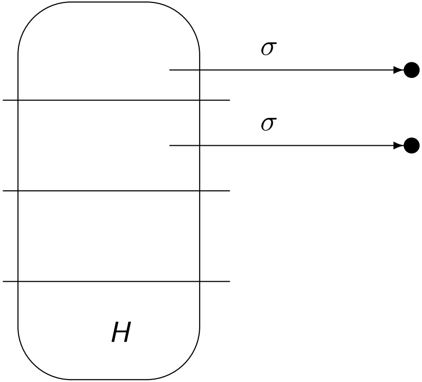
Bounded strategy



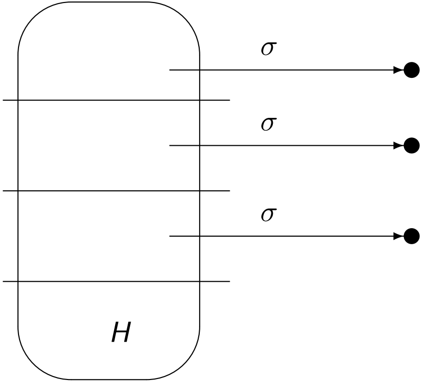
Bounded strategy



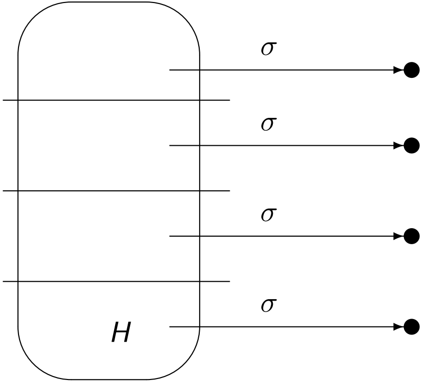
Bounded strategy



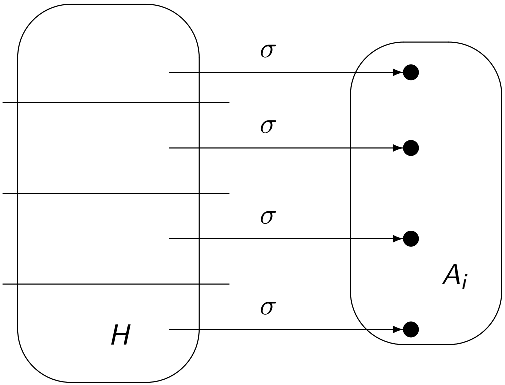
Bounded strategy



Bounded strategy



Bounded strategy



Factor-based strategy

Let H denote the set of all finite histories in a supergame G^{∞}, i.e., $H = A^{<\mathbf{N}}$.

Factor-based strategy

Let H denote the set of all finite histories in a supergame G^{∞}, i.e., $H = A^{<\mathbf{N}}$.

Let X be a set and φ be a mapping from H to X.

Factor-based strategy

Let H denote the set of all finite histories in a supergame G^∞, i.e., $H = A^{<\mathbf{N}}$.

Let X be a set and φ be a mapping from H to X.

We say that a strategy σ is a *factor-based strategy with factor* φ (φ-based strategy for short) for player i in the supergame G^∞

Factor-based strategy

Let H denote the set of all finite histories in a supergame G^∞, i.e., $H = A^{<\mathbf{N}}$.

Let X be a set and φ be a mapping from H to X.

We say that a strategy σ is a *factor-based strategy with factor* φ (φ-based strategy for short) for player i in the supergame G^∞

if there is a *factor-action function* $\omega : X \to A_i$

Factor-based strategy

Let H denote the set of all finite histories in a supergame G^∞, i.e.,

$H = A^{<\mathbf{N}}$.

Let X be a set and φ be a mapping from H to X.

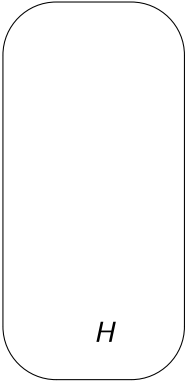
We say that a strategy σ is a *factor-based strategy with factor* φ (φ-based strategy for short) for player i in the supergame G^∞

if there is a *factor-action function* $\omega : X \to A_i$

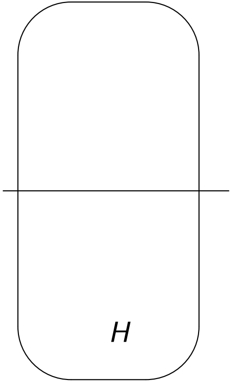
such that $\sigma = \omega \circ \varphi$.

MAX-PLANCK-GESELLSCHAFT

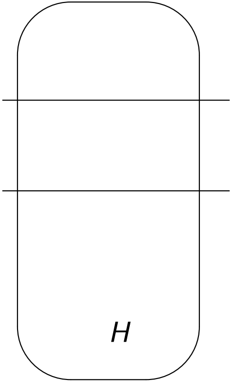
Factor-based strategy



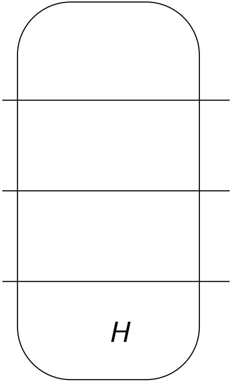
Factor-based strategy



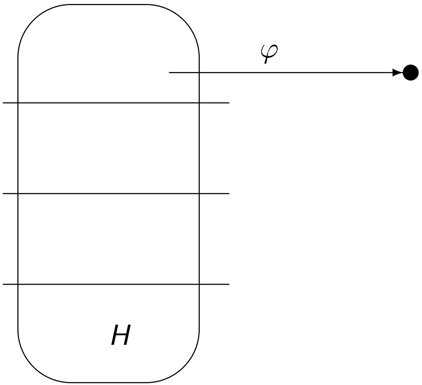
Factor-based strategy



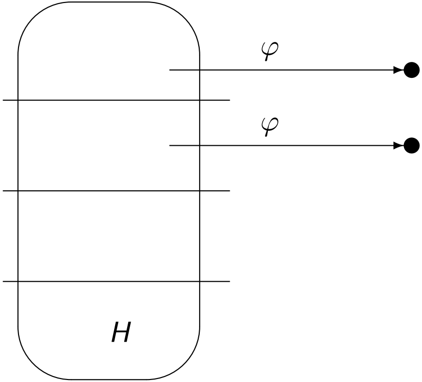
Factor-based strategy



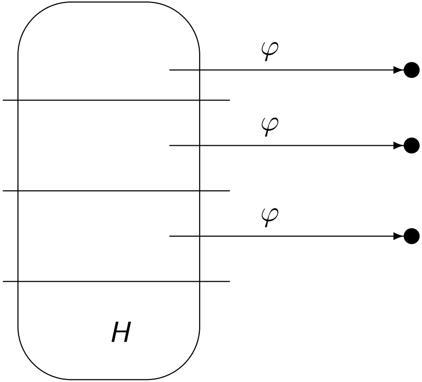
Factor-based strategy



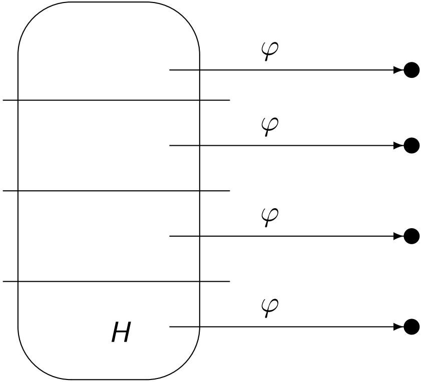
Factor-based strategy



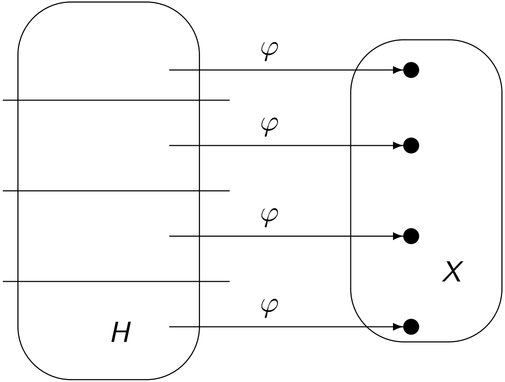
Factor-based strategy



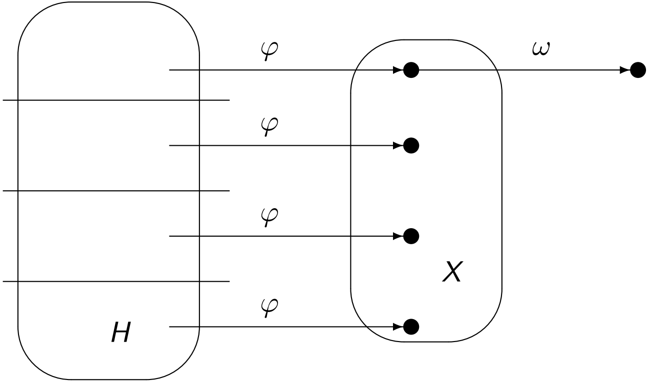
Factor-based strategy



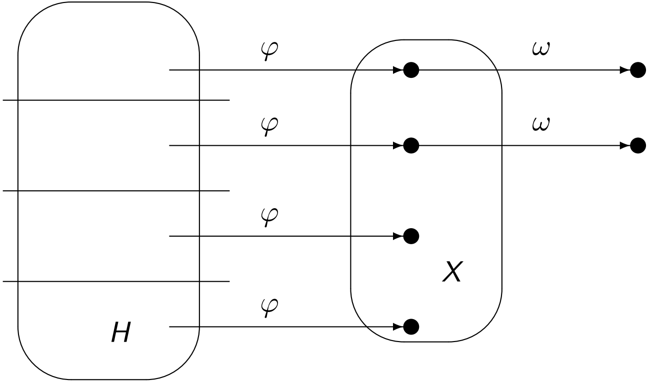
Factor-based strategy



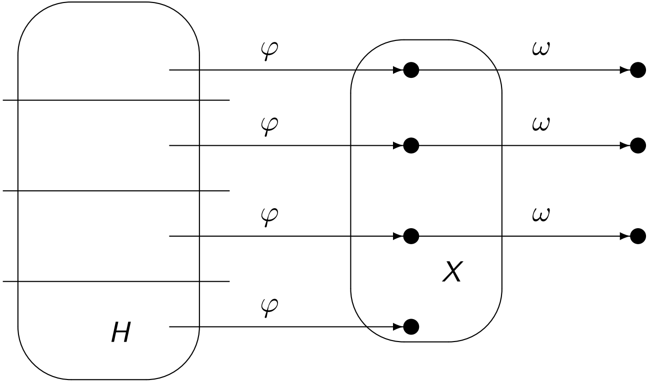
Factor-based strategy



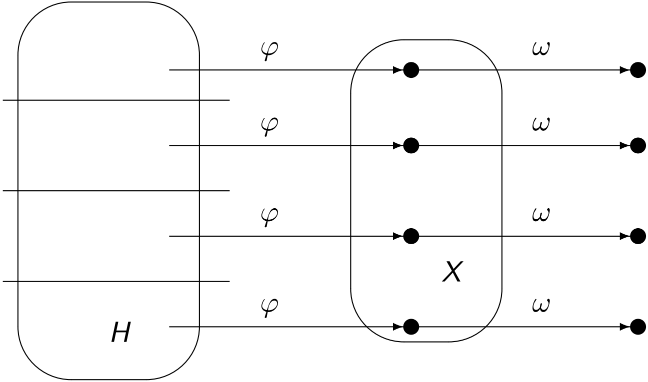
Factor-based strategy



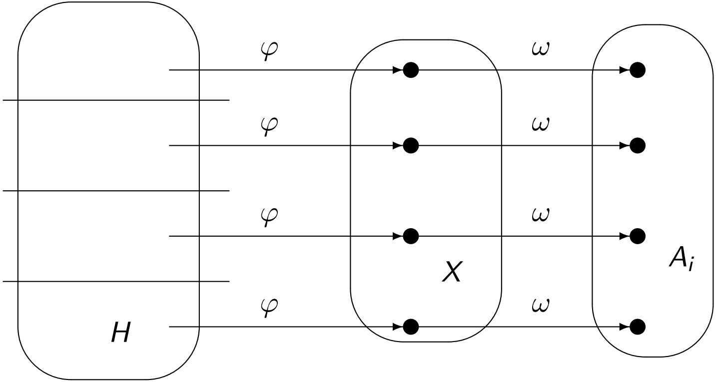
Factor-based strategy



Factor-based strategy



Factor-based strategy



Recursive factor

The factor φ is called *recursive*

Recursive factor

The factor φ is called *recursive* if there is a function $g : X \times A \to X$ such that

Recursive factor

The factor φ is called *recursive* if there is a function $g : X \times A \to X$ such that $\varphi(a_1, \ldots, a_t) =$

Recursive factor

The factor φ is called *recursive* if there is a function $g : X \times A \to X$ such that $\varphi(a_1, \ldots, a_t) = g(\qquad , \quad)$.

Recursive factor

The factor φ is called *recursive* if there is a function $g : X \times A \to X$ such that $\varphi(a_1, \ldots, a_t) = g(\varphi(a_1, \ldots, a_{t-1}), \quad)$.

Recursive factor

The factor φ is called *recursive* if there is a function $g : X \times A \to X$ such that $\varphi(a_1, \ldots, a_t) = g(\varphi(a_1, \ldots, a_{t-1}), a_t)$.

Recursive factor

The factor φ is called *recursive* if there is a function $g : X \times A \to X$ such that $\varphi(a_1, \ldots, a_t) = g(\varphi(a_1, \ldots, a_{t-1}), a_t)$.

Recursivity captures the fact that **what was forgotten can't be learnt once more.**

Examples of recursive factor based strategies

- Automata
- SBR strategies
- Imperfect monitoring (red-green blindness)

Stochastic games

A two-person stochastic game with finite action sets is 5-tuple

$\Gamma = \langle S, A, u, p, \mu \rangle$ such that

- a *state space* S is a nonempty set,

Stochastic games

A two-person stochastic game with finite action sets is 5-tuple

$\Gamma = \langle S, A, u, p, \mu \rangle$ such that

- a *state space* S is a nonempty set,
- $A(z) = A_1(z) \times A_2(z)$ is an *action set*: for every state $z \in S$, $A_i(z)$ is a nonempty finite set of actions for player i ($i = 1, 2$) at the state z,

MAX-PLANCK-GESELLSCHAFT

Stochastic games

A two-person stochastic game with finite action sets is 5-tuple

$\Gamma = \langle S, A, u, p, \mu \rangle$ such that

- a *state space* S is a nonempty set,
- $A(z) = A_1(z) \times A_2(z)$ is an *action set*: for every state $z \in S$, $A_i(z)$ is a nonempty finite set of actions for player i ($i = 1, 2$) at the state z,
- $u = (u_1, u_2)$ is a *payoff function*, where $u_i(z, a)$ is the payoff function of player i, ($z \in S, a \in A(z)$),

MAX-PLANCK-GESELLSCHAFT

Stochastic games

A two-person stochastic game with finite action sets is 5-tuple

$\Gamma = \langle S, A, u, p, \mu \rangle$ such that

- a *state space* S is a nonempty set,
- $A(z) = A_1(z) \times A_2(z)$ is an *action set*: for every state $z \in S$, $A_i(z)$ is a nonempty finite set of actions for player i ($i = 1, 2$) at the state z,
- $u = (u_1, u_2)$ is a *payoff function*, where $u_i(z, a)$ is the payoff function of player i, ($z \in S, a \in A(z)$),
- p is a *transition function*: for each state $z \in S$ and each action profile $a \in A(z)$, $p(z, a) \in \Delta(S)$ is the probability of the next state, and

MAX-PLANCK-GESELLSCHAFT

Stochastic games

A two-person stochastic game with finite action sets is 5-tuple

$\Gamma = \langle S, A, u, p, \mu \rangle$ such that

- a *state space* S is a nonempty set,
- $A(z) = A_1(z) \times A_2(z)$ is an *action set*: for every state $z \in S$, $A_i(z)$ is a nonempty finite set of actions for player i ($i = 1, 2$) at the state z,
- $u = (u_1, u_2)$ is a *payoff function*, where $u_i(z, a)$ is the payoff function of player i, ($z \in S, a \in A(z)$),
- p is a *transition function*: for each state $z \in S$ and each action profile $a \in A(z)$, $p(z, a) \in \Delta(S)$ is the probability of the next state, and
- $\mu \in \Delta(S)$ is a distribution of the initial state.

MAX-PLANCK-GESELLSCHAFT

Strategy in stochastic games

A play of the stochastic game Γ^∞ is a sequence of states and actions $(z_1, a_1, \ldots, z_t, a_t, z_{t+1},\ a_{t+1}, \ldots)$ with $a_t \in A(z_t)$.

Strategy in stochastic games

A play of the stochastic game Γ^{∞} is a sequence of states and actions $(z_1, a_1, \ldots, z_t, a_t, z_{t+1},\ a_{t+1}, \ldots)$ with $a_t \in A(z_t)$.

A pure strategy of player i in the stochastic game with perfect monitoring specifies her action $a_t^i \in A_i(z_t)$

Strategy in stochastic games

A play of the stochastic game Γ^{∞} is a sequence of states and actions $(z_1, a_1, \ldots, z_t, a_t, z_{t+1},\ a_{t+1}, \ldots)$ with $a_t \in A(z_t)$.

A pure strategy of player i in the stochastic game with perfect monitoring specifies her action $a_t^i \in A_i(z_t)$ as a function of the past state z_t and action profiles $(z_1, a_1, \ldots, a_{t-1})$.

Strategy in stochastic games

A play of the stochastic game Γ^∞ is a sequence of states and actions $(z_1, a_1, \dots, z_t, a_t, z_{t+1},\ a_{t+1}, \dots)$ with $a_t \in A(z_t)$.

A pure strategy of player i in the stochastic game with perfect monitoring specifies her action $a_t^i \in A_i(z_t)$ as a function of the past state z_t and action profiles $(z_1, a_1, \dots, a_{t-1})$.

Similarly, a behavioral strategy of player i is a function of the past state z_t and action profiles $(z_1, a_1, \dots, a_{t-1})$

Strategy in stochastic games

A play of the stochastic game Γ^∞ is a sequence of states and actions $(z_1, a_1, \ldots, z_t, a_t, z_{t+1},\ a_{t+1}, \ldots)$ with $a_t \in A(z_t)$.

A pure strategy of player i in the stochastic game with perfect monitoring specifies her action $a_t^i \in A_i(z_t)$ as a function of the past state z_t and action profiles $(z_1, a_1, \ldots, a_{t-1})$.

Similarly, a behavioral strategy of player i is a function of the past state z_t and action profiles $(z_1, a_1, \ldots, a_{t-1})$ and specifies the probability that an action $a_t^i \in A_i(z_t)$ is played.

Factor-based strategies in stochastic games

The choice of distribution of action a_t^i depends on $\varphi(z_1, a_1, \ldots, z_{t-1}, a_{t-1})$ and on the actual state z_t.

Factor-based strategies in stochastic games

The choice of distribution of action a_t^i depends on $\varphi(z_1, a_1, \ldots, z_{t-1}, a_{t-1})$ and on the actual state z_t.This means that $\omega : S \times X \to \Delta(A_i)$ and

Factor-based strategies in stochastic games

The choice of distribution of action a_t^i depends on $\varphi(z_1, a_1, \ldots, z_{t-1}, a_{t-1})$ and on the actual state z_t.This means that $\omega : S \times X \to \Delta(A_i)$ and

$$\sigma(z_1, a_1, \ldots, z_t) = \omega(z_t, \varphi(z_1, a_1, \ldots, z_{t-1}, a_{t-1})).$$

Factor-based strategies in stochastic games

The choice of distribution of action a^i_t depends on $\varphi(z_1, a_1, \ldots, z_{t-1}, a_{t-1})$ and on the actual state z_t.This means that $\omega : S \times X \to \Delta(A_i)$ and

$$\sigma(z_1, a_1, \ldots, z_t) = \omega(z_t, \varphi(z_1, a_1, \ldots, z_{t-1}, a_{t-1})).$$

The factor φ in the case of a stochastic game is called *recursive*

Factor-based strategies in stochastic games

The choice of distribution of action a_t^i depends on $\varphi(z_1, a_1, \ldots, z_{t-1}, a_{t-1})$ and on the actual state z_t.This means that $\omega : S \times X \to \Delta(A_i)$ and

$$\sigma(z_1, a_1, \ldots, z_t) = \omega(z_t, \varphi(z_1, a_1, \ldots, z_{t-1}, a_{t-1})).$$

The factor φ in the case of a stochastic game is called *recursive* if there is a function $g : X \times S \times A \to X$ such that

MAX-PLANCK-GESELLSCHAFT

Factor-based strategies in stochastic games

The choice of distribution of action a_t^i depends on $\varphi(z_1, a_1, \ldots, z_{t-1}, a_{t-1})$ and on the actual state z_t.This means that $\omega : S \times X \to \Delta(A_i)$ and

$$\sigma(z_1, a_1, \ldots, z_t) = \omega(z_t, \varphi(z_1, a_1, \ldots, z_{t-1}, a_{t-1})).$$

The factor φ in the case of a stochastic game is called *recursive* if there is a function $g : X \times S \times A \to X$ such that
$\varphi(z_1, a_1, \ldots, z_t, a_t) = g(\varphi(z_1, a_1, \ldots, z_{t-1}, a_{t-1}), z_t, a_t).$

Payoff in Stochastic games

A pair of strategies σ^1 and σ^2 of players 1 and 2 defines a probability distribution P_{σ^1,σ^2} on the space of plays of the stochastic game.

Payoff in Stochastic games

A pair of strategies σ^1 and σ^2 of players 1 and 2 defines a probability distribution P_{σ^1,σ^2} on the space of plays of the stochastic game. The expectation w.r.t. this probability distribution is denoted by E_{σ^1,σ^2}.

Payoff in Stochastic games

A pair of strategies σ^1 and σ^2 of players 1 and 2 defines a probability distribution P_{σ^1,σ^2} on the space of plays of the stochastic game. The expectation w.r.t. this probability distribution is denoted by E_{σ^1,σ^2}. Given a discount factor $0 < \beta < 1$ the (unnormalized) β-discounted payoff to player i is defined by

MAX-PLANCK-GESELLSCHAFT

Payoff in Stochastic games

A pair of strategies σ^1 and σ^2 of players 1 and 2 defines a probability distribution P_{σ^1,σ^2} on the space of plays of the stochastic game. The expectation w.r.t. this probability distribution is denoted by E_{σ^1,σ^2}. Given a discount factor $0 < \beta < 1$ the (unnormalized) β-discounted payoff to player i is defined by

$$V^i_\beta(\sigma^1, \sigma^2) = E_{\sigma^1,\sigma^2}\left(\sum_{t=1}^{\infty} \beta^{t-1} u_i(z_t, a_t)\right)$$

Payoff in Stochastic games

A pair of strategies σ^1 and σ^2 of players 1 and 2 defines a probability distribution P_{σ^1,σ^2} on the space of plays of the stochastic game. The expectation w.r.t. this probability distribution is denoted by E_{σ^1,σ^2}. Given a discount factor $0 < \beta < 1$ the (unnormalized) β-discounted payoff to player i is defined by

$$V^i_\beta(\sigma^1, \sigma^2) = E_{\sigma^1,\sigma^2}\left(\sum_{t=1}^{\infty} \beta^{t-1} u_i(z_t, a_t)\right)$$

and the *normalized β-discounted payoff* to player i is defined by

Payoff in Stochastic games

A pair of strategies σ^1 and σ^2 of players 1 and 2 defines a probability distribution P_{σ^1,σ^2} on the space of plays of the stochastic game. The expectation w.r.t. this probability distribution is denoted by E_{σ^1,σ^2}. Given a discount factor $0 < \beta < 1$ the (unnormalized) β-discounted payoff to player i is defined by

$$V^i_\beta(\sigma^1,\sigma^2) = E_{\sigma^1,\sigma^2}\left(\sum_{t=1}^{\infty} \beta^{t-1} u_i(z_t, a_t)\right)$$

and the *normalized β-discounted payoff* to player i is defined by

$$v^i_\beta(\sigma^1,\sigma^2) = (1-\beta) V^i_\beta(\sigma^1,\sigma^2).$$

Main Result

Let $\Gamma = \langle S, A, u, p, \mu \rangle$ be a two-person stochastic game with countably many states, finitely many actions at each state, and a bounded payoff function u_2.

Main Result

Let $\Gamma = \langle S, A, u, p, \mu \rangle$ be a two-person stochastic game with countably many states, finitely many actions at each state, and a bounded payoff function u_2. Let σ^1 be a φ-based behavioral strategy of player 1 in Γ^∞.

Main Result

Let $\Gamma = \langle S, A, u, p, \mu \rangle$ be a two-person stochastic game with countably many states, finitely many actions at each state, and a bounded payoff function u_2. Let σ^1 be a φ-based behavioral strategy of player 1 in Γ^∞. If φ is recursive, then the following hold.

Main Result

Let $\Gamma = \langle S, A, u, p, \mu \rangle$ be a two-person stochastic game with countably many states, finitely many actions at each state, and a bounded payoff function u_2. Let σ^1 be a φ-based behavioral strategy of player 1 in Γ^∞. If φ is recursive, then the following hold.

(i) For every $\beta \in (0, 1)$ there exists a φ-based pure strategy σ^2

Main Result

Let $\Gamma = \langle S, A, u, p, \mu \rangle$ be a two-person stochastic game with countably many states, finitely many actions at each state, and a bounded payoff function u_2. Let σ^1 be a φ-based behavioral strategy of player 1 in Γ^∞. If φ is recursive, then the following hold.

(i) For every $\beta \in (0, 1)$ there exists a φ-based pure strategy σ^2 such that for every behavioral strategy ρ of player 2 in Γ^∞

Main Result

Let $\Gamma = \langle S, A, u, p, \mu \rangle$ be a two-person stochastic game with countably many states, finitely many actions at each state, and a bounded payoff function u_2. Let σ^1 be a φ-based behavioral strategy of player 1 in Γ^∞. If φ is recursive, then the following hold.

(i) For every $\beta \in (0, 1)$ there exists a φ-based pure strategy σ^2 such that for every behavioral strategy ρ of player 2 in Γ^∞ we have $v^2_\beta(\sigma^1, \sigma^2) \geq v^2_\beta(\sigma^1, \rho)$.

Main Result

(ii) If S and the range of φ are, in addition, finite,

Main Result

(ii) If S and the range of φ are, in addition, finite, then there is a φ-based pure strategy σ^2 and a discount factor $\beta_0 \in (0, 1)$ such that

Main Result

(ii) If S and the range of φ are, in addition, finite, then there is a φ-based pure strategy σ^2 and a discount factor $\beta_0 \in (0,1)$ such that

- for every behavioral strategy ρ (of player 2 in Γ^∞)

Main Result

(ii) If S and the range of φ are, in addition, finite, then there is a φ-based pure strategy σ^2 and a discount factor $\beta_0 \in (0, 1)$ such that

- for every behavioral strategy ρ (of player 2 in Γ^∞) and every $\beta \in [\beta_0, 1)$,

Main Result

(ii) If S and the range of φ are, in addition, finite, then there is a φ-based pure strategy σ^2 and a discount factor $\beta_0 \in (0, 1)$ such that

- for every behavioral strategy ρ (of player 2 in Γ^∞) and every $\beta \in [\beta_0, 1)$, we have $v^2_\beta(\sigma^1, \sigma^2) \geq v^2_\beta(\sigma^1, \rho)$;

Main Result

(ii) If S and the range of φ are, in addition, finite, then there is a φ-based pure strategy σ^2 and a discount factor $\beta_0 \in (0,1)$ such that

- for every behavioral strategy ρ (of player 2 in Γ^∞) and every $\beta \in [\beta_0, 1)$, we have $v^2_\beta(\sigma^1, \sigma^2) \geq v^2_\beta(\sigma^1, \rho)$;
- for every behavioral strategy ρ we have

Main Result

(ii) If S and the range of φ are, in addition, finite, then there is a φ-based pure strategy σ^2 and a discount factor $\beta_0 \in (0,1)$ such that

- for every behavioral strategy ρ (of player 2 in Γ^∞) and every $\beta \in [\beta_0, 1)$, we have $v_\beta^2(\sigma^1, \sigma^2) \geq v_\beta^2(\sigma^1, \rho)$;
- for every behavioral strategy ρ we have

$$E_{\sigma^1,\sigma^2}\left(\liminf_{n\to\infty} \frac{1}{n}\sum_{t=1}^{n} u_2(z_t, a_t)\right) \geq E_{\sigma^1,\rho}\left(\limsup_{n\to\infty} \frac{1}{n}\sum_{t=1}^{n} u_2(z_t, a_t)\right).$$

MAX-PLANCK-GESELLSCHAFT

Conclusion

- A new approach to modeling strategies of bounded complexity is offered: factor-based strategies.

Conclusion

- A new approach to modeling strategies of bounded complexity is offered: factor-based strategies.
- The player's perception of the set of histories H is represented by a factor $\varphi : H \to X$, where X reflects the "cognitive complexity" of the player. The factor-based strategy is defined just on the elements of the set X.

Conclusion

- A new approach to modeling strategies of bounded complexity is offered: factor-based strategies.
- The player's perception of the set of histories H is represented by a factor $\varphi : H \to X$, where X reflects the "cognitive complexity" of the player. The factor-based strategy is defined just on the elements of the set X.
- Various strategies (as strategies played by finite automata, strategies with bounded recall as well as strategies based on imperfect monitoring) can be now jointly analysed in the same framework.

MAX-PLANCK-GESELLSCHAFT

Conclusion

- If the factor φ satisfies a natural additional condition (recursivity), then for every profile of factor-based strategies there is a best reply that is a pure factor-based strategy.

Conclusion

- If the factor φ satisfies a natural additional condition (recursivity), then for every profile of factor-based strategies there is a best reply that is a pure factor-based strategy.
- Besides other consequences we get that, in general, private strategies does not fare better than the public strategies against public strategies.

Should you remember more than me?

Should you remember more than me?

No, you do not have to!!!

Should you remember more than me?

No, you do not have to!!!

Thank you for your attention!