Zpracování měření a dat

tutorial

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rev. 2023, Josef Blažej

1. Definition of terms

1.1 List all the fundamental units.

[kg, m, s, mol, K, Cd, A]

1.2 Define the volume formula of the

(a) sphere
(b) cube
(c) cylinder

by known parameters radius (r), edge (a) and height (h).

[$\frac{4}{3}\pi r^3$ | a^3 | $\pi r^2 h$]

1.3 Which quantity represent these derived units?

(a) $m \cdot s^{-2}$
(b) A
(c) $kg \cdot m^{-3}$
(d) $kg \cdot m^2 \cdot s^{-2}$

[acceleration | is not derived unit | density | quantity of work]

1.4 Decide on the type of error?
Data = {1, 15, 30, 16, 35, 17, 10, 18}

(a) 35 in the range of $\langle 0,30 \rangle$
(b) 1, 30, 10 in the range of $\langle 0,30 \rangle$
(c) 15, 16, 17, 18 if you presume constant values

[raw | random | systematic]

1.5 Decide if the following measurements are direct or indirect?

(a) measure the edge of the table by the scale
(b) measure the surface of the square table by measurement (a)
(c) measure the voltage by the voltmeter
(d) people counting (51% of 1M voted)

[direct | indirect | indirect | direct]

1.6 Decide on the term?
Possibilities: accuracy, precision, bias

(a) The table is measured by the scale bought in the shop 1000 times.
(b) The table is compared with the standard 1000 times.
(c) What is the difference between (a) and (b).

[precision | accuracy | bias]

1.7 What type of measurement reduce the systematic error?

[comparative or compensation]

2. Normal error distribution and others

2.1 Write the Gaussian distribution formula.

$$\left[p(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x-x_0)^2}{2\sigma^2}} \right]$$

2.2 If σ is 40. How much is FWHM?

[96]

2.3 Data = {10, 20, 23, 14, 5, 33, 24, 11}
What is the most probable value?

$$x_0 = \frac{1}{n} \cdot \sum_{i=1}^{n} x_i = 17{,}5$$

2.4 How will be precision changed if we increase 100 times number of measurements?

$$s \approx \frac{1}{\sqrt{n}} = 0{,}1$$

the precision will increase 10 times.

2.5 Write the RMS formula?

2.6 Write the definition of probability (by the integral)?

2.7 What is the standard deviation of the following measurement with random errors?
Data = {10, 20, 23, 14, 5, 23, 24, 21, 4}

$$x_0 = \frac{1}{n} \cdot \sum_{i=1}^{n} x_i = 16$$

$$RMS \approx \sigma = \sqrt{\frac{\sum_{i=1}^{n} (x_i - x_0)^2}{n}} = 7{,}5$$

2.8 Data = {200cm, 203cm, 201cm, 199cm, 198cm, 199cm}
Mentioned data are the radius of sphere.
What is the standard deviation of the radius (r) and the volume (V)?

$r_0 = 200\text{cm}$

$$\sigma_r = \sqrt{\frac{\sum_{i=1}^{n} (x_i - x_0)^2}{n}} = 1{,}63\,cm$$

$$\sigma_V = \sqrt{\sum_{i=1}^{n} [(\frac{\partial V}{\partial r})^2 \cdot \sigma_r^2]} = \frac{\partial V}{\partial r} \cdot \sigma_r = \frac{\partial(\frac{4}{3}\pi r^3)}{\partial r} \cdot \sigma_r = 4\pi r_0^2 \sigma_r = 0{,}82\,m^3$$

2.9 What is the precision of circumference of a triangle with edges a_1, a_2, a_3 ?

$$\text{Data} = \{ \quad \begin{matrix} a_1 = (40\text{cm}, 41\text{cm}, 42\text{cm}), \\ a_2 = (20\text{cm}, 19\text{cm}, 19\text{cm}, 22\text{cm}), \\ a_3 = (30\text{cm}, 31\text{cm}, 31\text{cm}, 31\text{cm}, 27\text{cm}) \end{matrix} \quad \}$$

$a_{01} = 40\text{cm}$
$a_{02} = 20\text{cm}$
$a_{03} = 30\text{cm}$

$\sigma_{a1} = 1{,}29\,cm$
$\sigma_{a2} = 1{,}22\,cm$
$\sigma_{a3} = 1{,}55\,cm$

$$\sigma_C = \sqrt{\sigma_{a1}^2 + \sigma_{a2}^2 + \sigma_{a3}^2} = 2{,}36\,cm$$

2.10 What is the precision of circumference of rectangle with edges a_1, a_2 ?
Data = { $a_1 = (40\text{cm}, 41\text{cm}, 42\text{cm}), a_2 = (20\text{cm}, 19\text{cm}, 19\text{cm}, 22\text{cm})$ }

$a_{01} = 40\text{cm}$
$a_{02} = 20\text{cm}$

$\sigma_{a1} = 1{,}29\,cm$
$\sigma_{a2} = 1{,}22\,cm$

$$\sigma_S = a_{01} \cdot a_{02} \cdot \sqrt{(\frac{\sigma(a_1)}{a_{01}})^2 + (\frac{\sigma(a_1)}{a_{01}})^2} = 55{,}38\,cm^2$$

2.11 The histogram shows the registered photons. In each column is registered N_i photons. Write the probability formula of detection in *i-th* channel. Presume that the measurement started in the past and the total number of measurement N_{tot} is known.

$$\left[p_i = \frac{N_i}{N_{tot} - \sum_{k=1}^{i-1} N_k} \right]$$

2.12 How much parameters has a polynomial of degree 10.

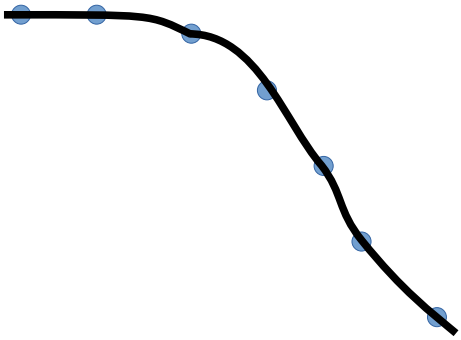
[11]

2.13 Consider 32 points equidistantly measured in time. What is the maximum degree of best fitting polynomial with the lowest possible RMS? Presume that we have no more additional information.

- points are evenly distributed without abnormal gaps
- pol. degree should be $\ll 32$
- pol. degree should be < 5.7

[5]

2.14 Why the curve displayed is not a spline?



[the curve is continuous, but its derivation has discontinuity in the third point]