# Description of D–branes invariant under Poisson–Lie T–plurality, [arXiv:0806.0963]

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## I. Gluing matrix

Sigma model on $\Sigma = \mathbb{R}\times\langle 0,\pi\rangle$ given by the action

$$S_{\mathcal{F}}[\phi] = \int_\Sigma d^2x\,\partial_-\phi\cdot\mathcal{F}(\phi)\cdot\partial_+\phi^t + \int_{\sigma=0} A\cdot\frac{\partial\phi^t}{\partial\tau}d\tau - \int_{\sigma=\pi} A\cdot\frac{\partial\phi^t}{\partial\tau}d\tau \qquad (1)$$

with boundary conditions of the form

$$\partial_- g|_{\sigma=0,\pi} = \mathcal{R}\,\partial_+ g|_{\sigma=0,\pi}, \qquad \sigma\equiv x_+ - x_-. \qquad (2)$$

$\mathcal{R}$ ... gluing operator ... in principle encodes configuration of D–branes through projector onto brane tangent space $\mathcal{N}=\mathcal{N}^2$ defined by

$$\mathcal{N}\circ(\mathcal{R}+id) = (\mathcal{R}+id),\ \ \mathrm{Ran}\,\mathcal{N} = \mathrm{Ran}\,(\mathcal{R}+id)$$

## II. Physical requirements on $\mathcal{R}$

- conformality (vanishing stress tensor)[a]

$$R\cdot(\mathcal{F}+\mathcal{F}^t)\cdot R^t = (\mathcal{F}+\mathcal{F}^t) \qquad (3)$$

- D–brane a submanifold, i.e. Ran$\mathcal{N}$ integrable

$$N_\kappa{}^\mu N_\lambda{}^\nu\partial_{[\mu}N_{\nu]}{}^\rho = 0 \qquad (4)$$

- boundary equation of motion

$$N\cdot\Big((\mathcal{F}+\Delta) - (\mathcal{F}+\Delta)^t\cdot R^t\Big) = 0 \qquad (5)$$

where $\Delta = \mathrm{d}A$, i.e. satisfies

$$N_\kappa{}^\nu N_\lambda{}^\rho N_\mu{}^\sigma\partial_{[\nu}\Delta_{\rho\sigma]} = 0. \qquad (6)$$

[a] non–script letters = matrices of corresponding operators in a coordinate basis, acting from the right

## III. Equivalently, but without $\mathcal{N}$

If we assume that $\Delta$ is defined by (5) (its non–uniqueness doesn't affect the sigma model dynamics) we can find an equivalent version of (6) in the form[a]

$$0 = \frac{\partial}{\partial y^\vartheta}\left(\mathcal{F}^t\cdot R^t - \mathcal{F}\right)_{\rho[\nu}(R+\mathbf{1})_\mu{}^\rho(R+\mathbf{1})_{\lambda]}{}^\vartheta - \left(\mathcal{F}^t\cdot R^t - \mathcal{F}\right)_{\rho[\nu}\frac{\partial}{\partial y^\vartheta}(R+\mathbf{1})_\mu{}^\rho(R+\mathbf{1})_{\lambda]}{}^\vartheta. \qquad (7)$$

This together with (4) written as Frobenius integrability condition

$$[\mathrm{Ran}\,(\mathcal{R}+id), \mathrm{Ran}\,(\mathcal{R}+id)]\subset \mathrm{Ran}\,(\mathcal{R}+id) \qquad (8)$$

allows to study Eq. (3-6) without knowledge of explicit form of the projector $\mathcal{N}$.

[a] antisymmetrization in $\lambda,\mu,\nu$

## IV. Poisson–Lie T–plurality I [1, 2]

Target space ... isotropic subgroup $G$ of Drinfel'd double $(G|\tilde G)$ – Lie group $D$ whose Lie algebra $\mathfrak{d}$ admits a decomposition $\mathfrak{d} = \mathfrak{g}\dot{+}\tilde{\mathfrak{g}}$ into a pair of subalgebras maximally isotropic with respect to a symmetric ad-invariant non-degenerate bilinear form $\langle\,.\,,.\,\rangle$.

$\mathcal{F}$ written as $\mathcal{F} = e(g)\cdot F(g)\cdot e(g)^t$ where $e(g)$ are components of right–invariant Maurer–Cartan form $dgg^{-1}$ and

$$F(g) = (E_0^{-1}+\Pi)^{-1},\ \ \Pi = b(g)\cdot a(g)^{-1} \qquad (9)$$

where $E_0$ is a constant matrix and $a(g), b(g)$ are submatrices of the adjoint representation of the group $G$ on $\mathfrak{d}$. $\Pi$ defines the Poisson structure on the Poisson–Lie group $G$.

## V. Poisson–Lie T–plurality II

Equations of motion of such sigma models allow the lift $l : \Sigma\to D$ of the solution $g:\Sigma\to G$ into the Drinfel'd double which satisfies [1]

$$\langle\,\partial_\pm l l^{-1}, \mathcal{E}^\pm\,\rangle = 0, \qquad (10)$$

where $l = g\tilde h$ and $\mathcal{E}^\pm$ are two orthogonal subspaces in $\mathfrak{d}$, spanned by $T+E_0\cdot\tilde T$, $T-E_0^t\cdot\tilde T$, respectively. (10) is invariant w.r.t. choice of the decomposition $\mathfrak{g}\dot{+}\tilde{\mathfrak{g}}$ of $\mathfrak{d}$.

Decomposing $\mathfrak{d}$ into other pairs $\hat{\mathfrak{g}}\dot{+}\bar{\mathfrak{g}}$, $l = \hat g\bar h$ we get solutions of sigma models defined on $\hat G$ with the tensor $\hat F(\hat g) = (\hat E_0^{-1}+\hat\Pi)^{-1}$. This procedure was called Poisson–Lie T–plurality in [2].

## VI. Transformation of $\mathcal{R}$ [3]

If the original boundary conditions are prescribed in the form

$$R = e(g)\cdot F^t(g)\cdot C\cdot F^{-1}(g)\cdot e^{-1}(g), \qquad (11)$$

where $C$ is a constant matrix orthogonal w.r.t $E_0^{-1}+E_0^{-t}$ due to (3), then the transformed solutions $\hat g$ satisfy boundary conditions in a form similar to (11) (i.e. with proper replacements by $\hat F(\hat g), \hat e(\hat g)$) with

$$\hat C = M_-^{-1}\cdot C\cdot M_+, \qquad (12)$$

where matrices $M_+, M_-$ and $\hat E_0$ are explicitly given in terms of $E_0$ and the transformation matrix between the pairs of dual bases of $\mathfrak{g},\tilde{\mathfrak{g}}$ and $\hat{\mathfrak{g}},\bar{\mathfrak{g}}$ only.

## VII. Lifted D–branes

D–branes defined by the gluing matrix (11) can be lifted to the Drinfel'd double. The lifted boundary conditions take the form

$$\partial_\tau l\, l^{-1}|_{\sigma=0,\pi}\in V_{\mathcal{D}} = \mathrm{span}(A\cdot T + B\cdot\tilde T), \qquad (13)$$

where the matrices $A$ and $B$ are

$$A = E_0^{-t} + C\cdot E_0^{-1},\quad B = C - \mathbf{1}. \qquad (14)$$

Due to (3) $V_{\mathcal{D}}$ is maximally isotropic. The cond. (7) is equivalent to $V_{\mathcal{D}}$ being a subalgebra. Therefore, the lifted D–branes are cosets $\mathfrak{D}l$ where $Lie(\mathfrak{D}) = V_{\mathcal{D}}$ as predicted in [4] and the integrability (8) of D–branes in $G$ follows by coset projection. Conversely, any maximally isotropic subalgebra can be written[a] in the form $V_{\mathcal{D}}$.

[a] upon some regularity condition, generically satisfied

## VIII. Summary

We have shown that the set of physical constraints on the gluing operator $\mathcal{R}$ as postulated in (3-6) is preserved under Poisson Lie T–duality(plurality) transformation provided the transformed electromagnetic field strength $\Delta$ is found from (5). On the other hand, if one has imposed the richer set of constraints stated in [5] then they would not be invariant.

We have also proved that our formulation is equivalent to the description originally discovered by C. Klimčík and P. Ševera in [4]. Both approaches can be considered complementary; their being more geometric; in our it is easier to write both the original and transformed boundary conditions.

## References

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