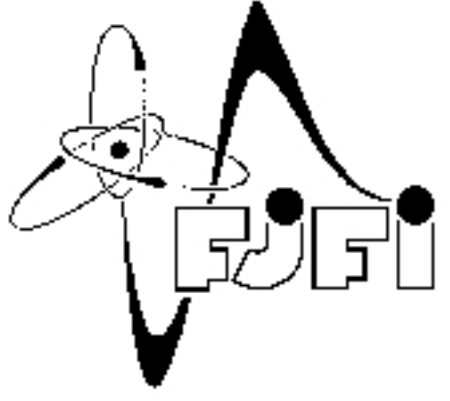
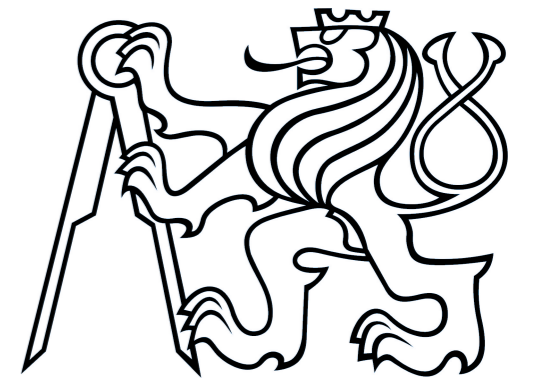


# Description of D–branes invariant under Poisson–Lie T–plurality, [arXiv:0806.0963]



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## I. Gluing matrix

Sigma model on  $\Sigma = \mathbb{R} \times \langle 0, \pi \rangle$  given by the action

$$S_{\mathcal{F}}[\phi] = \int_{\Sigma} d^2x \partial_- \phi \cdot \mathcal{F}(\phi) \cdot \partial_+ \phi^t + \int_{\sigma=0} A \cdot \frac{\partial \phi^t}{\partial \tau} d\tau - \int_{\sigma=\pi} A \cdot \frac{\partial \phi^t}{\partial \tau} d\tau \quad (1)$$

with boundary conditions of the form

$$\partial_- g|_{\sigma=0, \pi} = \mathcal{R} \partial_+ g|_{\sigma=0, \pi}, \quad \sigma \equiv x_+ - x_- \quad (2)$$

$\mathcal{R} \dots$  **gluing operator**  $\dots$  in principle encodes configuration of D–branes through **projector** onto brane tangent space  $\mathcal{N} = \mathcal{N}^2$  defined by

$$\mathcal{N} \circ (\mathcal{R} + id) = (\mathcal{R} + id), \quad \text{Ran } \mathcal{N} = \text{Ran } (\mathcal{R} + id)$$

## II. Physical requirements on $\mathcal{R}$

- **conformality** (vanishing stress tensor)<sup>a</sup>

$$R \cdot (\mathcal{F} + \mathcal{F}^t) \cdot R^t = (\mathcal{F} + \mathcal{F}^t) \quad (3)$$

- **D–brane a submanifold**, i.e.  $\text{Ran } \mathcal{N}$  integrable

$$N_{\kappa}^{\mu} N_{\lambda}^{\nu} \partial_{[\mu} N_{\nu]}^{\rho} = 0 \quad (4)$$

- **boundary equation of motion**

$$N \cdot \left( (\mathcal{F} + \Delta) - (\mathcal{F} + \Delta)^t \cdot R^t \right) = 0 \quad (5)$$

where  $\Delta = dA$ , i.e. satisfies

$$N_{\kappa}^{\nu} N_{\lambda}^{\rho} N_{\mu}^{\sigma} \partial_{[\nu} \Delta_{\rho\sigma]} = 0. \quad (6)$$

<sup>a</sup>non-script letters = matrices of corresponding operators in a coordinate basis, acting from the right

## III. Equivalently, but without $\mathcal{N}$

If we assume that  $\Delta$  is defined by (5) (its non-uniqueness doesn't affect the sigma model dynamics) we can find an equivalent version of (6) in the form<sup>a</sup>

$$0 = \frac{\partial}{\partial y^{\theta}} (\mathcal{F}^t \cdot R^t - \mathcal{F})_{\rho[\nu} (R+1)_{\mu}^{\rho} (R+1)_{\lambda]}^{\theta} - (\mathcal{F}^t \cdot R^t - \mathcal{F})_{\rho[\nu} \frac{\partial}{\partial y^{\theta}} (R+1)_{\mu}^{\rho} (R+1)_{\lambda]}^{\theta}. \quad (7)$$

This together with (4) written as Frobenius integrability condition

$$[\text{Ran } (\mathcal{R} + id), \text{Ran } (\mathcal{R} + id)] \subset \text{Ran } (\mathcal{R} + id) \quad (8)$$

allows to study Eq. (3-6) without knowledge of explicit form of the projector  $\mathcal{N}$ .

<sup>a</sup>antisymmetrization in  $\lambda, \mu, \nu$

## IV. Poisson–Lie T–plurality I [1, 2]

**Target space**  $\dots$  isotropic subgroup  $G$  of **Drinfel'd double**  $(G|\hat{G})$  – Lie group  $D$  whose Lie algebra  $\mathfrak{d}$  admits a decomposition  $\mathfrak{d} = \mathfrak{g} \dot{+} \tilde{\mathfrak{g}}$  into a pair of subalgebras maximally isotropic with respect to a symmetric ad-invariant non-degenerate bilinear form  $\langle \cdot, \cdot \rangle$ .

$\mathcal{F}$  written as  $\mathcal{F} = e(g) \cdot F(g) \cdot e(g)^t$  where  $e(g)$  are components of right-invariant Maurer–Cartan form  $dg g^{-1}$  and

$$F(g) = (E_0^{-1} + \Pi)^{-1}, \quad \Pi = b(g) \cdot a(g)^{-1} \quad (9)$$

where  $E_0$  is a constant matrix and  $a(g), b(g)$  are submatrices of the adjoint representation of the group  $G$  on  $\mathfrak{d}$ .  $\Pi$  defines the Poisson structure on the Poisson–Lie group  $G$ .

## V. Poisson–Lie T–plurality II

Equations of motion of such sigma models allow the **lift**  $l : \Sigma \rightarrow D$  of the solution  $g : \Sigma \rightarrow G$  into the Drinfel'd double which satisfies [1]

$$\langle \partial_{\pm} l l^{-1}, \mathcal{E}^{\pm} \rangle = 0, \quad (10)$$

where  $l = g \tilde{h}$  and  $\mathcal{E}^{\pm}$  are two orthogonal subspaces in  $\mathfrak{d}$ , spanned by  $T + E_0 \cdot \tilde{T}, T - E_0^t \cdot \tilde{T}$ , respectively. (10) is invariant w.r.t. choice of the decomposition  $\mathfrak{g} \dot{+} \tilde{\mathfrak{g}}$  of  $\mathfrak{d}$ .

Decomposing  $\mathfrak{d}$  into other pairs  $\hat{\mathfrak{g}} \dot{+} \bar{\mathfrak{g}}, l = \hat{g} \bar{h}$  we get solutions of sigma models defined on  $\hat{G}$  with the tensor  $\hat{F}(\hat{g}) = (\hat{E}_0^{-1} + \hat{\Pi})^{-1}$ . This procedure was called **Poisson–Lie T–plurality** in [2].

## VI. Transformation of $\mathcal{R}$ [3]

If the original boundary conditions are prescribed in the form

$$R = e(g) \cdot F^t(g) \cdot C \cdot F^{-1}(g) \cdot e^{-1}(g), \quad (11)$$

where  $C$  is a constant matrix orthogonal w.r.t  $E_0^{-1} + E_0^{-t}$  due to (3), then the **transformed solutions**  $\hat{g}$  satisfy boundary conditions in a form similar to (11) (i.e. with proper replacements by  $\hat{F}(\hat{g}), \hat{e}(\hat{g})$ ) with

$$\hat{C} = M_-^{-1} \cdot C \cdot M_+, \quad (12)$$

where matrices  $M_+, M_-$  and  $\hat{E}_0$  are explicitly given in terms of  $E_0$  and the transformation matrix between the pairs of dual bases of  $\mathfrak{g}, \tilde{\mathfrak{g}}$  and  $\hat{\mathfrak{g}}, \bar{\mathfrak{g}}$  only.

## VII. Lifted D–branes

D–branes defined by the gluing matrix (11) can be lifted to the Drinfel'd double. The lifted boundary conditions take the form

$$\partial_{\tau} l l^{-1}|_{\sigma=0, \pi} \in V_{\mathcal{D}} = \text{span}(A \cdot T + B \cdot \tilde{T}), \quad (13)$$

where the matrices  $A$  and  $B$  are

$$A = E_0^{-t} + C \cdot E_0^{-1}, \quad B = C - \mathbf{1}. \quad (14)$$

Due to (3)  $V_{\mathcal{D}}$  is maximally isotropic. The cond. (7) is equivalent to  $V_{\mathcal{D}}$  being a subalgebra. Therefore, the lifted D–branes are **cosets  $\mathcal{D}l$**  where  $\text{Lie}(\mathcal{D}) = V_{\mathcal{D}}$  as predicted in [4] and the integrability (8) of D–branes in  $G$  follows by coset projection. Conversely, any maximally isotropic subalgebra can be written<sup>a</sup> in the form  $V_{\mathcal{D}}$ .

<sup>a</sup>upon some regularity condition, generically satisfied

## VIII. Summary

We have shown that **the set of physical constraints on the gluing operator  $\mathcal{R}$  as postulated in (3-6) is preserved under Poisson Lie T–duality(plurality)** transformation provided the transformed electromagnetic field strength  $\Delta$  is found from (5). On the other hand, if one has imposed the richer set of constraints stated in [5] then they would not be invariant.

We have also proved that **our formulation is equivalent to the description originally discovered by C. Klimčík and P. Ševera in [4]**. Both approaches can be considered complementary; their being more geometric; in our it is easier to write both the original and transformed boundary conditions.

## References

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