



# Jiří Patera

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## Jiří Patera (1936 – 2022)

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- born in **Zdice**, Czechoslovakia (10 October 1936)
- graduated from high school in **Děčín** (1955)
- admitted to the *Faculty of Technical and Nuclear Physics*, **Charles University**, Prague (1955)
- continued his studies at the **Leningrad State University** (1956–1957)
- received an equivalent of M.Sc. degree in *Theoretical Nuclear Physics* at the **Moscow State University** (1958–1961)
- affiliated with *Physics Institute* of the **Czech Academy of Sciences**, Prague (1961–1969)

- realized scientific stays
  - ▶ Institute of Nuclear Physics, Dubna, USSR
  - ▶ California Institute of Technology, Pasadena, USA
- received an equivalent of Ph.D. degree at the *Faculty of Nuclear Sciences and Physical Engineering*, **Czech Technical University**, Prague (1964)
- realized postdoctoral stays
  - ▶ Université de Montréal, Canada
  - ▶ Imperial College of Science and Technology, London, UK
- member of *Centre des Recherches Mathématiques*, **Université de Montréal**, Canada (1969–2021)
- became a full professor at the *Département de mathématiques et de statistique, Faculté des arts et des sciences*, **Université de Montréal**, Canada (1983)
- died in **Montréal** (3 January 2022)

- **awards**

- ▶ Killam Research Fellowship (1983–1984)
- ▶ CAP-CRM Prize in Theoretical and Mathematical Physics (2004)
- ▶ Honoris Causa Doctoral Degree, Czech Technical University, Prague (2005)

- **memberships**

- ▶ MIND Institute, USA

- **grants**

- ▶ Natural Sciences and Engineering Research Council of Canada (NSERC)
- ▶ NATO
- ▶ MITACS
- ▶ Lockheed Martin Canada
- ▶ OODA Technologies

- **patents**

- ▶ *Aperiodic encryption for digital data*, with E. Pelantová, Z. Masáková (2000)
- ▶ *Continuous extension of discrete transform for data processing*, with A. Atoyan (2010)

- **publications**

- ▶ 5 books
- ▶ 237 articles
- ▶ 4 391 citations,  $h$ -index 33 (Scopus)

- **supervision**

- ▶ M.Sc., Ph.D., postdocs

- **branching rules for simple Lie algebras**

- original algorithm for reduction of representations of simple Lie algebras to direct sum of representations of their maximal subalgebras
  - ▶ J. Patera, D. Sankoff, *Branching rules for representations of simple Lie algebras*, Presses Université de Montréal, Montréal (1973)
  - ▶ W. G. McKay, J. Patera, *Tables of dimensions, indices, and branching rules for representations of simple Lie algebras*, Marcel Dekker, New York (1981)
  - ▶ W. G. McKay, J. Patera, D. Rand, *Tables of representations of simple Lie algebras, Vol. 1: Exceptional simple Lie algebras*, Les Publications CRM, Montréal (1990)

- **dominant weight multiplicities**

- original algorithm for fast computation of the multiplicities of dominant weight of irreducible representations of simple Lie algebras and affine Kac–Moody algebras
  - ▶ M. R. Bremner, R. V. Moody, J. Patera, *Tables of dominant weight multiplicities for representations of simple Lie algebras*, Marcel Dekker, New York (1985)
  - ▶ S. Kass, R. V. Moody, J. Patera, R. Slansky, *Affine Kac–Moody Algebras, Weight Multiplicities and Branching Rules, Vol.1 & 2*, University of California Press, Berkeley (1991)

- **subalgebras of Lie algebras**
- methods for classifying subalgebras of a given Lie algebra under the action of groups of inner or outer automorphisms
  - ▶ J. Patera, R. T. Sharp, P. Winternitz, H. Zassenhaus, *Subgroups of the similitude group of three-dimensional Minkowski space*, *Canad. J. Phys.* **54** (1976) 950–961
  - ▶ J. Patera, P. Winternitz, H. Zassenhaus, *Quantum numbers for particles in de Sitter space*, *J. Math. Phys.* **7** (1976) 717–728
  - ▶ J. Patera, P. Winternitz, *Subalgebras of real three and four dimensional Lie algebras*, *J. Math. Phys.* **18** (1977) 1449–1455
- **maximal Abelian subalgebras**
- results specialized to maximal Abelian subalgebras of the classical Lie algebras
  - ▶ J. Patera, P. Winternitz, H. Zassenhaus, *On the maximal Abelian subgroups of the linear classical algebraic groups*, *Math. Rep. Acad. Sci.* **2** (1980) 231–236
  - ▶ J. Patera, P. Winternitz, H. Zassenhaus, *Maximal Abelian subalgebras of real and complex symplectic Lie algebras*, *J. Math. Phys.* **24** (1983) 1973–1985

- **elements of finite order in simple Lie groups**
- properties of characters of conjugacy classes of the elements of finite order in simple Lie groups
  - ▶ R. V. Moody, J. Patera, *Characters of elements of finite order in simple Lie groups*, SIAM J. on Algebraic and Discrete Methods **5** (1984) 359–383
  - ▶ R. V. Moody, J. Patera, *Computation of character decompositions of class functions on compact semisimple Lie groups*, Mathematics of Computation **48** (1987) 799–827
  - ▶ W. G. McKay, R. V. Moody, J. Patera, A. Pianzola, *The 785 conjugacy classes of rational elements of finite order in  $E_8$* , Contemporary Math. **110** (1990) 79–123
  - ▶ S. Grimm and J. Patera, *Decomposition of tensor products of the fundamental representations of  $E_8$* , in Advances in Mathematical Sciences – CRM's 25 Years, ed. L. Vinet, CRM Proc. Lecture Notes, vol. 11, Amer. Math. Soc., Providence, RI (1997) 329–355.
- **character generators for elements of finite order**
  - ▶ R. V. Moody, J. Patera, R. T. Sharp, *Character generators for elements of finite order in simple Lie groups  $A_1, A_2, A_3, B_2$  and  $G_2$* , J. Math. Phys. **24** (1983) 2387–2397

- **gradings of simple Lie algebras**

- classification of fine gradings of classical simple Lie algebras

- ▶ J. Patera, H. Zassenhaus, *The Pauli matrices in  $n$ -dimensions and finest gradings of Lie algebras of type  $A_{n-1}$* , J. Math. Phys. **29** (1988) 665–673
- ▶ J. Patera, H. Zassenhaus, *On Lie gradings I*, Linear Algebra & Appl. **112** (1989) 87–159
- ▶ M. Havlíček, J. Patera, E. Pelantová, *On the fine gradings of simple classical Lie algebras*, Int. J. Mod. Physics A **12** (1997) 189–194
- ▶ M. Havlíček, J. Patera, and E. Pelantová, *On Lie gradings II. (dedicated to the memory of Hans Zassenhaus)*, Linear Algebra & Appl. **277** (1998) 97–125
- ▶ M. Havlíček, J. Patera, and E. Pelantová, *On Lie gradings III. Gradings of the real forms of classical Lie algebras, (dedicated to the memory of Hans Zassenhaus)*, Linear Algebra & Appl. **314** (2000) 1–47

- **gradings of representations**

- ▶ J. Patera, R. T. Sharp, J. Van der Jeugt, *New gradings of  $\mathfrak{sl}(3, \mathbb{C})$  representations*, J. Math. Phys. **30** (1989) 2763–2769

- **classification and symmetries of fine gradings**

- ▶ M. Havlíček, J. Patera, E. Pelantová, J. Tolar, *On fine gradings and their symmetries*, Czech. J. Phys. **51** (2001) 383–391
- ▶ J. Patera, E. Pelantová, M. Svobodová, *Fine gradings of  $\mathfrak{o}(5, \mathbb{C})$ ,  $\mathfrak{sp}(4, \mathbb{C})$  and of their real forms*, J. Math. Phys. **42** (2001) 3839–3853
- ▶ M. Havlíček, J. Patera, E. Pelantová, J. Tolar, *Automorphisms of a finest grading of  $\mathfrak{sl}(n, \mathbb{C})$* , J. Math. Phys. **43** (2002) 1083–1094
- ▶ J. Patera, E. Pelantová, M. Svobodová, *The eight fine gradings of  $\mathfrak{sl}(4, \mathbb{C})$  and  $\mathfrak{o}(6, \mathbb{C})$* , J. Math. Phys. **43** (2002) 6353–6378
- ▶ J. Patera, E. Pelantová, M. Svobodová, *Fine gradings of  $\mathfrak{o}(4, \mathbb{C})$* , J. Math. Phys. **45** (2004) 2188–2198

- **graded contractions of simple Lie algebras**

- classification of contractions induced by fine gradings of low-dimensional simple Lie algebras

- ▶ M. de Montigny, J. Patera, *Discrete and continuous graded contractions of Lie algebras and superalgebras*, J. Phys. A: Math. Gen. **24** (1991) 525–547
- ▶ M. Couture, J. Patera, R. T. Sharp, P. Winternitz, *Graded contractions of  $\mathfrak{sl}(3, \mathbb{C})$* , J. Math. Phys. **32** (1991) 2310–2318
- ▶ M. de Montigny, J. Patera, J. Tolar, *Graded contractions of kinematical groups of space time*, J. Math. Phys. **35** (1994) 405–425
- ▶ J. Hrivnák, P. Novotný, J. Patera, J. Tolar, *Graded contractions of the Pauli graded  $\mathfrak{sl}(3, \mathbb{C})$* , Linear Algebra & Appl. **418** (2006) 498–550

- **graded contractions of representations**

- ▶ R. V. Moody, J. Patera, *Discrete and continuous graded contractions of representations of Lie algebras*, J. Phys. A: Math. Gen. **24** (1991) 2227–2258
- ▶ X. Leng, J. Patera, *Graded contractions of representations of  $\mathfrak{sl}(n, \mathbb{C})$  with respect to the maximal parabolic subalgebras*, J. Phys. A: Math. Gen. **27** (1994) 1233–1250

- properties of **quasicrystals** studied from viewpoint of applications
  - ▶ R. V. Moody, J. Patera, *Quasicrystals and icosians*, J. Phys. A: Math. Gen. **26** (1993) 2829–2853
  - ▶ L. Chen, R. V. Moody, J. Patera, *Non-crystallographic root systems*, in Quasicrystals and Discrete Geometry, Fields Institute Monograph Series **10** 135–178, ed. J. Patera, Amer. Math. Soc. (1998)
  - ▶ Z. Masáková, J. Patera, E. Pelantová, *Inflation centers in cut and project quasicrystals*, J. Phys. A: Math. Gen. **31** (1998) 1443–1453
  - ▶ Z. Masáková, J. Patera, E. Pelantová, *Minimal distances in quasicrystals*, J. Phys. A: Math. Gen. **31** (1998) 1539–1552
  - ▶ Z. Masáková, J. Patera, E. Pelantová, *Selfsimilar Delone sets and cut and project quasicrystals*, J. Phys. A: Math. Gen. **31** (1998) 4927–4946
  - ▶ Z. Masáková, J. Patera, J. Zich, *Classification of distinct Voronoi and Delone tiles in two-dimensional quasicrystals: General Method*, J. Phys. A: Math. Gen. **36** (2003) 1869–1894
  - ▶ M. Grundland, J. Patera, Z. Masáková, N. Dodgson, *Image sampling on quasicrystals*, SIGMA **5** (2009) 075

- **carbon nanostructures** with non-crystallographic symmetry groups
  - ▶ B. Champagne, M. Kjiri, J. Patera, R. T. Sharp, *Description of reflection generated polytopes using decorated Coxeter diagrams*, Can. J. Phys. **73** (1995) 566–584
  - ▶ R. V. Moody, J. Patera, *Voronoi domains and dual cells in the generalized kaleidoscope with applications to root and weight lattices*, (dedicated to H. S. M. Coxeter), Can. J. Math., **47** (1995) 573–605
  - ▶ M. Bodner, J. Patera, M. Szajewska,  $C_{70}$ ,  $C_{80}$ ,  $C_{90}$ , and carbon nanotubes by breaking of the icosahedral symmetry of  $C_{60}$ , Acta Cryst. A: Foundations of Crystallography **69** (2013) 583–591
  - ▶ M. Bodner, J. Patera, M. Szajewska, *Breaking of Icosahedral Symmetry:  $C_{60}$  to  $C_{70}$* , PLoS ONE **9** (2014) e84079
  - ▶ M. Bodner, E. Bourret, J. Patera, M. Szajewska, *Icosahedral symmetry breaking:  $C_{60}$  to  $C_{78}$ ,  $C_{96}$  and to related nanotubes*, Acta Cryst. A: Foundations of Crystallography **70** (2014) 650–655
  - ▶ M. Bodner, E. Bourret, J. Patera, M. Szajewska, *Icosahedral symmetry breaking:  $C_{60}$  to  $C_{84}$ ,  $C_{108}$  and to related nanotubes*, Acta Cryst. A: Foundations of Crystallography **71** (2015) 297–300

- **discrete orthogonality and transforms** of Weyl orbit functions

- ▶ J. Patera, A. Zaratsyan, *Discrete and continuous cosine transform generalized to the Lie groups  $SU(2) \times SU(2)$  and  $O(5)$* , J. Math. Phys. **46** (2005) 053514
- ▶ J. Patera, A. Zaratsyan, *Discrete and continuous cosine transform generalized to Lie groups  $SU(3)$  and  $G(2)$* , J. Math. Phys. **46** (2005) 113506
- ▶ J. Patera, *Compact simple Lie groups and theirs  $C^-$ ,  $S^-$ , and  $E^-$ -transforms*, SIGMA **1** (2005) 025
- ▶ A. U. Klimyk, J. Patera, *Orbit functions*, SIGMA **2** (2006) 006
- ▶ R. V. Moody, J. Patera, *Orthogonality within the families of  $C^-$ ,  $S^-$ , and  $E^-$  functions of any compact semisimple Lie group*, SIGMA **2** (2006) 076
- ▶ A. U. Klimyk, J. Patera, *(Anti)symmetric multidimensional trigonometric functions and the corresponding Fourier transforms*, J. Math. Phys. **48** (2007) 093504
- ▶ A. U. Klimyk, J. Patera, *Antisymmetric orbit functions*, SIGMA **3** (2007) 023
- ▶ A. U. Klimyk, J. Patera,  *$E^-$ -orbit functions*, SIGMA **4** (2008) 002
- ▶ A. U. Klimyk, J. Patera, *Alternating multivariate trigonometric functions and corresponding Fourier transforms*, J. Phys. A: Math. Theor. **41** (2008) 145205

- **discrete orthogonality and transforms** of Weyl orbit functions

- ▶ J. Hrivnák, J. Patera, *On discretization of tori of compact simple Lie groups*, J. Phys. A: Math. Theor. **42** (2009) 385208
- ▶ J. Hrivnák, J. Patera, *On  $E$ -discretization of tori of compact simple Lie groups*, J. Phys. A: Math. Theor. **43** (2010) 165206
- ▶ J. Hrivnák, L. Motlochová, J. Patera, *Two-dimensional symmetric and antisymmetric generalizations of sine functions*, J. Math. Phys. **51** (2010) 073509
- ▶ J. Hrivnák, J. Patera, S. Pošta, *Three-variable exponential functions of the alternating group*, J. Phys. A: Math. Theor. **45** (2012) 045201
- ▶ L. Háková, J. Hrivnák, J. Patera, *Six types of  $E$ -functions of Lie group  $O(5)$  and  $G(2)$*  J. Phys. A: Math. Theor. **43** (2012) 165206
- ▶ J. Hrivnák, L. Motlochová, J. Patera, *On discretization of tori of compact simple Lie groups: II*, J. Phys. A: Math. Theor. **45** (2012) 255201
- ▶ L. Háková, J. Hrivnák, J. Patera, *Four families of Weyl group orbit functions of  $B_3$  and  $C_3$* , J. Math. Phys. **54** (2013) 083501
- ▶ J. Hrivnák, M. Myronova, J. Patera, *Central Splitting of  $A_2$  Discrete Fourier–Weyl Transforms*, Symmetry **12** (2020) 1828

- **orthogonality, transforms and cubature formulas** of associated Chebyshev-like multivariate polynomials
  - ▶ M. Nesterenko, J. Patera, M. Szajewska, A. Tereszkievicz, *Orthogonal polynomials of compact simple Lie groups. The branching rules for polynomials*, J. Phys. A: Math. Theor. **43** (2010) 495207
  - ▶ R. V. Moody, J. Patera, *Cubature formulae for orthogonal polynomials in terms of elements of finite order of compact simple Lie groups*, Advances of Applied Math. **47** (2011) 509–535
  - ▶ R. V. Moody, L. Motlochová, J. Patera, *Gaussian cubature arising from hybrid characters of simple Lie groups*, J. Fourier Anal. Appl. **20** (2014) 1257–1290
  - ▶ J. Hrivnák, L. Motlochová, J. Patera, *Cubature formulas of multivariate polynomials arising from symmetric orbit functions*, Symmetry **8** (2016) 63
  - ▶ A. Bezubik, J. Hrivnák, J. Patera, S. Pošta, *Three–variable symmetric and antisymmetric exponential functions and orthogonal polynomials*, Math. Slovac. **67** (2017), 427–446
  - ▶ T. Czyżycki, J. Hrivnák, J. Patera, *Generating Functions for Orthogonal Polynomials of  $A_2$ ,  $C_2$  and  $G_2$* , Symmetry **10** (2018) 354

"...Personally I was lucky to participate in that collaboration [with J. Patera], to see the motivation and energy how he asks new questions, his taste in choosing new problems. I witnessed his efforts to bring to completion particular result to possible applications, computer programs, user friendly tables, etc. I also had the chance to learn from him how to attract new students for science, how to boost their self-confidence and launch their scientific careers."

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*E. Pelantová*

"...On the scientific front, they [J. Patera and P. Winternitz] have written papers that will keep being touchstones for major areas of mathematics and theoretical physics as well as springboards for many discoveries to come. To all the people they have trained, inspired and befriended, sharing their knowledge, intelligence and culture, they have offered something of themselves that will be transmitted through generations."

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*L. Vinet*

"...In the end we wrote some 25 papers together, and our interactions and friendship extended until the very end.[...] In short, George was a huge component of my research and social life. He deeply influenced my appreciation of mathematical physics."

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*R. V. Moody*

- selected photographs









- *In memoriam - Jiří Patera (1936-2022)*, CRM, Montréal, Canada (2022)
- I. Kraus, V. Kučerová, *Čestní doktoři Českého vysokého učení technického v Praze*, CTU Publishing House, Prague, Czech Republic (2022)
- J. Patera, *My scientific career* (2017) (unpublished)
- E. Pelantová, *A speech for the ceremony of the Doctorate Honoris Causa of Jiří Patera* (2005) (unpublished)
- Y. Saint-Aubin, L. Vinet, *Algebraic methods in physics: A symposium for the 60th birthdays of Jiří Patera and Pavel Winternitz*, Springer, New York, USA (2012)
- L. Vinet, *Two entangled and scientifically impactful lives: Jiří Patera, Pavel Winternitz and the Montréal school of mathematical physics*, SciPost Phys. Proc. **14** (2023) 012