

## Memories of Pavel Winternitz



By **Libor Šnobl**, using excerpts from various texts by Pavel  
Superintegrability, Exact-Solvability and Representation Theory,  
Prague, November 22, 2024

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**1** Brief CV

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# Pavel's life in brief I

- Pavel Winternitz was born in Prague on July 25th, 1936. During the Second World War he lived with his parents in emigration in England.
- In 1954 he began his studies at the Faculty of Mathematics and Physics of Charles University in Prague. In 1955 he received a scholarship from the Czechoslovak government to study in the Soviet Union.
- In 1955 – 1960 he studied at Leningrad State University where he specialized in the theory of atomic nucleus and defended his thesis under the supervision of G. F. Drukaryev.

## Pavel's life in brief II

- In 1960 he joined the Institute of Nuclear Research of the Czechoslovak Academy of Sciences, Department of Theoretical Physics, dealing with the problems of polarization of nucleons in scattering and weak interactions (especially the capture of  $\mu^-$  mesons by atomic nuclei).
- In 1962 Pavel's and Milada's twins Peter and Michal were born.
- In 1962 – 1968 he worked in the Laboratory of Theoretical Physics of JINR in Dubna, in the group of Ya. A. Smorodinsky. In 1966 he defended his PhD thesis (i.e. CSc.) there.
- After Soviet occupation of Czechoslovakia in 1968 he emigrated first to the UK (Rutherford Lab), then to the USA (University of Pittsburgh).

## Pavel's life in brief III

- In 1972 he joined J. Patera at Centre de recherches mathématiques and the University of Montréal where he was appointed professor in 1984. In 2020 he retired and in September 2020 was awarded the title professor emeritus.
- Pavel died February 13th, 2021 in Montréal.

# Pavel and Milada at the workshop on the occasion of their 80th birthday



# How did Pavel become mathematical physicist?

*My MSc. degree was on polarization phenomena in elementary particle scattering. My PhD supervisor was Ya. A. Smorodinsky at the Joint Institute for Nuclear Research in Dubna. Ironically, I came to Dubna to work in a field that Smorodinsky and his group had essentially created: nucleon-nucleon scattering phenomenology and the complete reconstruction of the scattering matrix from experimental data. However, his interests had shifted towards mathematical physics, more specifically applications of group theory in physics. Still more specifically representations of the Lorentz group realized in different bases, harmonic analysis on the Lorentz group and its homogeneous spaces.*

# How did Pavel become mathematical physicist?

*I was greatly influenced by Smorodinsky and my stay in Dubna.*

*Among the things that I learned from him are that if you are doing mathematical physics you must use “real” cutting edge mathematics. If you are doing phenomenology, you must have real back and forward interactions with experimentalists.*

# Most fruitful research interests I

## 1 Superintegrability in classical and quantum mechanics

*“My interest in this field was sparked by lectures on quantum mechanics that Drukaryev gave while I was an undergraduate in Leningrad. They concerned Pauli’s, Fock’s and Bargmann’s  $O(4)$  symmetries of the hydrogen atom.”*

In Dubna, Pavel with collaborators started a systematic search for **superintegrable systems** in 2 and 3 dimensions, i.e. **integrable systems possessing additional independent integrals of motion**, leading to results on the connection between superintegrability and multiseparability. They also found infinite families of such systems, now called Smorodinsky-Winternitz potentials.

## Most fruitful research interests II

After several decades Pavel presented further results on superintegrable systems in a series of papers *sometimes referred to as **TTW***. One of the *T*'s is *A. Turbiner*, the other either *F. Tremblay* (a student of Pavel, in 2009–10) or *P. Tempesta* (postdoc of Pavel in 2001).

## Most fruitful research interests III

In *J. Math. Phys.* 42 (2001) 4248–4257 Tempesta, Turbiner and Pavel have focused on the relation of **superintegrability and exact solvability** in quantum mechanics, i.e. the existence of a flag of subspaces preserved by the Hamiltonian. (A *flag* is formed by an infinite set of vector subspaces ordered in such a way that each of them contains the previous one as a subspace.)

They have shown that all four superintegrable quantum systems on the Euclidean plane with integrals of motion at most quadratic in the momenta possess the same underlying hidden algebra  $\mathfrak{sl}(3)$ . Consequently, after a gauge

## Most fruitful research interests IV

transformation, i.e. multiplication by the ground state wavefunction

$$H \rightarrow \tilde{H}' : \tilde{H}\psi = \frac{1}{\psi_0} H(\psi_0 \cdot \psi), \quad (1)$$

and a suitable change of coordinates each of these Hamiltonians, as well as its integrals of motion preserve a flag of polynomials which corresponds to highest-weight finite-dimensional representations of the Lie algebra  $\mathfrak{sl}(3)$  realized by first order differential operators.

This lead to a [conjecture that the property of exact solvability will remain valid for higher dimensional superintegrable systems of this type.](#)

# Most fruitful research interests V

In *J. Phys. A: Math. Theor.* **42** (2009) 242001 Tremblay, Turbiner and Pavel presented the potential in the 2D plane

$$V(r, \phi) = \omega^2 r^2 + \frac{\alpha k^2}{r^2 \cos^2 k\phi} + \frac{\beta k^2}{r^2 \sin^2 k\phi} \quad (2)$$

which generalizes the already known systems like Smorodinsky–Winternitz system ( $k = 1$ ), Wolfes model ( $k = 3$ ) or Calogero system ( $\alpha = 0$ ) and conjectured that it is both classically and quantum mechanically superintegrable for arbitrary positive parameter  $k$ . Next, in *J. Phys. A: Math. Theor.* **43** (2010) 015202 they have shown that in the classical mechanics the potential (2) implies closed trajectories for both integer and rational values of  $k$  and the period is independent of  $k$ .

## Most fruitful research interests VI

Each of these results sparked a significant number of follow-up articles by various authors, attempting to prove the conjectures and extend their validity. Pavel often half-jokingly mentioned that a well formulated and interesting conjecture brings much more citations than an article with full proof.

In last two decades Pavel worked also on superintegrability with higher order integrals of motion, their polynomial algebras and exotic potentials expressed in terms of Painlevé transcendents (with [A. Escobar Ruiz](#), [I. Marquette](#), [S. Post](#), [I. Yurdusen](#) and others), and on superintegrable systems with vector potentials or particles with spin (with [A. Marchesiello](#), [D. Riglioni](#), [I. Yurdusen](#) and [L. Šnobl](#)).

# Most fruitful research interests VII

## 2 Classification and recognition of Lie algebras.

Starting with the computation of Casimir invariants of various Lie algebras with J. Patera and H. Zassenhaus, followed by research with H. Zassenhaus and D. Rand on the algorithms for recognition of Lie algebras in *On the identification of a Lie algebra given by its structure constants. I. Direct decompositions, Levi decompositions, and nilradicals* *Linear Algebra Appl.* 109 (1988), 197–246. together with their implementation in PASCAL, and series of papers on the classification of Lie algebras with nilradicals belonging to various application–relevant series (Abelian, Heisenberg etc.) with J.F. Ndogmo, J. Rubin, S. Tremblay and L. Šnobl, culminating in the monograph *Libor Šnobl and Pavel Winternitz, Classification and Identification of Lie Algebras.*

# Most fruitful research interests VIII

- 3 Classification of the subalgebras of Lie algebras, two-variable expansions of scattering amplitudes, separation of variables and special functions as basis functions for group representations.

Started in Dubna with I. Friš on subalgebras of  $\mathfrak{o}(3, 1)$ , continued with Ya. A. Smorodinsky and M. Sheftel in relation with relativistic expansions of scattering amplitudes. This work was continued in Dublin and at the Rutherford High Physics Laboratory, in Pittsburgh and finally Montreal in a series of articles with L. O’Raifeartaigh, P. Pajas, J. Belinfante, N. Macfadyen, J. Patera, R.T. Sharp, H. Zassenhaus, V. Hussin, W. Miller Jr, E. Kalnins, M. del Olmo, M.A. Rodriguez, L. Gagnon, Z. Thomova and others.

# Most fruitful research interests IX

While working on this topic, Pavel had introduced his postdoc E. Kalnins to W. Miller, then on sabbatical at CRM, initiating their intense live–long collaboration.

## 4 Nucleon–nucleon scattering formalism.

Pavel started to work on the problems of scattering of nucleons already during his studies at Leningrad State University, leading to his diploma thesis and the first publications: P. VINTERNITTS, Scattering matrix for the scattering of nucleons upon particles with spin 1, J. Exptl. Theoret. Phys. 12 (1961) 1025; Elastic scattering of nucleons on targets with spin 1. Cz. J. Phys. B 12, 1962, p. 482–489.

## Most fruitful research interests X

On the request of his friends and colleagues [F. Lehar](#) and [Z. Janout](#) who were looking for the interpretation of their accelerator experiments at JINR he focused on the problem of nucleon–nucleon scattering and continued to work on it even after his emigration and arrival to Canada. This work culminated in the paper [J. Bystrický, F. Lehar and P. Winternitz, Formalism of nucleon-nucleon elastic-scattering experiments. Journal de physique 39 \(1978\) 1–32](#) where a detailed exposition of the nucleon-nucleon elastic scattering formalism is presented. Till today it is a standard reference for experimentalists and for phenomenologists interested in the reconstruction of scattering amplitudes from data.

# Most fruitful research interests XI

## 5 Lie point symmetries of differential, difference and differential delay equations.

Systematic study of point symmetries of differential equations with numerous collaborators (V. Dorodnitsyn, A.M. Grundland, F. Gungor, J. Harnad, R. Kozlov, L. Martina, D. Levi, M.A. Rodriguez, and many others) lead Pavel together with D. Levi to introduce a novel notion of **conditional symmetry** and later to propose a scheme in which difference equations can possess continuous symmetries by allowing not only the equation but also the lattice on which it is defined to transform and the related topic of symmetry preserving discretization of differential equations. Now it is a topic to which a whole series of international conferences is devoted, **SIDE Symmetries and Integrability of Difference Equations**.

## Most fruitful research interests XII

Many results obtained by Pavel with Decio Levi and others were presented together in the monograph **Decio Levi, Pavel Winternitz and Ravi I. Yamilov, Continuous Symmetries and Integrability of Discrete Equations.**

### 6 Nonlinear differential and difference equations with superposition formulas.

Pavel with his collaborators (S. Shnider, R.L. Anderson, J. Harnad, V. Hussin, M. del Olmo, M.A. Rodriguez, Z. Thomova and others) identified many nonlinear ODEs with superposition formulas, studied their properties as well as their difference analogues.

that is, the change in the damping can become quite appreciable. As time goes on, the radiation remains only at those angles corresponding to minimum damping.

In conclusion, we give an equation for the amplitude  $\chi(k)$  of the radiation from a crystal with cubic lattice at a point the direction to which is given by the vector  $\mathbf{m}$ ,  $|\mathbf{m}| = 1$ :

$$\begin{aligned} \ddot{\chi}(t) + \beta \sum_{\mathbf{k}} \chi(\mathbf{k}) \ddot{\chi} \left[ t - \frac{c}{v} (|\mathbf{k} + \mathbf{km}|) \right] \\ - \beta \sum_{\mathbf{k}} \chi(\mathbf{k}) \ddot{\chi} \left[ t - \frac{c}{v} (|\mathbf{k} + \mathbf{km}|) \right] \\ + 2\gamma \dot{\chi}(t) + \omega^2 \chi(t) = 0. \end{aligned} \quad (12)$$

The similarity between (7) and (12) gives grounds for expecting that their solutions will also be essentially similar. The described variation in frequency can be observed, in principle, by the procedure proposed by Mössbauer.<sup>1,2</sup>

The authors are grateful to V. I. Ogilevetskii, V. M. Pain, Ya. I. Khanin, D. S. Chernavskii, and F. L. Shapiro for participating in a discussion of this work and for significant comments.

<sup>1</sup>For the approximations made in the derivation of (7), it is also necessary that the condition  $1 \pm \cos \theta \geq 2\gamma v/c$  be satisfied; this condition is violated only for angles  $\theta$  which are exceedingly close to  $\theta = 0$  and  $\theta = \pi$ .

<sup>2</sup>M. I. Podguretskii and I. I. Rofzen, preprint, Joint Institute for Nuclear Research, R-344g.

<sup>3</sup>R. L. Mössbauer, *Z. Phys.* **151**, 124 (1958).

<sup>4</sup>R. L. Mössbauer, *Z. Naturforsch.* **14a**, 211 (1959).

Translated by J. G. Adashko  
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## SCATTERING MATRIX OF NUCLEONS ON A TARGET WITH SPIN 1

P. VINTERNITZ

Leningrad State University

Submitted to JETP editor July 8, 1960

J. Exptl. Theoret. Phys. (U.S.S.R.) **39**, 1476 (November, 1960)

THE article by Budyanski<sup>1</sup> contains the general form of the scattering matrix of a particle with spin 1 by a particle with spin 1, and 12 observable quantities are calculated, making up one of

the possible complete sets of experiments. It appears to us that this matrix has not been written down quite correctly.

Using the Oehme method<sup>2</sup> we can write the matrix for the scattering of nucleons on any target in the form

$$M = a + b m + c \sigma m + d \sigma \quad (1)$$

(the notation is the same as used by Oehme<sup>2</sup>). The coefficients in (1) should have the following form:

$$a = \alpha_1 I + \alpha_2 S_1 + \alpha_3 S_1^2 + \alpha_4 S_1^3,$$

$$b = \beta_1 I + \beta_2 S_1 + \beta_3 S_1^2 + \beta_4 S_1^3,$$

$$c = \gamma_1 S_1 + \gamma_2 (S_1 S_1 + S_1 S_1), \quad d = \delta_1 S_1 + \delta_2 (S_1 S_1 + S_1 S_1). \quad (2)$$

If we transform the scattering matrix given by Budyanski<sup>1</sup> to the form (1), we obtain

$$a = A_1 I + A_2 S_1 + A_3 (S_1^2 + S_1^3),$$

$$b = B_1 I + B_2 S_1 + B_3 (S_1^2 + S_1^3),$$

$$c = C_1 S_1 + C_2 S_1 S_1 + C_3 S_1 S_1,$$

$$d = D_1 S_1 + D_2 S_1 S_1 + D_3 S_1 S_1. \quad (3)$$

It is obvious that no general considerations lead to  $\alpha_1 = \alpha_4$  and  $\beta_1 = \beta_4$ . On the other hand, the complete system of orthogonal basis matrices of the spin space of the particle with spin 1 do not contain expressions of the type  $S_1 S_1$  and  $S_1 S_1$  individually, but of the type  $S_1 S_1 + S_1 S_1$  (see, for example, reference 3), and consequently one must assume  $C_1 = C_2$  and  $D_1 = D_2$ . The number of complex scalar coefficients should actually be 12 (this follows from general relations given by Puzikov<sup>4</sup>), and this is satisfied by the scattering matrix both in form (2) and (3). It is naturally necessary to use the formula (2), and not (3), for all calculations and suitably correct all the expressions for the cross sections, the polarization, and the correlation functions obtained in reference 1.

<sup>1</sup>G. M. Budyanski, *JETP* **33**, 889 (1957), Soviet Phys. *JETP* **6**, 684 (1958).

<sup>2</sup>R. Oehme, *Phys. Rev.* **96**, 147 (1955).

<sup>3</sup>A. S. Davydov, *Теория атомного ядра* (Theory of the Atomic Nucleus), M., Fizmatgiz, 1958.

<sup>4</sup>L. D. Puzikov, *JETP* **34**, 947 (1958), Soviet Phys. *JETP* **7**, 655 (1958).

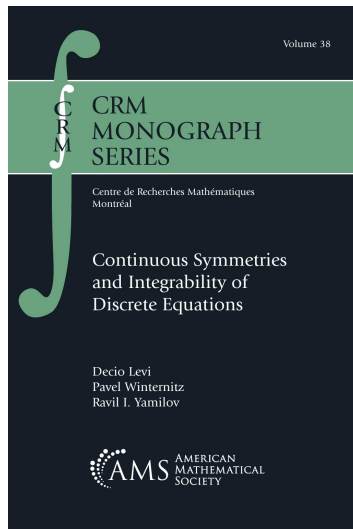
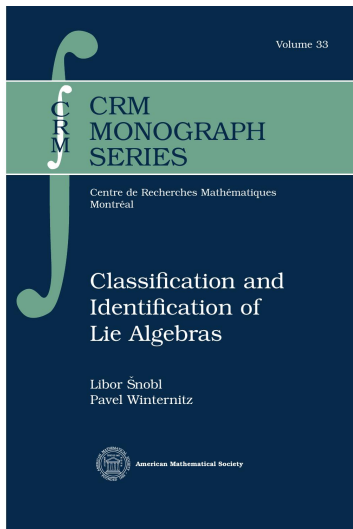
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# Scientific output

Pavel has published

- more than 300 research papers,
- more than 120 proceedings contributions,
- two books
  - Libor Šnobl and Pavel Winternitz, [Classification and Identification of Lie Algebras](#). CRM Monograph Series, 33. American Mathematical Society, 2014.
  - Decio Levi, Pavel Winternitz and Ravil I. Yamilov, [Continuous Symmetries and Integrability of Discrete Equations](#). CRM Monograph Series, 38. American Mathematical Society, 2022.
- and was (co-)editor of various proceedings and special issues.

# Pavel's books



# Scientific output

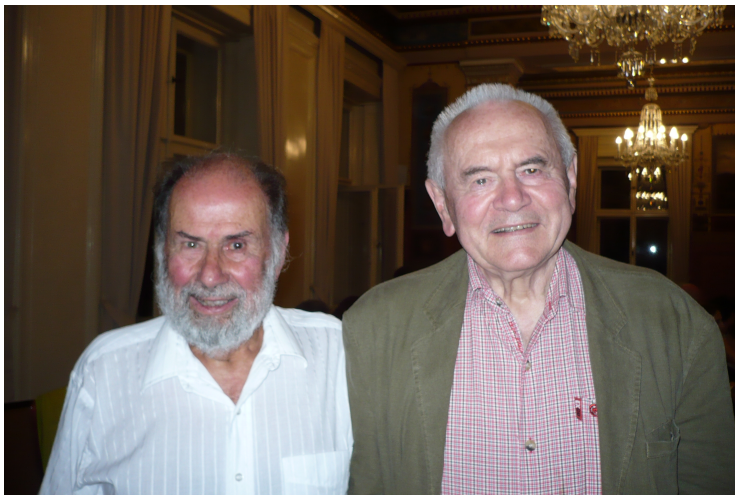
- His papers were cited more than 8 500 times.
- Five of his papers have collected more than 200 citations:
  1. J. Patera, R.T. Sharp, P. Winternitz, H. Zassenhaus, Invariants of real low dimension Lie algebras, J. Math. Phys. 17 (1975), 986–994,
  2. A.A. Makarov, Ya.A. Smorodinsky, Kh. Valiev, P. Winternitz, A systematic search for nonrelativistic systems with dynamical symmetries - Part I: The integrals of motion, Nuovo Cimento A 52 (1967), 1061–1084,
  3. J. Bystrický, F. Lehar, P. Winternitz, Formalism of nucleon–nucleon elastic scattering experiments. J. Phys. France 39 (1978), 1–32,
  4. J. Friš, V. Mandrosov, , Ya.A. Smorodinsky, M. Uhlíř, P. Winternitz, On higher symmetries in quantum mechanics, Phys. Lett. 16 (1965) 354–356,
  5. D. Levi, P. Winternitz, Non–classical symmetry reduction: Example of the Boussinesq equation, J. Phys. A 22 (1989) 2915–2924.
- More than 10 of his papers were cited more than 100 times.
- Current H–index as computed by Web of Science is 57.

# Pavel's collaborators

Pavel had literally worldwide web of collaborators. Actually, some of his sabbaticals he arranged using a round-the-world flight ticket. Among others,

- in **Europe**: J. Bystrický, V. Dorodnitsyn, R. Kozlov, [F. Lehar \(20\)](#), [D. Levi \(43\)](#), A. Marchesiello, L. Martina, M. del Olmo, L. O'Raiheartaigh, F. Perrot, [M.A. Rodriguez \(13\)](#), Ya. A. Smorodinsky, L. Šnobl, P. Tempesta, R. Yamilov
- in **Canada**: [M. Grundland \(14\)](#), [J. Harnad \(11\)](#), V. Hussin, [J. Patera \(32\)](#), R.T. Sharp, L. Vinet
- in **US**: J. G. Belinfante, W. Miller Jr., S. Post, Z. Thomova, F. Tremblay, [H. Zassenhaus \(16\)](#)
- elsewhere in **America**: M. Moshinsky, K. Tenenblat, A. V. Turbiner
- in **Asia**: M. Sheftel, İ Yurduşen
- in **Australia and New Zealand**: E. Kalnins, I. Marquette

# Jirka and Pavel at the workshop on the occasion of Pavel's and Patera's 80th birthday



# And group photo from the same workshop



# Public recognition

- Wigner Medal 2018,
- Patria Prize in the Czech Head competition 2011 (i.e. winner in the category of Czech scientists working abroad),
- doctor honoris causa of the Czech Technical University in Prague 2006,
- foreign member of the Mexican Academy of Sciences since 2003,
- CAP–CRM Prize in Theoretical and Mathematical Physics 2002.

# Pavel receiving Česká hlava Award in 2011



# Pavel receiving Wigner Medal in 2018



# Pavel adventurous

Pavel lived not only life of a scientist but also enjoyed many other activities. He loved to read or to spend good time with his friends. He was also keen on travelling and outdoor activities. Till his late age he was going skiing, canoeing and swimming. Among his most notable achievements in this aspect belong:

- **Swimming across Bosphorus Strait.** He used to recollect that the current is so strong that while swimming approx. 1 km across he was drifted by some 4 km aside. He also refuted frequent concerns of his Turkish colleagues – by then the water in Bosphorus wasn't as polluted as now.

# Pavel at winter school in Jánské Lázně during his last sabbatical in 2015



# Parachuting

*One day in 1966 I read on the bulletin board in the JINR Dubna that a parachute club had been founded and that it was possible to join it. I called the number given and was told that training would be organised for three evenings, for a week, and then on Saturday a small plane would be flown over Dubna with a jump from the plane. But then they added that they had to find out if the club can also accept foreigners. In the end they found out that foreigners can participate. In the meantime, I missed two of the three training sessions. But it didn't matter, they told me that one evening was enough. I jumped from that plane three times, with automatic parachute opening, from an altitude of 800 meters. The next jumps were to be with manual parachute opening and from an altitude of 3,000 metres, but by then I had already left for Trieste. After returning to Dubna, I learned that one parachutist had killed herself on the next jump and the club was closed down.*

## Ascent to Lenin's Peak, 7 134 metres a.s.l., Pamir

*Among other things, we learned that in a country where everything is either forbidden or mandatory, exceptions to this rule can be arranged. For example, we managed to organize a climb of five members of the Czechoslovak group in Dubna to [Lenin's Peak in the Pamir Mountains](#). The participants were Mirek Malý, his wife Marta, Franta Lehar, Roša Caletka and me. Today the mountain is located on the border between Kyrgyzstan and Tajikistan and has changed its name, in Tajikistan it is now called Avicenna Shield.*

At the time, this ascent established a group expedition altitude record by Czech climbers and equated the individual one (to significant consternation and displeasure of climbers officially organized in Czechoslovak Union of Physical Education and Sports).

# Ascent to Lenin's Peak, 7 134 metres a.s.l., Pamir

A [few curiosities](#) about the expedition:

- While the expedition members obtained necessary travel permits into restricted area by misleading the KGB authorities with a suitable “typo” in the destination’s name, Pavel’s wife Milada managed to reach the base camp without any permit, just arguing with the officers at each checkpoint.
- Crampons were handmade by Dubna blacksmiths based on some photos in journals (in exchange for a few bottles of vodka, of course). However, the expedition members forgot to remind them that they should also have forward directed spikes and thus had difficulties in steep climbs.
- It turned out that leather hiking boots were not the “best choice” for glacier climbing and some of the team members suffered serious frostbite.

# Czech expedition on the top of Lenin's Peak



# Pavel on the top of Lenin's Peak



*Finally I will reveal the secret of scientific longevity:  
collaborate with people who are smarter than you  
and from some point on also younger than you.<sup>1</sup>*

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<sup>1</sup>Concluding sentence of Pavel's Wigner Medal acceptance speech. < ≡ > ≡ ↺ ↻



You will be greatly missed by  
many, Pavel !