On Poisson–Lie T–plurality of boundary conditions

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Outline

1. Elements of Poisson–Lie T–plurality
2. Consistent boundary conditions
3. Poisson–Lie T–plurality transformation of the gluing operator

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Elements of Poisson–Lie T–plurality of σ–models

The σ–model given by the action

$$S_F[g] = \int_\Sigma d^2x\, \rho_-(g) \cdot F(g) \cdot \rho_+(g)^t = \int_\Sigma d^2x\, \partial_-\phi^\mu \mathcal{F}_{\mu\nu}(\phi)\partial_+\phi^\nu \quad (1)$$

where the map g maps $\Sigma = \langle 0, \pi \rangle \times \mathbb{R}$ into the group G whose Lie algebra has basis $\{T_a\}$,

$$\rho_\pm(g)^a \equiv (\partial_\pm g g^{-1})^a = \partial_\pm \phi^\mu e_\mu{}^a(g), \quad (\partial_\pm g g^{-1}) = \rho_\pm(g) \cdot T$$

$\phi^\mu : \Sigma \to \mathbb{R}^{\dim G}$ is the same map as g but written in some group coordinates, and x_+, x_- are the light–cone coordinates on Minkowski $\mathbb{R}^2$ $\tau = x_+ + x_-$, $\sigma = x_+ - x_-$.

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The basic idea of Poisson–Lie T–duality

C. Klimčík and P. Ševera, Phys. Lett. B 351 (1995) 455.

Under certain conditions the equations of motion in the bulk of the σ–model can be written as equations on

Drinfel'd double

$(G|\tilde{G})$ – Lie group D whose Lie algebra $\mathfrak{d}$ admits a decomposition $\mathfrak{d} = \mathfrak{g} \dot{+} \tilde{\mathfrak{g}}$ into a pair of subalgebras maximally isotropic with respect to a symmetric ad-invariant nondegenerate bilinear form $\langle\, . \,,\, . \,\rangle$.

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If the metric together with the B–field are such that

$$F(g) = (E_0^{-1} + \Pi(g))^{-1}, \quad \Pi(g) = b(g) \cdot a(g)^{-1} = -\Pi(g)^t, \tag{2}$$

then the bulk equations of motion of the σ–model can be formulated as the equations on the Drinfel'd double

$$\langle \partial_\pm l \, l^{-1} , \mathcal{E}^\pm \rangle = 0,$$

where $l = g\tilde{h} \in D, \; g \in G, \; \tilde{h} \in \tilde{G}$ and

$$\mathcal{E}^+ = \mathrm{span}\left(T + E_0 \cdot \tilde{T}\right), \qquad \mathcal{E}^- = \mathrm{span}\left(T - E_0^t \cdot \tilde{T}\right)$$

are two orthogonal subspaces in $\mathfrak{d}$.

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Poisson–Lie T–plurality

R. von Unge, J. High En. Phys. 02:07 (2002) 014.

Main idea:

In general there are several decompositions (Manin triples) of a Drinfel'd double.

Let $\hat{\mathfrak{g}} \dotplus \bar{\mathfrak{g}}$ be another decomposition of the Lie algebra $\mathfrak{d}$ into maximal isotropic subalgebras. The dual bases of $\mathfrak{g}, \tilde{\mathfrak{g}}$ and $\hat{\mathfrak{g}}, \bar{\mathfrak{g}}$ are related by the linear transformation

$$\begin{pmatrix} T \\ \tilde{T} \end{pmatrix} = \begin{pmatrix} p & q \\ r & s \end{pmatrix} \begin{pmatrix} \widehat{T} \\ \bar{T} \end{pmatrix}, \qquad (3)$$

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is defined analogously but with

$$\begin{aligned} \widehat{F}(\hat{g}) &= (\widehat{E}_0^{-1} + \widehat{\Pi}(\hat{g}))^{-1}, \quad \widehat{\Pi}(\hat{g}) = \widehat{b}(\hat{g}) \cdot \widehat{a}(\hat{g})^{-1} = -\widehat{\Pi}(\hat{g})^t, \\ \widehat{E}_0 &= (p + E_0 \cdot r)^{-1} \cdot (q + E_0 \cdot s) \end{aligned}$$

The relation between the classical solutions of equations of motion in the bulk of the two σ–models is obtained from two possible decompositions of $l \in D$

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But what about the boundary conditions? Does a solution with well–defined boundary conditions transform into another one?

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Gluing operator

The gluing operator $\mathcal{R}$

We impose the boundary condition in the form

$$\partial_- g|_{\sigma=0,\pi} = \mathcal{R}\partial_+ g|_{\sigma=0,\pi} \tag{4}$$

Explicitly we write in coordinates or in a frame e.g.

$$\partial_-\phi|_{\sigma=0,\pi} = \partial_+\phi \cdot R_\phi|_{\sigma=0,\pi}, \quad \rho_-(g)|_{\sigma=0,\pi} = \rho_+(g) \cdot R_\rho|_{\sigma=0,\pi} \tag{5}$$

We define the Dirichlet projector $\mathcal{Q}$ that projects vectors onto the space normal to the D–brane $\equiv -1$ eigenspace of $\mathcal{R}$ and Neumann projector $\mathcal{N}$ that projects onto the tangent space of the brane. The corresponding matrices Q, N are given by

$$Q^2 = Q, \quad Q \cdot R = R \cdot Q = -Q, \quad N = \mathbf{1} - Q. \tag{6}$$

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Consistency conditions on gluing operator

In addition to (6) we want the following conditions to hold, originally derived in C. Albertsson, U. Lindström and M. Zabzine, Nucl. Phys. B 678 (2004) 295, [hep-th/0202069] (in SUSY setting)

- conformal – to be consistent with the conformal constraint $\mathcal{T}_{++}|_{\sigma=0,\pi} = \mathcal{T}_{--}|_{\sigma=0,\pi}$ we need

$$R \cdot (\mathcal{F} + \mathcal{F}^t) \cdot R^t = (\mathcal{F} + \mathcal{F}^t) \tag{7}$$

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- integrability – $\mathrm{Im}(\mathcal{N})$ must form an integrable distribution, its integral submanifolds being the D–branes

$$N_\kappa{}^\mu N_\lambda{}^\nu \partial_{[\mu} N_{\nu]}{}^\rho = 0 \qquad (9)$$

- equivalence with the action principle – the boundary condition should be equivalent to the vanishing variation of the action on the boundary

$$N \cdot (\mathcal{F} - \mathcal{F}^t \cdot R^t) = 0 \qquad (10)$$

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PL T–plurality transformation of the gluing operator

We have found that the transformed solution $\hat{g}$ satisfies

$$\widehat{\rho}_-(\hat{g})|_{\sigma=0,\pi} = \widehat{\rho}_+(\hat{g}) \cdot \widehat{R_\rho}|_{\sigma=0,\pi} \tag{11}$$

where the transformed gluing operator is

$$\widehat{R_\rho} = \widehat{F}^t(\hat{g}) \cdot M_-^{-1} \cdot F^{-t}(g) \cdot R_\rho(g) \cdot F(g) \cdot M_+ \cdot \widehat{F}^{-1}(\hat{g}), \tag{12}$$

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$$M_+ = s + {E_0}^{-1} \cdot q, \quad M_- = s - {E_0}^{-t} \cdot q.$$

Good news

- The transformed gluing operator $\widehat{R_\rho}$ is found explicitly.
- $\widehat{R_\rho}$ satisfies the conformal condition (7) if and only if the original R_ρ does (proven)
- $\widehat{R_\rho}$ allows the definition of projectors (6) and satisfies the orthogonality condition (8) if and only if the original R_ρ does in all the examples investigated for the transitions inside the six–dimensional Drinfel'd doubles $(Bianchi\,5 \mid \mathbb{R}^3) \simeq (Bianchi\,6_0 \mid \mathbb{R}^3)$ and the semiabelian four–dimensional Drinfel'd double $(af(1) \mid \mathbb{R}^2) \simeq (af(1) \mid af(1))$.

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Not so good news

- $\widehat{R_\rho}$ defined by (12) may depend not only on $\hat{g}$ but also on g and consequently on $\bar{g}$.

Solution: $\widehat{R_\rho}$ is function of $\hat{g}$ only if the matrix–valued function $C(g) = F^{-t}(g) \cdot R_\rho(g) \cdot F(g)$ extended to a function on the whole Drinfel'd double as $C(g\tilde{h}) = C(g)$ satisfies

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Bad news

- The integrability condition (9) and equivalence with the action principle (10) are not preserved under the PL T–plurality transformation. (Explicit counterexamples were found.) Sometimes $\widehat{R_\rho}$ satisfies (9),(10) only in specific points or submanifolds of $\widehat{G}$. How to interpret this? We don't know yet.

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Thank you for you attention

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