On Poisson–Lie T–plurality of boundary conditions

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Outline

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Elements of Poisson–Lie T–plurality of $\sigma-$models

We consider the σ–model given by the action

$$S_F[g] = \int_\Sigma d^2x\, \rho_-(g) \cdot F(g) \cdot \rho_+(g)^t = \int_\Sigma d^2x\, \partial_-\phi^\mu \mathcal{F}_{\mu\nu}(\phi)\partial_+\phi^\nu \quad (1)$$

where the map g maps $\Sigma = \langle 0, \pi\rangle \times \mathbb{R}$ into the group G, whose Lie algebra has basis $\{T_a\}$,

$$\rho_\pm(g)^a \equiv (\partial_\pm g g^{-1})^a = \partial_\pm\phi^\mu e_\mu{}^a(g), \quad \partial_\pm g g^{-1} = \rho_\pm(g) \cdot T$$

$\phi^\mu : \Sigma \to \mathbb{R}^{\dim G}$ is the same map as g but written in some group coordinates.

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x_+, x_- are the light–cone coordinates on Minkowski $\mathbb{R}^2$:
$\tau = x_+ + x_-,\ \sigma = x_+ - x_-$.

The matrix F, or equivalently the tensor $\mathcal{F}_{\mu\nu}$, can be viewed as a combination of the metric (symmetric part) and the B–field (antisymmetric part) on the group G written in the appropriate basis.

N.B. The general setting works for the group G acting freely on the target $\mathcal{M}$. Then the coordinates on $\mathcal{M}/G$ (e.g. spacetime time) are the so–called spectator fields since they don't transform under the PL T–plurality transformation. We have assumed for simplicity that the target space coincides with the group G, i.e. there are no spectator fields.

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The basic idea of Poisson–Lie T–duality

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Under certain conditions the equations of motion in the bulk of the σ–model can be written as equations on

Drinfel'd double

$(G|\tilde{G})$ – Lie group D whose Lie algebra $\mathfrak{d}$ admits a decomposition $\mathfrak{d} = \mathfrak{g} \dot{+} \tilde{\mathfrak{g}}$ into a pair of subalgebras maximally isotropic with respect to a symmetric ad-invariant nondegenerate bilinear form $\langle \, . \, , . \, \rangle$. $G, \tilde{G}$ denote the corresponding Lie subgroups.

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Locally (i.e. in the vicinity of the group unit), there exists a unique decomposition $l = g\tilde{g},\ l \in D,\ g \in G,\ \tilde{g} \in \tilde{G}$ on the Drinfel'd double D. For the so–called perfect Drinfel'd doubles it is defined globally and we shall for simplicity consider only these.

If the metric together with the B–field are such that

$$F(g) = (E_0^{-1} + \Pi(g))^{-1}, \quad \Pi(g) = b(g) \cdot a(g)^{-1} = -\Pi(g)^t, \tag{2}$$

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$$\langle \partial_\pm l\, l^{-1}, \mathcal{E}^\pm \rangle = 0,$$

where $l = g\tilde{h} : \Sigma \to D,\ g : \Sigma \to G,\ \tilde{h} : \Sigma \to \tilde{G}$ and

$$\mathcal{E}^+ = \mathrm{span}\left(T + E_0 \cdot \tilde{T}\right), \qquad \mathcal{E}^- = \mathrm{span}\left(T - E_0^t \cdot \tilde{T}\right)$$

are two orthogonal subspaces in $\mathfrak{d}$. The map $\tilde{h} : \Sigma \to \tilde{G}$ is defined as a potential of a flat $\tilde{G}$–valued connection by

$$\begin{aligned} \partial_+ \tilde{h}\tilde{h}^{-1} &= -\rho_+(g) \cdot F(g)^t \cdot a^{-t}(g) \cdot \tilde{T}, \\ \partial_- \tilde{h}\tilde{h}^{-1} &= +\rho_-(g) \cdot F(g) \cdot a^{-t}(g) \cdot \tilde{T} \end{aligned}$$

where the flatness of the connection is equivalent to the equations of motion of g. Consequently, $\tilde{h}$ and l are determined by g up to the choice of a constant shift

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Poisson–Lie T–plurality

R. von Unge, J. High En. Phys. 02:07 (2002) 014.

Main idea:

In general there are several decompositions (Manin triples) of a Drinfel'd double.

Let $\hat{\mathfrak{g}} \dot{+} \bar{\mathfrak{g}}$ be another decomposition of the Lie algebra $\mathfrak{d}$ into maximal isotropic subalgebras. The dual bases of $\mathfrak{g}, \tilde{\mathfrak{g}}$ and $\hat{\mathfrak{g}}, \bar{\mathfrak{g}}$ are related by the linear transformation

$$\begin{pmatrix} T \\ \tilde{T} \end{pmatrix} = \begin{pmatrix} p & q \\ r & s \end{pmatrix} \begin{pmatrix} \widehat{T} \\ \bar{T} \end{pmatrix}. \quad (4)$$

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is defined analogously but with

$$\widehat{F}(\hat{g}) = (\widehat{E}_0^{-1} + \widehat{\Pi}(\hat{g}))^{-1}, \quad \widehat{\Pi}(\hat{g}) = \widehat{b}(\hat{g}) \cdot \widehat{a}(\hat{g})^{-1},$$
$$\widehat{E}_0 = (p + E_0 \cdot r)^{-1} \cdot (q + E_0 \cdot s) \qquad (5)$$

The relation between the classical solutions of equations of motion in the bulk of the two σ–models is obtained from two possible decompositions of $l : \Sigma \to D$

$$l = g\tilde{h} = \hat{g}\bar{h}. \qquad (6)$$

But what about the boundary conditions? Does a solution with well–defined boundary conditions transform into another one?

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Gluing operator

The gluing operator $\mathcal{R}$

We impose the boundary condition in the form

$$\partial_- g|_{\sigma=0,\pi} = \mathcal{R}\partial_+ g|_{\sigma=0,\pi} \tag{7}$$

Explicitly we write in coordinates or in a frame e.g.

$$\partial_-\phi|_{\sigma=0,\pi} = \partial_+\phi \cdot R_\phi|_{\sigma=0,\pi}, \quad \rho_-(g)|_{\sigma=0,\pi} = \rho_+(g) \cdot R_\rho|_{\sigma=0,\pi} \tag{8}$$

Because we want to reconstruct the D–brane configuration from the knowledge of the gluing operator $\mathcal{R}$ we have to assume that the gluing operator is defined everywhere on G, i.e. $\mathcal{R} \in \Sigma(TG \times T^*G)$.

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Such an assumption together with the consistency conditions postulated below means that G is foliated by D–branes. Other possible configurations are not included in our analysis.

We define the Dirichlet projector $\mathcal{Q}$ that projects vectors onto the space normal to the D–brane $\equiv -1$ eigenspace of $\mathcal{R}$ and also annihilates the time derivative on the boundary and Neumann projector $\mathcal{N}$ that projects onto the tangent space of the brane. The corresponding matrices Q, N are given by

$$Q^2 = Q, \quad Q \cdot R = R \cdot Q = -Q, \quad N = \mathbf{1} - Q. \qquad (9)$$

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Consistency conditions on gluing operator

In addition to (9) we want the following conditions to hold, originally derived in C. Albertsson, U. Lindström and M. Zabzine, Nucl. Phys. B 678 (2004) 295, [hep-th/0202069] (in SUSY setting)

- conformal – to be consistent with the conformal constraint $\mathcal{T}_{++}|_{\sigma=0,\pi} = \mathcal{T}_{--}|_{\sigma=0,\pi}$ we need

$$R \cdot (\mathcal{F} + \mathcal{F}^t) \cdot R^t = (\mathcal{F} + \mathcal{F}^t) \qquad (10)$$

- orthogonality – Neumann and Dirichlet directions must be indeed orthogonal

$$N \cdot (\mathcal{F} + \mathcal{F}^t) \cdot Q^t = 0 \qquad (11)$$

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- integrability – $\mathrm{Im}(\mathcal{N})$ must form an integrable distribution, its integral submanifolds being the D–branes

$$N_{\kappa}{}^{\mu} N_{\lambda}{}^{\nu} \partial_{[\mu} N_{\nu]}{}^{\rho} = 0 \qquad (12)$$

- equivalence with the action principle – the boundary condition should be equivalent to the vanishing variation of the action on the boundary

$$N \cdot (\mathcal{F} - \mathcal{F}^t \cdot R^t) = 0 \qquad (13)$$

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PL T–plurality transformation of the gluing operator

We have found that the transformed solution $\hat{g}$ satisfies

$$\widehat{\rho}_-(\hat{g})|_{\sigma=0,\pi} = \widehat{\rho}_+(\hat{g}) \cdot \widehat{R_\rho}|_{\sigma=0,\pi} \qquad (14)$$

where the transformed gluing operator is

$$\widehat{R_\rho} = \widehat{F}^t(\hat{g}) \cdot M_-^{-1} \cdot F^{-t}(g) \cdot R_\rho(g) \cdot F(g) \cdot M_+ \cdot \widehat{F}^{-1}(\hat{g}), \qquad (15)$$

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Good news

- The transformed gluing operator $\widehat{R_\rho}$ is found explicitly.
- $\widehat{R_\rho}$ satisfies the conformal condition (10) if and only if the original R_ρ does (proven)
- $\widehat{R_\rho}$ allows the definition of projectors (9) and satisfies the orthogonality condition (11) if and only if the original R_ρ does in all the examples investigated for the transitions inside the six–dimensional Drinfel'd doubles $(\mathit{Bianchi}\,5 \mid \mathbb{R}^3) \simeq (\mathit{Bianchi}\,6_0 \mid \mathbb{R}^3)$ and the semiabelian four–dimensional Drinfel'd double $(af(1) \mid \mathbb{R}^2) \simeq (af(1) \mid af(1))$. Similarly, the integrability condition (12) was preserved in these examples.

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- $\widehat{R_\rho}$ satisfies the conformal condition (10) if and only if the original R_ρ does (proven)
- $\widehat{R_\rho}$ allows the definition of projectors (9) and satisfies the orthogonality condition (11) if and only if the original R_ρ does in all the examples investigated for the transitions inside the six–dimensional Drinfel'd doubles $(Bianchi\,5 \mid \mathbb{R}^3) \simeq (Bianchi\,6_0 \mid \mathbb{R}^3)$ and the semiabelian four–dimensional Drinfel'd double $(af(1) \mid \mathbb{R}^2) \simeq (af(1) \mid af(1))$. Similarly, the integrability condition (12) was preserved in these examples.

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Not so good news

- $\widehat{R_\rho}$ defined by (15) may depend not only on $\hat{g}$ but also on g and consequently on $\bar{h}$.

  Solution: if the matrix–valued function $C(g) = F^{-t}(g) \cdot R_\rho(g) \cdot F(g)$ extended to a function on the whole Drinfel'd double as $C(g\tilde{h}) = C(g)$ satisfies

  $$C(\hat{g}\bar{h}) = C(\hat{g}). \tag{16}$$

  then $\widehat{R_\rho}$ is function of $\hat{g}$ only.

Note that $C \equiv \mathbf{1}$ corresponds to free (Neumann) boundary conditions, i.e. restriction to gluing operators satisfying (16) appears to be quite reasonable. (Because we allow nonvanishing B–field, free boundary conditions are not $R_\rho = \mathbf{1}$ but $R_\rho(g) = F^t(g) \cdot F^{-1}(g)$.)

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For example, consider the free boundary conditions on G, i.e. $C = \mathbf{1}$, $R_\rho(g) = F^t(g) \cdot F^{-1}(g)$. If one performs the PL T–duality transformation, i.e. $G \leftrightarrow \tilde{G}$, one finds $\widehat{R_\rho}$ corresponding to D–branes contained in the symplectic leaves on the Poisson–Lie group $\tilde{G}$. Such $\widehat{R_\rho}$ satisfies all the consistency conditions except (13). However, this condition is satisfied only in points where $\widehat{N} = 0$, i.e. where $\widehat{R_\rho} = -\mathbf{1}$.

Way out: introduce electromagnetic interaction acting on the endpoints, i.e. effectively changing the new B–field by an addition of an exact 2–form. This doesn't affect the bulk equations of motion but may help on the boundary.

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Preliminary conclusion: it seems that this helps at least in the acse of the PL T–duality of Neumann boundary condition.

But the correction needed is singular is some points (typically where $\widehat{R_\rho} = -\mathbf{1}$). Does that imply that the dual to free boundary conditions contains specific electro/magnetic charge configuration ?

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Thank you for your attention