

$$\mathcal{T}_2 [Q_i^1 F_i^1, \rho_S^I(t_0) \rho_R] = \mathcal{T}_2 \{ Q_i^1 F_i^1 \rho_S^I(t_0) \rho_R - \rho_S^I(t_0) \rho_R Q_i^1 F_i^1 \} = \text{Buzian}$$

$$= Q_i^1 \rho_S^I(t_0) \langle F_i^1 \rangle_R - \rho_S^I(t_0) Q_i^1 \langle F_i^1 \rangle_R = [Q_i^1, \rho_S^I(t_0)] \langle F_i^1 \rangle_R$$

$$\mathcal{T}_2 [Q_i^1 F_i^1, [Q_j^2 F_j^2, \rho_S^I(t_0) \rho_R]] =$$

$$= \mathcal{T}_2 \{ Q_i^1 F_i^1 Q_j^2 F_j^2 \rho_S^I(t_0) \rho_R - Q_i^1 F_i^1 \rho_S^I(t_0) \rho_R Q_j^2 F_j^2 - Q_j^2 F_j^2 \rho_S^I(t_0) \rho_R Q_i^1 F_i^1 + \rho_S^I(t_0) \rho_R Q_j^2 F_j^2 Q_i^1 F_i^1 \}$$

$$= Q_i^1 Q_j^2 \rho_S^I(t_0) \langle F_i^1 F_j^2 \rangle_R - Q_i^1 \rho_S^I(t_0) Q_j^2 \langle F_j^2 F_i^1 \rangle_R - Q_j^2 \rho_S^I(t_0) Q_i^1 \langle F_i^1 F_j^2 \rangle_R + \rho_S^I(t_0) Q_j^2 Q_i^1 \langle F_j^2 F_i^1 \rangle_R$$

$$= \langle F_i^1 F_j^2 \rangle_R (Q_i^1 Q_j^2 \rho_S^I(t_0) - Q_j^2 \rho_S^I(t_0) Q_i^1) - \langle F_j^2 F_i^1 \rangle_R (Q_i^1 \rho_S^I(t_0) Q_j^2 - \rho_S^I(t_0) Q_j^2 Q_i^1)$$

$$\rho_S^I(t) - \rho_S^I(t_0) = -i \sum_i \int_{t_0}^t dt_1 \mathcal{T}_2 [Q_i^1 F_i^1, \rho_S^I(t_0) \rho_R] - \sum_{ij} \int_{t_0}^t dt_1 \int_{t_0}^{t_1} dt_2 \mathcal{T}_2 [Q_i^1 F_i^1, [Q_j^2 F_j^2, \rho_S^I \rho_R]]$$

$$= -i \sum_i \int_{t_0}^t dt_1 [Q_i^1, \rho_S^I(t_0)] \langle F_i^1 \rangle_R - \sum_{ij} \int_{t_0}^t dt_1 \int_{t_0}^{t_1} dt_2 \{ (Q_i^1 Q_j^2 \rho_S^I - Q_j^2 \rho_S^I Q_i^1) \langle F_i^1 F_j^2 \rangle_R - (Q_i^1 \rho_S^I Q_j^2 - \rho_S^I Q_j^2 Q_i^1) \langle F_j^2 F_i^1 \rangle_R \}$$