Odvození rovnic (3.16),(3.17)

$$\vec{E} = \vec{i_y}\, \mathcal{E}(z,t)\cos(\omega t - kz + \phi(z,t)) \qquad (3.1)$$

$$\vec{P} = \vec{i_y}\, \{\mathcal{P}_1(z,t)\cos(\omega t - kz + \phi(z,t)) + \mathcal{P}_2(z,t)\sin(\omega t - kz + \phi(z,t))\} \qquad (3.2)$$

vlnová rovnice: $\Delta\vec{E} - \frac{1}{c_0^2}\frac{\partial^2\vec{E}}{\partial t^2} = \mu_0 \frac{\partial^2\vec{P}}{\partial t^2}$ (2.13), pomalu proměnná amplituda a fáze

• dosazení (3.1) do levé části (2.13):

[sin, resp. cos označuje $\sin(\omega t - kz + \phi(z,t))$, resp. $\cos(\omega t - kz + \phi(z,t))$]

derivace: $$\frac{\partial E}{\partial z} = \frac{\partial \mathcal{E}(z,t)}{\partial z}\cos + \mathcal{E}(z,t)\left(k - \frac{\partial\phi(z,t)}{\partial z}\right)\sin$$

$$\frac{\partial E}{\partial t} = \frac{\partial \mathcal{E}(z,t)}{\partial t}\cos - \mathcal{E}(z,t)\left(\omega + \frac{\partial\phi(z,t)}{\partial t}\right)\sin$$

$$\frac{\partial^2 E}{\partial z^2} = \frac{\partial^2\mathcal{E}}{\partial z^2}\cos + 2\frac{\partial\mathcal{E}}{\partial z}\left(k - \frac{\partial\phi}{\partial z}\right)\sin - \mathcal{E}\left(k - \frac{\partial\phi}{\partial z}\right)^2\cos - \mathcal{E}\frac{\partial^2\phi}{\partial z^2}\sin$$

$$\frac{\partial^2 E}{\partial t^2} = \frac{\partial^2\mathcal{E}}{\partial t^2}\cos - 2\frac{\partial\mathcal{E}}{\partial t}\left(\omega + \frac{\partial\phi}{\partial t}\right)\sin - \mathcal{E}\left(\omega - \frac{\partial\phi}{\partial t}\right)^2\cos - \mathcal{E}\frac{\partial^2\phi}{\partial t^2}\sin$$

dosazení:

$$\frac{\partial^2 E}{\partial z^2} - \frac{1}{c^2}\frac{\partial^2 E}{\partial t^2} = \frac{1}{c^2}\Big(\cos c^2\frac{\partial^2\mathcal{E}}{\partial z^2} + \underline{2\sin c^2 k\frac{\partial\mathcal{E}}{\partial z}} - 2\sin c^2\frac{\partial\mathcal{E}}{\partial z}\frac{\partial\phi}{\partial z} - \underline{\mathcal{E}\cos c^2 k^2} +$$

$$+ \underline{2\mathcal{E}\cos c^2 k\frac{\partial\phi}{\partial z}} - \mathcal{E}\cos c^2\left(\frac{\partial\phi}{\partial z}\right)^2 - \mathcal{E}\sin c^2\frac{\partial^2\phi}{\partial z^2} - \cos\frac{\partial^2\mathcal{E}}{\partial t^2} + \underline{2\omega\sin\frac{\partial\mathcal{E}}{\partial t}} +$$

$$+ 2\sin\frac{\partial\mathcal{E}}{\partial t}\frac{\partial\phi}{\partial t} + \underline{\mathcal{E}\cos\omega^2} + \underline{2\omega\cos\mathcal{E}\frac{\partial\phi}{\partial t}} + \cos\mathcal{E}\left(\frac{\partial\phi}{\partial t}\right)^2 + \sin\mathcal{E}\frac{\partial^2\phi}{\partial t^2}\Big)$$

zanedbáme všechny členy s derivacemi druhého řádu, podtržené členy zůstanou

zachováme:

$$\frac{\partial^2 E}{\partial z^2} - \frac{1}{c^2}\frac{\partial^2 E}{\partial t^2} \cong \frac{1}{c^2}\Big(2\sin c^2 k\frac{\partial\mathcal{E}}{\partial z} - \mathcal{E}\cos(c^2k^2 - \omega^2) + 2\mathcal{E}\cos c^2 k\frac{\partial\phi}{\partial z} +$$

$$\cancel{+2\mathcal{E}\cos c^2 k\frac{\partial\phi}{\partial z}} + 2\omega\sin\frac{\partial\mathcal{E}}{\partial t} + 2\omega\mathcal{E}\cos\frac{\partial\phi}{\partial t}\Big)$$

zavedeme $\Delta k = \frac{\omega}{c} - k$, signál blízko rezonance $\omega \cong \omega_{21}$

$$\frac{\partial^2 E}{\partial z^2} - \frac{1}{c^2}\frac{\partial^2 E}{\partial t^2} \simeq \frac{1}{c^2}\Big(2\sin\underbrace{c\omega}_{c^2k}\frac{\partial\mathcal{E}}{\partial z} + \underbrace{c\omega\Delta k}_{c^2k^2-\omega^2}\mathcal{E}\cos + 2\mathcal{E}\cos\underbrace{c\omega}_{c^2k}\frac{\partial\phi}{\partial z} + 2\omega\sin\frac{\partial\mathcal{E}}{\partial t} + 2\omega\mathcal{E}\cos\frac{\partial\phi}{\partial t}\Big) =$$

$$= \frac{2\omega}{c}\left(\sin\left[\frac{\partial\mathcal{E}}{\partial z} + \frac{1}{c}\frac{\partial\mathcal{E}}{\partial t}\right] + \mathcal{E}\cos\left[\frac{\Delta k}{2} + \frac{\partial\phi}{\partial z} + \frac{1}{c}\frac{\partial\phi}{\partial t}\right]\right)$$

• dosazení (3.2) do pravé strany (2.13)

derivace:

$$\frac{\partial P}{\partial t} = \frac{\partial\mathcal{P}_1}{\partial t}\cos - \mathcal{P}_1\left(\omega + \frac{\partial\phi}{\partial t}\right)\sin + \frac{\partial\mathcal{P}_2}{\partial t}\sin + \mathcal{P}_2\left(\omega + \frac{\partial\phi}{\partial t}\right)\cos$$

$$\frac{\partial^2 P}{\partial t^2} = \frac{\partial^2\mathcal{P}_1}{\partial t^2}\cos \underline{- 2\frac{\partial\mathcal{P}_1}{\partial t}\sin\omega} - 2\frac{\partial\mathcal{P}_1}{\partial t}\frac{\partial\phi}{\partial t}\sin \underline{- \mathcal{P}_1\cos\omega^2 - 2\mathcal{P}_1\cos\omega\frac{\partial\phi}{\partial t}} - \mathcal{P}_1\cos\left(\frac{\partial\phi}{\partial t}\right)^2 -$$

$$- \mathcal{P}_1\sin\frac{\partial^2\phi}{\partial t^2} + \frac{\partial^2\mathcal{P}_2}{\partial t^2}\sin \underline{+ 2\frac{\partial\mathcal{P}_2}{\partial t}\cos\omega} + 2\frac{\partial\mathcal{P}_2}{\partial t}\cos\omega + 2\frac{\partial\mathcal{P}_2}{\partial t}\frac{\partial\phi}{\partial t}\cos -$$

$$\underline{- \mathcal{P}_2\sin\omega^2} \underline{- 2\mathcal{P}_2\sin\omega\frac{\partial\phi}{\partial t}} - \mathcal{P}_2\sin\left(\frac{\partial\phi}{\partial t}\right)^2 + \mathcal{P}_2\cos\frac{\partial^2\phi}{\partial t}$$

Nepodtržené členy opět zanedbáme:

$$\frac{\partial^2 P}{\partial t^2} \approx -2\frac{\partial P_1}{\partial t}\sin\omega - P_1\cos\omega^2 - 2P_1\cos\omega\frac{\partial\phi}{\partial t} + 2\frac{\partial P_2}{\partial t}\cos\omega - P_2\sin\omega^2 - 2P_2\sin\omega\frac{\partial\phi}{\partial t}$$

Předpoklady o pomalých změnách v prostoru a čase amplitudy a fáze lze zapsat:

$$\left|\frac{\partial \mathcal{E}}{\partial t}\frac{1}{\mathcal{E}}\right| \ll \omega \,, \quad \left|\frac{\partial\phi}{\partial t}\right| \ll \omega \qquad (3.3)$$

$$\left|\frac{\partial P_1}{\partial t}\frac{1}{P_1}\right| \ll \omega \,, \quad \left|\frac{\partial P_2}{\partial t}\frac{1}{P_2}\right| \ll \omega \qquad (3.5)$$

Tyto předpoklady využijeme k dalšímu zjednodušení výrazu pro $\frac{\partial^2 P}{\partial t^2}$:

$$\frac{\partial^2 P(z,t)}{\partial t^2} \approx -P_1\cos\omega^2 - P_2\sin\omega^2$$ (neuvažujeme závislost polarizace na čase)

- Dosazení do vlnové rovnice:

$$\frac{2\omega}{c}\left(\sin\left[\frac{\partial\mathcal{E}}{\partial z} + \frac{1}{c}\frac{\partial\mathcal{E}}{\partial t}\right] + \mathcal{E}\cos\left[\Delta k + \frac{\partial\phi}{\partial z} + \frac{1}{c}\frac{\partial\phi}{\partial t}\right]\right) = -\mu_0\omega^2\left(P_1\cos + P_2\sin\right)$$

Porovnáním členů u cos a sin dostaneme pro signál blízko rezonance rovnice (3.16) a (3.17)

$$\left\{\frac{\partial\phi}{\partial z} + \frac{1}{c}\frac{\partial\phi}{\partial t} + \frac{\Delta k}{2}\right\}\mathcal{E} = -\frac{\mu_0\omega_{21}c}{2}P_1 \qquad (3.16)$$

$$\frac{\partial\mathcal{E}}{\partial z} + \frac{1}{c}\frac{\partial\mathcal{E}}{\partial t} = -\frac{\mu_0\omega_{21}c}{2}P_2 \qquad (3.17)$$