Fyzika laserů – cvičení

Rovnice poloklasické teorie interakce látky a záření

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Pauliho rovnice pro tlumený dvouhladinový atom ve vnějším poli

- Pauliho rovnice pro tlumený dvouhladinový atom v silném vnějším elektromagnetickém poli:

$$\frac{\partial \rho_{11}}{\partial t} = \Gamma_2 \rho_{22} - \Gamma_1 \rho_{11} + \mathrm{i}\frac{\vec{E}}{\hbar} \cdot (\vec{d}_{12}\rho_{21} - \vec{d}_{21}\rho_{12}) \tag{1}$$

$$\frac{\partial \rho_{22}}{\partial t} = \Gamma_1 \rho_{11} - \Gamma_2 \rho_{22} + \mathrm{i}\frac{\vec{E}}{\hbar} \cdot (\vec{d}_{21}\rho_{12} - \vec{d}_{12}\rho_{21}) \tag{2}$$

$$\frac{\partial \rho_{12}}{\partial t} = -\left(\Gamma_{21} - \mathrm{i}\omega_{21}\right)\rho_{12} + \mathrm{i}\frac{\vec{E}}{\hbar} \cdot \vec{d}_{12}(\rho_{22} - \rho_{11}) \tag{3}$$

$$\frac{\partial \rho_{21}}{\partial t} = -\left(\Gamma_{21} + \mathrm{i}\omega_{21}\right)\rho_{21} - \mathrm{i}\frac{\vec{E}}{\hbar} \cdot \vec{d}_{21}(\rho_{22} - \rho_{11}) \tag{4}$$

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- Cílem je napsat rovnice pro měřitelné veličiny – polarizaci prostředí $\langle \hat{\vec{d}} \rangle = \vec{d}_{21} \varrho_{12} + \vec{d}_{12} \varrho_{21}$ a inverzi populaci hladin $\langle \hat{n} \rangle = (\varrho_{22} - \varrho_{11})$

Odvození rovnice pro polarizaci

- Rovnice pro nediagonální prvky

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- vynásobíme skalárně $\vec{d}_{21}$, resp $\vec{d}_{12}$:

$$\left[\left(\frac{\partial}{\partial t} + \Gamma_{21}\right) - \mathrm{i}\omega_{21}\right]\vec{d}_{21}\varrho_{12} = \mathrm{i}\frac{\vec{E}}{\hbar}\left|\vec{d}_{21}\right|^2(\varrho_{22} - \varrho_{11}), \quad (7)$$

$$\left[\left(\frac{\partial}{\partial t} + \Gamma_{21}\right) + \mathrm{i}\omega_{21}\right]\vec{d}_{12}\varrho_{21} = -\mathrm{i}\frac{\vec{E}}{\hbar}\left|\vec{d}_{21}\right|^2(\varrho_{22} - \varrho_{11}) \quad (8)$$

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- Tyto rovnice sečteme a odečteme, abychom získali členy odpovídající $\langle\hat{\vec{d}}\,\rangle$:

$$\left(\frac{\partial}{\partial t} + \Gamma_{21}\right)\overbrace{(\vec{d}_{21}\varrho_{12} + \vec{d}_{12}\varrho_{21})}^{\langle\hat{\vec{d}}\,\rangle} + \mathrm{i}\omega_{21}(\vec{d}_{12}\varrho_{21} - \vec{d}_{21}\varrho_{12}) = 0, \tag{9}$$

$$\left(\frac{\partial}{\partial t} + \Gamma_{21}\right)(\vec{d}_{21}\varrho_{12} - \vec{d}_{12}\varrho_{21}) - \mathrm{i}\omega_{21}\underbrace{(\vec{d}_{12}\varrho_{21} + \vec{d}_{21}\varrho_{12})}_{\langle\hat{\vec{d}}\,\rangle} = \mathrm{i}\frac{2\vec{E}}{\hbar}\left|\vec{d}_{21}\right|^2(\varrho_{22} - \varrho_{11}) \tag{10}$$

Odvození rovnice pro polarizaci

- Máme

$$\left(\frac{\partial}{\partial t} + \Gamma_{21}\right)\langle\hat{\vec{d}}\,\rangle + \mathrm{i}\omega_{21}(\vec{d}_{12}\varrho_{21} - \vec{d}_{21}\varrho_{12}) = 0, \tag{11}$$

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- Rovnici (11) vynásobíme zleva $\left(\frac{\partial}{\partial t} + \Gamma_{21}\right)$ a dosadíme z (12) tak, abychom se zbavili členu $(\vec{d}_{21}\varrho_{12} - \vec{d}_{12}\varrho_{21})$:

$$\left(\frac{\partial}{\partial t} + \Gamma_{21}\right)^2\langle\hat{\vec{d}}\,\rangle = \mathrm{i}\omega_{21}\left[\mathrm{i}\omega_{21}\langle\hat{\vec{d}}\,\rangle + 2\mathrm{i}\frac{\vec{E}}{\hbar}\left|\vec{d}_{21}\right|^2\underbrace{(\varrho_{22} - \varrho_{11})}_{\langle\hat{n}\rangle}\right] \tag{13}$$

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- Po úpravě dostáváme rovnici pro polarizaci:

$$\left[\left(\frac{\partial}{\partial t} + \frac{1}{T_2}\right)^2 + \omega_{21}^2\right]\langle \hat{\vec{d}}\,\rangle = -\omega_{21}\frac{2\vec{E}}{\hbar}\left|\vec{d}_{21}\right|^2\langle \hat{n}\rangle \tag{14}$$

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- $T_2 = \Gamma_{21}^{-1}$ je relaxační doba polarizace (příčná).