Random processes

Winter term 2021/22, FJFI ČVUT

Exercises 2

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1. Let $X = (X_t)_{t\geq 0}$ be a Gaussian process. Show that also

   a) $Y_t = 2X_t + 1$,

   b) $Z_t = X_{t^2}$

   are Gaussian processes.

2. Let $W = (W_t)_{t\geq 0}$ be the standard Wiener process and put $M_t := \max\limits_{0\leq s\leq t} W_s$. Show, that also the following processes are standard Wiener processes:

   a) $(-W_t)_{t\geq 0}$;

   b) $(W_{s+t} - W_s)_{t\geq 0}$, $s > 0$ fixed;

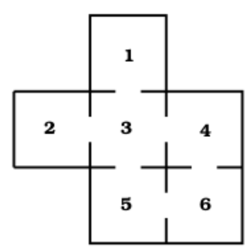
   c) $W^* = (W^*_t)_{t\geq 0} = (aW(t/a^2))_{t\geq 0}$;

   d) $(tW_{1/t})_{t\geq 0}$.

   Further show that $M^*_t := \max_{0\leq s\leq t} W^*_s = \max_{0\leq s\leq t} aW_{s/a^2} = aM_{t/a^2}$

3. Find the transition matrix for the random walk on $\mathbb{Z}$ and for the gambler's ruin.

4. There is the rat in the maze. In each step it runs randomly through one of the doors in the room, in which it is right now.



   (i) Write the transition matrix of the corresponding Markov chain.

   (ii) Find the stationary distribution.

   (iii) The rat starts in the first room. What is the average time of return to the first room?

   (iv) The rat starts in the first room, in the fifth room, there is a trap with cheese. What is the average time of the first (and the last...) arrival into the fifth room?

5. We toss the coin repeatedly. The outcome might be "Orel" (O) or "Hlava" (H). How many tosses we need on average before the sequence HOHH appears?