Theory of random processes

ZS 2021/22, FJFI ČVUT

List of topics:

Essentially, all definitions and theorems from the lecture.

Z1. Definition of a random process, its paths, finite-dimensional distribution functions, consistent system of X in times $(t_1, \dots, t_n)$, Daniell-Kolmogorov theorem

Z2. Mean value, autocovariance function, autocorrelation function of X, positive semi-definite functions, Gauss processes

Z3. Strongly and weakly stationary processes, Markov processes

Z4. Wiener process W: definition, $\mu_W(t), C_W(s,t)$, Markov property

Z5. Time of first passage of the Wiener process, its density

Z6. Quadratic variance $\langle X \rangle_t$, $\langle W \rangle_t = t$

Z7. Poisson process N: definition, $\mu_N(t), C_N(s,t)$, Markov property

Z8. Limit, continuity and derivative of a random process: definitions, characterizations, examples

Z9. Process with orthogonal increments, distribution function F, $L_2(F)$, definition of $\int_0^t f(s)dW_s$

Z10. Definition of the Riemann integral $\int_a^b X_t dt$, characterize the convergence

Z11. Ergodic theorem for weakly stationary processes

Z12. Karhunen-Loève expansion

Z13. Chapman-Kolmogorov equations for Markov chains with discrete and continuous time

Z14. Exercises

Z15. Lévy's construction of the Wiener process - main idea

List of topics - extension

R1. Poisson process: density of the waiting time, law of large numbers

R2. Random walk in $\mathbb{R}^d$: transitive for $\mathbb{E}N < +\infty$, recurrent for $\mathbb{E}N = +\infty$

R3. Random walk in $\mathbb{R}^d$: convergence/divergence of $\mathbb{E}N$ for simple walk in $d \geq 1$

R4. Details of the definition of $\int_0^t f(s)dW_s$, proof of basic properties

R5. $\int_0^t W_s dW_s$, $\int_0^t W_s ds$ from (the Riemannian) definition

R6. Karhunen-Loève: proof

R7. Karhunen-Loève for the Wiener process

R8. Karhunen-Loève for the Wiener bridge

R9. Yule process - solution with the help of the generating function

R10. Yule process - solution with the help of e^{Gt}

R11. Ergodic Markov processes (Pavliotis, pp. 37-39)

R12. Numeric simulation of paths of the Brownian motion: from the definition and with the help of Karhunen-Loève decomposition

R13. Quantum Markov Chains (according to S. Gudder - Journal of Mathematical Physics, 2008; definition, chap. 2, examples)

R14. Lévy's construction of the Wiener process - details