Theory of random processes

ZS 2022/23, FJFI ČVUT

List of topics:

Essentially, all definitions and theorems from the lecture.

1. Definition of a random process, its paths, finite-dimensional distribution functions, consistent system of X in times $(t_1, \ldots, t_n)$, Daniell-Kolmogorov theorem
2. Mean value, autocovariance function, autocorrelation function of X, positive semi-definite functions, Gauss processes
3. Strongly and weakly stationary processes, Markov processes
4. Wiener process W: definition, $\mu_W(t), C_W(s,t)$, Markov property
5. Lévy's construction of the Wiener process - main idea
6. Time of first passage of the Wiener process, its density
7. Quadratic variance $\langle X \rangle_t$, $\langle W \rangle_t = t$
8. Poisson process N: definition, $\mu_N(t), C_N(s,t)$, Markov property
9. Poisson process: density of the waiting time, law of large numbers
10. Limit, continuity and derivative of a random process: definitions, characterizations, examples
11. Process with orthogonal increments, distribution function F, $L_2(F)$, definition of $\int_0^t f(s)dW_s$, $\int_0^t W_s dW_s$
12. Definition of the Riemann integral $\int_a^b X_t dt$, characterize the convergence
13. Ergodic theorem for weakly stationary processes
14. Karhunen-Loève expansion
15. Karhunen-Loève for the Wiener process
16. Chapman-Kolmogorov equations for Markov chains with discrete and continuous time
17. Yule process - problem, main idea of the solution
18. Conditional probability density, Chapman-Kolmogorov equations for Markov processes
19. Kolmogorov differential equations for Markov chains with continuous time
20. Diffusion processes