Theory of random matrices

ZS 2021/22, FJFI ČVUT

List of topics

1. Basic facts about matrices: Spectral norm, Frobenius norm, trace, singular value decomposition, orthogonal matrices
2. Random matrices, Gaussian orthogonal ensemble, $\varrho(H) = \varrho(H_{1,1}, H_{1,2}, \ldots, H_{N,N})$
3. Rotational invariance of GOE
4. Wigner's surmise ($N = 2$): δ-function notation, classical reformulation
5. Distance function from $X_1, \ldots, X_N$ i.i.d. variables on $[0, 1]$, limit for $N \to \infty$
6. jpdf $\varrho(x_1, \ldots, x_N)$
7. Wigner's surmise ($N = 2$) from $\varrho(x_1, x_2)$
8. Definition of the spectral density $\varrho(x)$ from $\varrho(x_1, \ldots, x_N)$, counting function $n(x)$, $n(A)$, $\mathbb{E}n(A)$
9. Weak limit of $n_N(x)$, Wigner's semicircle law (statement)
10. volumes and areas A_{n-1}, V_n in $\mathbb{R}^n$, "volume" $\mathbb{V}_n$ and (directly) $\mathbb{V}_2$
11. Vandermond: $|J(H \to \{x, O\})|$ - formulation of the general problem; $|J(H \to \{x, O\})|$ for $N = 2$
12. jpdf $\varrho(x_1, \ldots, x_N)$ from $\varrho(H_{1,1}, \ldots, H_{N,N})$: statement and sketch of the proof
13. Exponential of a matrix: definition, basic properties, spectrum, ordering of matrices
14. Trace and "trace is cyclic", Lie product formula, Golden-Thompson inequality (statement)
15. Scalar Bernstein inequality (with proof)
16. Non-commutative Bernstein inequality - statement and basic idea of the proof
17. Matrix sparsification - setting, the algorithm, error estimate
18. Matrix multiplication - setting, the algorithm, error estimate
19. Singular value decomposition - setting and the algorithm

Topics: extensions

R1. Vandermond: $|J(H \to \{x, O\})|$ - details for general N

R2. Lie product formula & Golden-Thompson inequality (with notes)

R3. Non-commutative Bernstein inequality - details of the proof

R4. Matrix sparsification - proof and numerical implementation

R5. Non-commutative Bernstein inequality from the Lieb's theorem

R6. Lieb's theorem - proof

R7. Randomized matrix multiplication - proof and implementation

R8. Randomized SVD - proof and implementation