
List of topics

1. Basic facts about matrices: Spectral norm, Frobenius norm, trace, singular value decomposition, orthogonal matrices
2. Random matrices, Gaussian orthogonal ensemble, $\varrho(H) = \varrho(H_{1,1}, H_{1,2}, \dots, H_{N,N})$
3. Rotational invariance of GOE
4. Wigner's surmise ($N = 2$): δ -function notation, classical reformulation
5. Distance function from X_1, \dots, X_N i.i.d. variables on $[0, 1]$, limit for $N \rightarrow \infty$
6. jpdf $\varrho(x_1, \dots, x_N)$
7. Wigner's surmise ($N = 2$) from $\varrho(x_1, x_2)$
8. Definition of the spectral density $\varrho(x)$ from $\varrho(x_1, \dots, x_N)$, counting function $n(x)$, $n(A)$, $\mathbb{E}n(A)$
9. Weak limit of $n_N(x)$, Wigner's semicircle law (statement)
10. volumes and areas A_{n-1}, V_n in \mathbb{R}^n , "volume" \mathbb{V}_n and (directly) \mathbb{V}_2
11. Vandermond: $|J(H \rightarrow \{x, O\})|$ - formulation of the general problem; $|J(H \rightarrow \{x, O\})|$ for $N = 2$
12. jpdf $\varrho(x_1, \dots, x_N)$ from $\varrho(H_{1,1}, \dots, H_{N,N})$: statement and proof
13. Vandermond: $|J(H \rightarrow \{x, O\})|$ - details for general N
14. Exponential of a matrix: definition, basic properties, spectrum, ordering of matrices
15. Trace and "trace is cyclic", Lie product formula, Golden-Thompson inequality (statement)
16. Scalar Bernstein inequality
17. Non-commutative Bernstein inequality
18. Non-commutative Bernstein inequality from the Lieb's theorem
19. Matrix sparsification - setting, the algorithm, error estimate
20. Matrix multiplication - setting, the algorithm, error estimate
21. Singular value decomposition - setting and the algorithm