~~+. Vandermonde~~

$H \longleftrightarrow (X, O)$; $H = OXO^T$

- Přepočítat H jako funkci (X,O) a vyjádřit dH

- $N=2$

$$H = \begin{pmatrix} H_{11} & H_{12} \\ H_{12} & H_{22} \end{pmatrix} = \begin{pmatrix} \cos\varphi & \sin\varphi \\ -\sin\varphi & \cos\varphi \end{pmatrix} \begin{pmatrix} X_1 & 0 \\ 0 & X_2 \end{pmatrix} \begin{pmatrix} \cos\varphi & -\sin\varphi \\ \sin\varphi & \cos\varphi \end{pmatrix}$$

$$= \begin{pmatrix} \cos\varphi & \sin\varphi \\ -\sin\varphi & \cos\varphi \end{pmatrix} \begin{pmatrix} X_1\cos\varphi & -X_1\sin\varphi \\ X_2\sin\varphi & X_2\cos\varphi \end{pmatrix}$$

$$= \begin{pmatrix} X_1\cos^2\varphi + X_2\sin^2\varphi \,;\; (X_2-X_1)\sin\varphi\cos\varphi \\ (X_2-X_1)\sin\varphi\cos\varphi \,;\; X_1\sin^2\varphi + X_2\cos^2\varphi \end{pmatrix}$$

$$\begin{pmatrix} \frac{\partial H_{11}}{\partial X_1} ; \frac{\partial H_{11}}{\partial X_2} ; \frac{\partial H_{11}}{\partial \varphi} \\ \frac{\partial H_{12}}{\partial X_1} ; \frac{\partial H_{12}}{\partial X_2} ; \frac{\partial H_{12}}{\partial \varphi} \\ \frac{\partial H_{22}}{\partial X_1} ; \frac{\partial H_{22}}{\partial X_2} ; \frac{\partial H_{22}}{\partial \varphi} \end{pmatrix} = \begin{pmatrix} \cos^2\varphi \;;\; \sin^2\varphi \;; 2(X_2-X_1)\cos\varphi\sin\varphi \\ -\sin\varphi\cos\varphi ; \sin\varphi\cos\varphi ; (X_2-X_1)(\cos^2\varphi-\sin^2\varphi) \\ \sin^2\varphi ; \cos^2\varphi ; (X_1-X_2)2\sin\varphi\cos\varphi \end{pmatrix}$$

$$\det \overset{I+III\to I}{=} (X_2-X_1)\cdot\det\begin{pmatrix} 1 \;;\; 1 ; \;0 \\ -\sin\varphi\cos\varphi, \sin\varphi\cos\varphi, \cos^2\varphi-\sin^2\varphi \\ \sin^2\varphi, \cos^2\varphi ; -2\sin\varphi\cos\varphi \end{pmatrix}$$

$$\overset{II-I\to II}{=} (X_2-X_1)\det\begin{pmatrix} 1 \;;\; 0 \;;\; 0 \\ -\sin\varphi\cos\varphi ; 2\sin\varphi\cos\varphi ; \cos^2\varphi-\sin^2\varphi \\ \sin^2\varphi ; \cos^2\varphi-\sin^2\varphi ; -2\sin\varphi\cos\varphi \end{pmatrix}$$

$$= (X_2-X_1)\left\{-4\sin^2\varphi\cos^2\varphi - (\cos^2\varphi-\sin^2\varphi)^2\right\} = (X_2-X_1)\left\{-\cos^4\varphi - 2\sin^2\varphi\cos^2\varphi - \sin^4\varphi\right\}$$

$$= \underline{-(X_2-X_1).}$$

$H = \mathcal{O} X \mathcal{O}^T$

$$\boxed{\partial(\mathcal{O}^T) = (\partial \mathcal{O})^T}$$

- $\partial H = (\partial\mathcal{O}) X \mathcal{O}^T + \mathcal{O}(\partial X)\mathcal{O}^T + \mathcal{O} X (\partial \mathcal{O}^T)$

- $\mathbb{1} = \mathcal{O}\mathcal{O}^T \ldots \; \emptyset = (\partial\mathcal{O})\mathcal{O}^T + \mathcal{O}\cdot(\partial\mathcal{O}^T)$

$$\partial\mathcal{O}^T = -\mathcal{O}^T(\partial\mathcal{O})\mathcal{O}^T$$

- $\partial H = (\partial\mathcal{O}) X \mathcal{O}^T + \mathcal{O}(\partial X)\mathcal{O}^T - \mathcal{O} X \mathcal{O}^T(\partial\mathcal{O})\mathcal{O}^T$

$$= \mathcal{O}[\underbrace{\mathcal{O}^T}_{\partial\mathcal{O}} X \cancel{\mathcal{O}} + (\partial X) - X\mathcal{O}^T\partial\mathcal{O}]\mathcal{O}^T$$

$\partial\Omega = \mathcal{O}^T\partial\mathcal{O}$: $\quad \partial H = \mathcal{O}[\underbrace{\partial\Omega\cdot X + \partial X - X\partial\Omega}_{\partial\hat{H}}]\mathcal{O}^T$

- $\mathcal{O}^T\partial\mathcal{O}$ je antisymetrická matice

$$\ldots \; \mathbb{1} \simeq (\mathcal{O}+\partial\mathcal{O})^T(\mathcal{O}+\partial\mathcal{O}) = \mathcal{O}^T\mathcal{O} + (\partial\mathcal{O})^T\mathcal{O} + \mathcal{O}^T(\partial\mathcal{O})$$

$$\Rightarrow (\partial\mathcal{O})^T\mathcal{O} + \mathcal{O}^T(\partial\mathcal{O}) = \emptyset$$

$$[\mathcal{O}^T(\partial\mathcal{O})]^T + \mathcal{O}^T(\partial\mathcal{O}) = \emptyset$$

- ∂H a $\partial\hat{H}$ se liší ortogonální transformací -- stačí najít

$$\partial\hat{H} \longrightarrow \{\partial X, \partial\Omega\}$$

- $\partial\hat{H} = \partial\Omega\cdot X + \partial X - X\partial\Omega$

$$\frac{d\hat{H}_{ij}}{dx_k} = \begin{cases} 1 & \text{pro } i=j=k \\ 0 & \text{jinak} \end{cases} ; \quad \frac{d\hat{H}_{ij}}{d\Omega_{k,l}} = \begin{cases} 0 \ldots & \text{pro } i\neq k \text{ nebo } j\neq l \\ x_j - x_i & \text{pro } i=k,\, j=l \end{cases}$$

$$
\begin{array}{c|ccc|ccc}
 & /dx_1 & \dots & /dx_N & /d\Omega_{12} & \dots & /d\Omega_{N-1,N} \\
\hline
dH_{11} & 1 & & 0 & & & \\
dH_{22} & & \ddots & & & 0 & \\
\vdots & & & & & & \\
dH_{NN} & 0 & & 1 & & & \\
\hline
dH_{12} & & & & x_2 - x_1 & & \\
\vdots & & 0 & & & \ddots & \\
dH_{N-1,N} & & & & & & x_N - x_{N-1}
\end{array}
$$

$$\Rightarrow \det = \prod_{i<j} x_j - x_i \; ; \quad |\det| = \prod_{i<j} |x_j - x_i| .$$

Obecný postup pro $N=2$

$$(H) \longleftrightarrow (x_1, x_2, \varphi)$$

$$X = \begin{pmatrix} x_1 & 0 \\ 0 & x_2 \end{pmatrix}; \quad O = \begin{pmatrix} \cos\varphi & +\sin\varphi \\ -\sin\varphi & \cos\varphi \end{pmatrix}$$

$$\delta X = \begin{pmatrix} dx_1 & 0 \\ 0 & dx_2 \end{pmatrix}; \quad \delta O = \begin{pmatrix} -\sin\varphi ; & \cos\varphi \\ -\cos\varphi ; & -\sin\varphi \end{pmatrix} d\varphi$$

$$O^T(\delta O) = \begin{pmatrix} \cos\varphi ; & -\sin\varphi \\ \sin\varphi ; & \cos\varphi \end{pmatrix} \cdot \; d\varphi = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} d\varphi$$

atd.