12RSEN Control Systems and Sensors

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CZECH TECHNICAL UNIVERSITY IN PRAGUE

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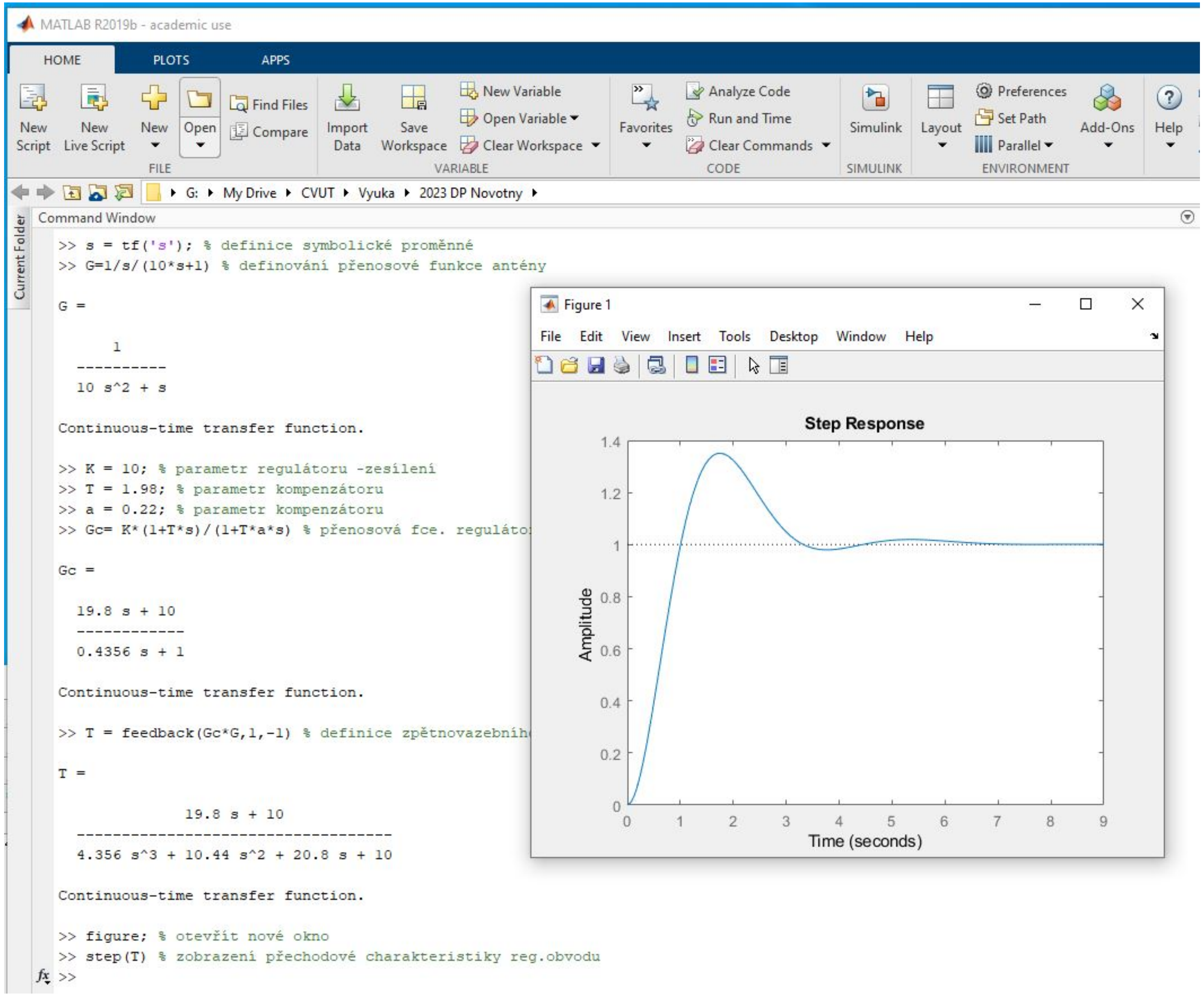
Course Requirements

- Complete a lab assignment and submit a report (mandatory for exam).
- The final exam is a written test.
- You must score at least 50% to pass.
- No aids (calculators, notes, etc.) are allowed during the exam.

MATLAB

- **MATLAB** is a proprietary programming language and numeric computing environment developed by MathWorks.
- Numerous examples of control system's analysis in MATLAB will be given throughout the lecture:
  - Plotting time/frequency response curves
  - Finding closed-form analytical solutions of various responses
  - Analyzing control system's stability

MATLAB


MATLAB R2019b - academic use
HOME
PLOTS
APPS
New Script
New Live Script
New
Open
Find Files
Compare
FILE
Import Data
Save Workspace
New Variable
Open Variable
Clear Workspace
VARIABLE
Favorites
Analyze Code
Run and Time
Clear Commands
CODE
Simulink
SIMULINK
Layout
Preferences
Set Path
Parallel
Add-Ons
ENVIRONMENT
Help
G: My Drive CVUT Vyuka 2023 DP Novotny
Current Folder
Command Window
>> s = tf('s'); % definice symbolické proměnné
>> G=1/s/(10*s+1) % definování přenosové funkce antény
G =
1
----
10 s^2 + s
Continuous-time transfer function.
>> K = 10; % parametr regulátoru -zesílení
>> T = 1.98; % parametr kompenzátoru
>> a = 0.22; % parametr kompenzátoru
>> Gc= K*(1+T*s)/(1+T*a*s) % přenosová fce. regulátor
Gc =
19.8 s + 10

0.4356 s + 1
Continuous-time transfer function.
>> T = feedback(Gc*G,1,-1) % definice zpětnovazebníh
T =
19.8 s + 10

4.356 s^3 + 10.44 s^2 + 20.8 s + 10
Continuous-time transfer function.
>> figure; % otevřít nové okno
>> step(T) % zobrazení přechodové charakteristiky reg.obvodu
>>
Figure 1
File
Edit
View
Insert
Tools
Desktop
Window
Help
Step Response
Amplitude
Time (seconds)
0
0.2
0.4
0.6
0.8
1
1.2
1.4
0
1
2
3
4
5
6
7
8
9

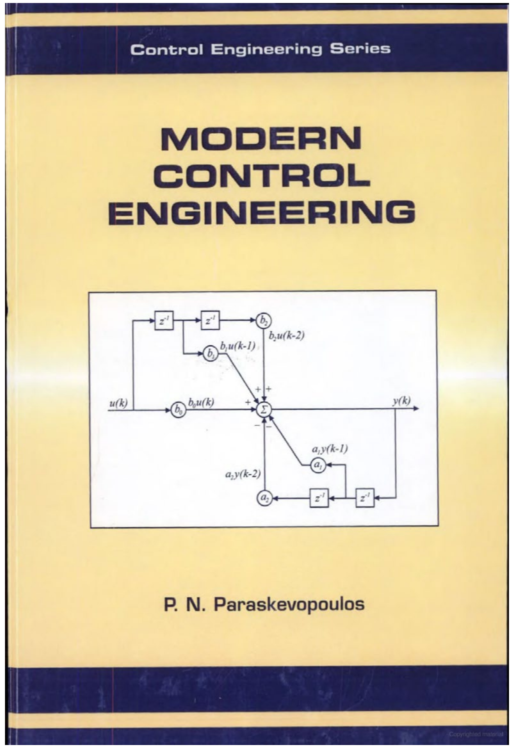
MATLAB

- Learning or understanding MATLAB syntax **is not required**.
- CTU student can (need to verify for ERASMUS students !!):
  - download MATLAB at https://download.cvut.cz
  - use Windows terminal server at **PCLabs.fjfi.cvut.cz** (logon with MS\username) via remote desktop
- MATLAB Toolboxes used:
  - Symbolic Math Toolbox
  - Control Systems Toolbox
  - System Identification Toolbox

Recommended Textbooks

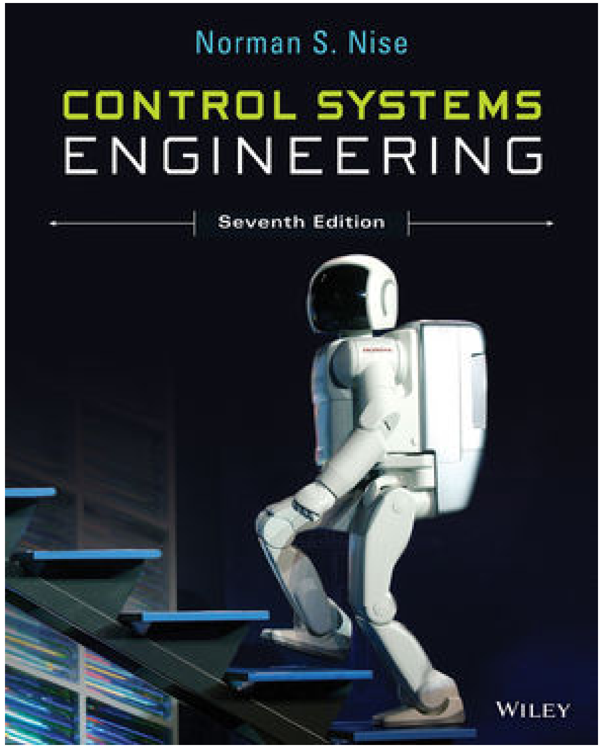
[Modern Control Engineering](#)

2002, P.N. Paraskevopoulos



[Control Systems Engineering](#)

7^{th} Ed., 2015, Norman. S. Nise



Course Syllabus

- Basic concepts in control theory
- Basic system classification
- Basic system properties
- System Identification
- Basic types of controllers

Course Syllabus

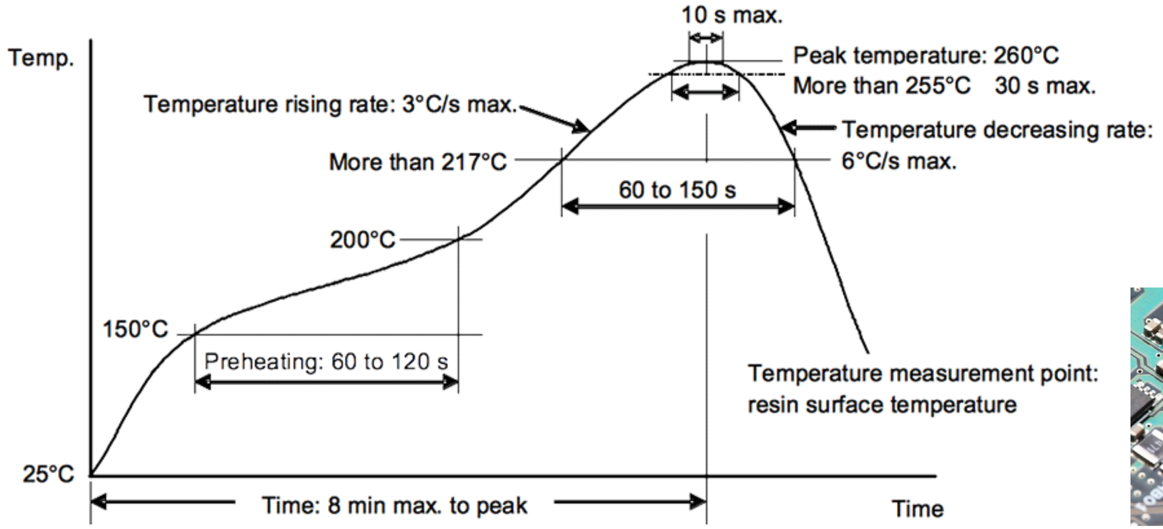
- Control quality evaluation
- Control systems stability
- Controller design methods
- Digital control
- Sensors

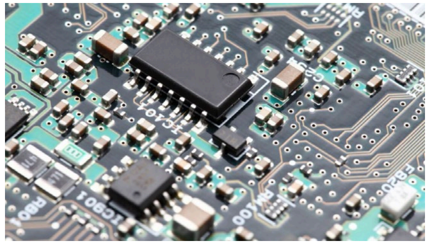
Why Study Control Systems?

- Control systems are **everywhere** in modern life:
  - Stabilized voltage power supplies (e.g. wall adapters, laptop chargers)
  - Washing machines and dishwashers
  - Room thermostats
  - Autonomous vehicles and drones
  - Robotics and industrial automation
  - Biomedical devices (prosthetics, insulin pumps)
  - Spacecraft and aerospace

Why Study Control Systems?

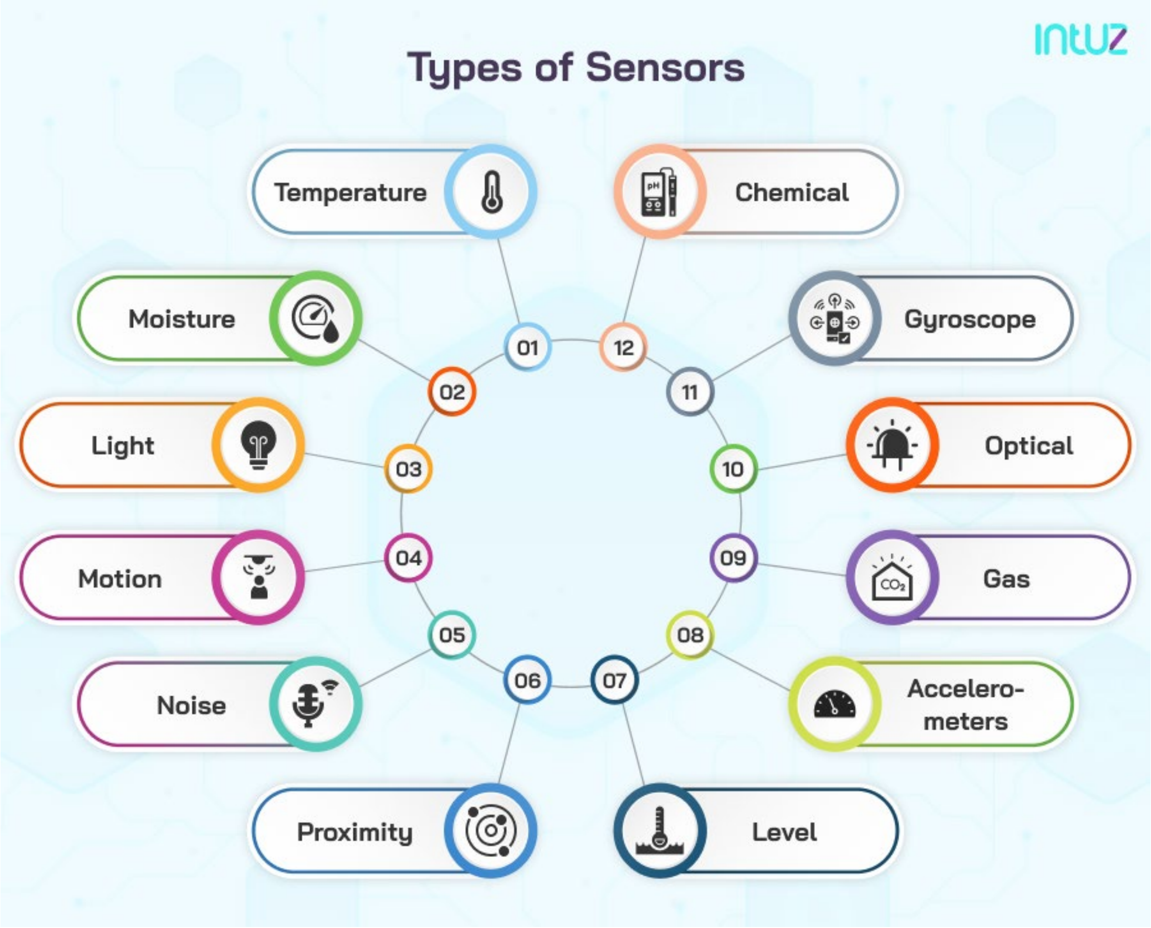
- As technology progressed, it became necessary to ensure that the output of a controlled system follows a specified value and temporal behavior in response to the given input signal.
- Example: Reflow soldering oven





Why Study Control Systems?

- **Sensors**, also known as **transducers**, are an essential part of any control system or circuit. Understanding how these components interact is crucial.
- Sensors convert physical quantities into electrical signals (voltage, resistance, current).



Why Study Control Systems?

- Even 2000 years ago, humans developed control mechanisms like **Clepsydra** (water clocks) – Ancient feedback-based timekeeping
- Until the year 1656, these were the most accurate and widely used instruments for measuring time (replaced by the pendulum clock)

Why Study Control Systems?

- The milestone of feedback systems was **Watt's centrifugal governor** (regulator) developed in 1788.

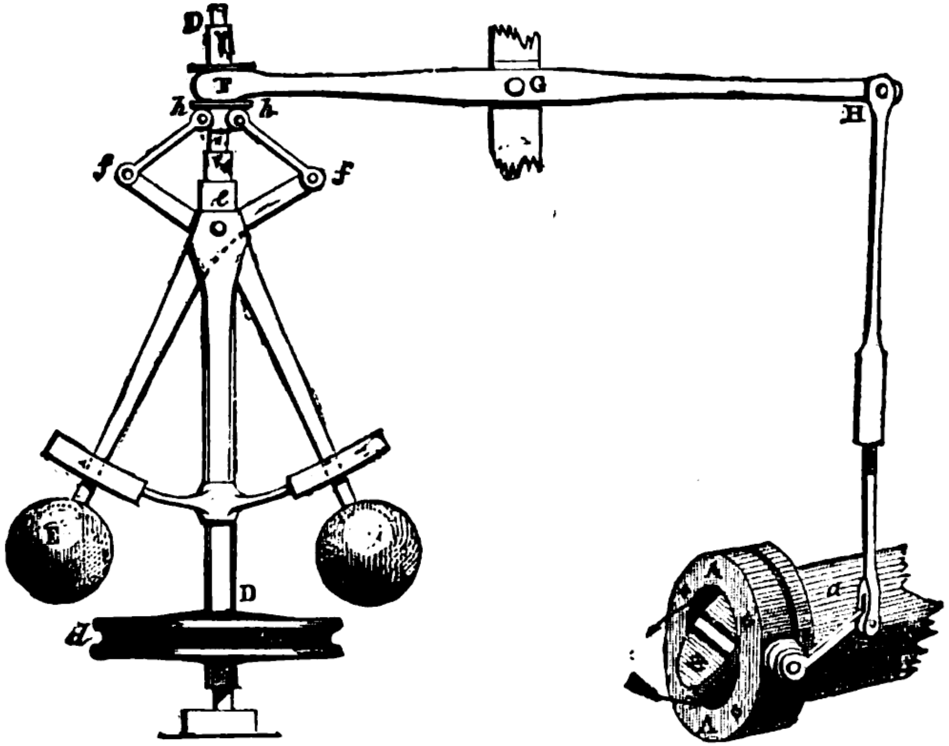
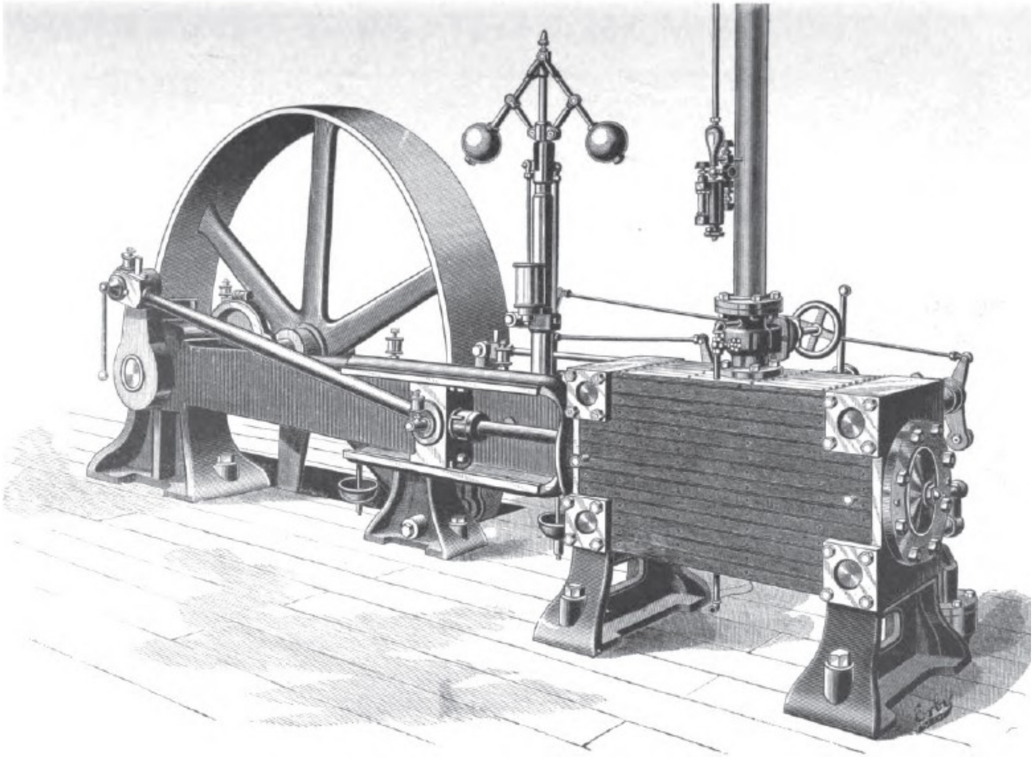


FIG. 4.—*Governor and Throttle-Valve.*



C. & G. COOPER & CO.'S CORLISS ENGINE. (FRONT VIEW.)

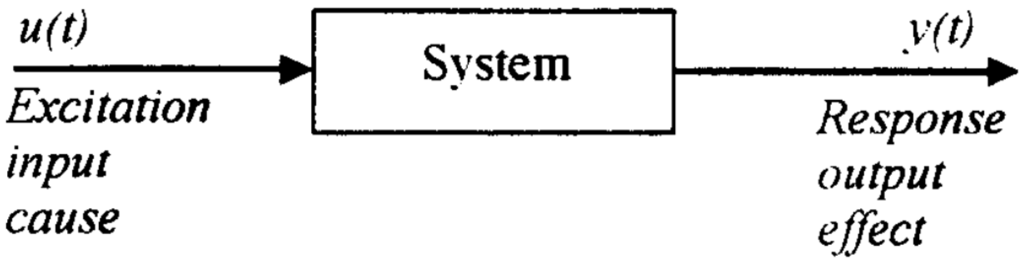
Course Syllabus

- **Basic concepts in control theory**
  - **Key Terms and Concepts**
  - Basic Structure of a Control System
- Basic system classification
- Basic system properties
- System Identification
- Basic types of controllers
- Control quality evaluation
- Control systems stability
- Controller design methods
- Digital control
- Sensors

Basic concepts in control theory
Key Terms and Concepts

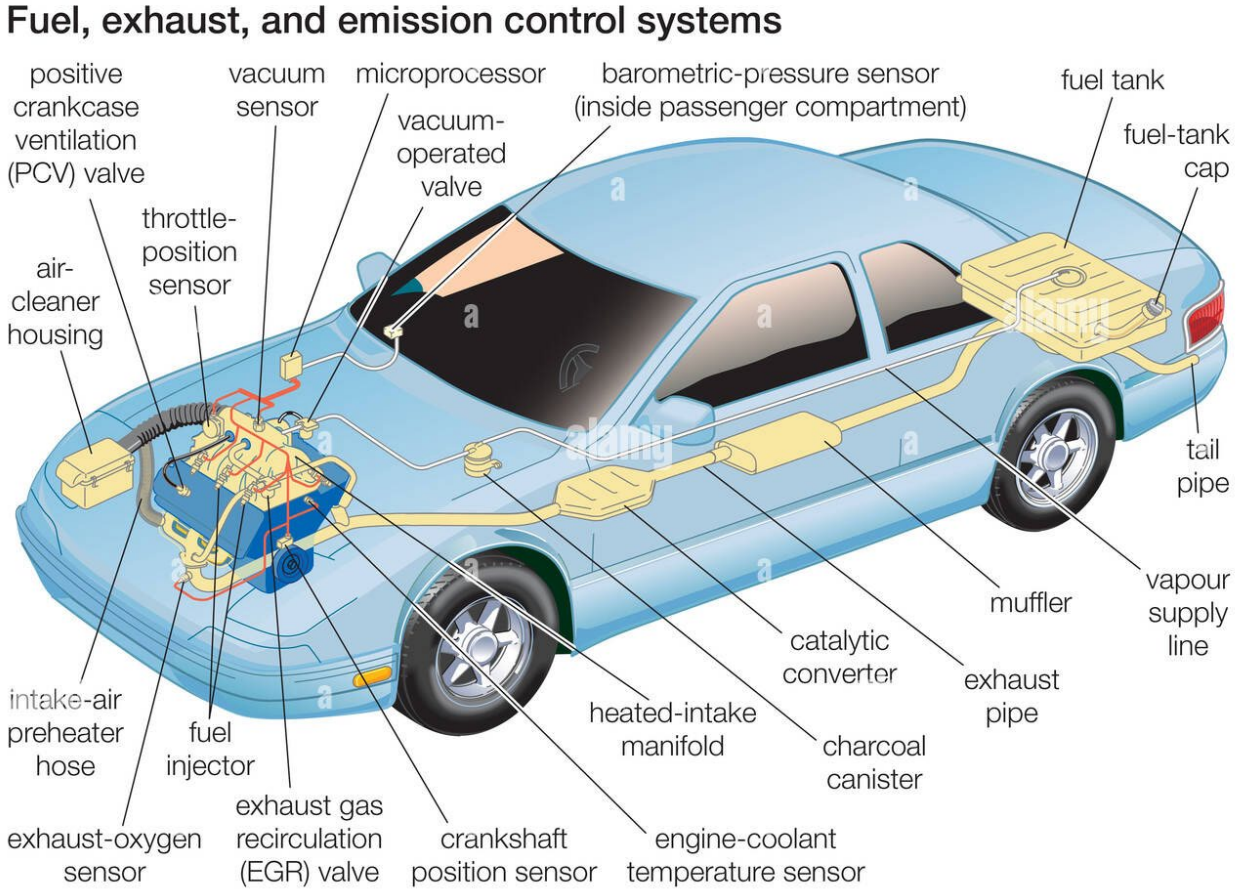
System (Plant)

- A system consists of subsystems and processes (or plants) assembled for the purpose of obtaining a desired *output* with desired performance, given a specified *input*.
- In simple terms: A collection of components we want to control
- Examples: robot arm, car engine, home heating system…



Basic concepts in control theory
Key Terms and Concepts

- **Car engine subsystems**
  - Starter
  - Ignition
  - Air intake
  - Fuel
  - Exhaust
  - Cooling
  - Electrical
  - Control
  - Diagnostic…



Basic concepts in control theory
Key Terms and Concepts

Dynamical System

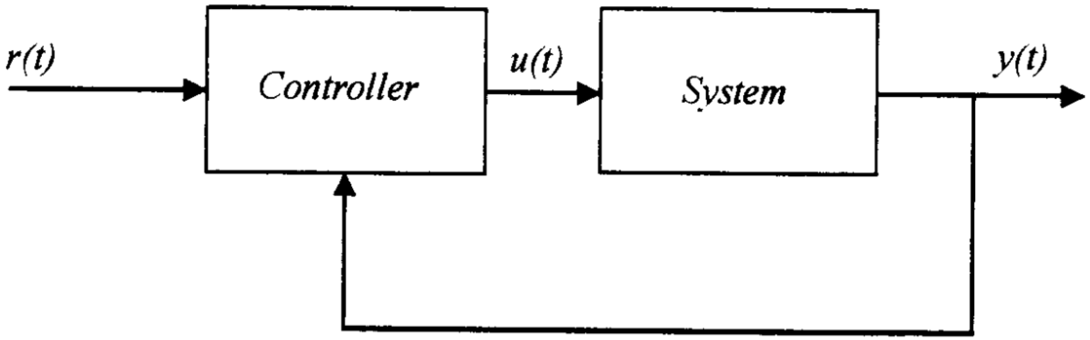
- Any system that changes with time (its quantities and variables change with time).
- Its current state and output depend not only on the immediate input but also on its previous (or initial) state.
- The total response is a combination of the system's *natural response* (how it dissipates or acquires energy) and its *forced response* (dependent on the input).

Basic concepts in control theory

Key Terms and Concepts

Control Theory

- Is a field of engineering and applied mathematics that uses feedback to influence the behavior of a system in order to achieve a desired outcome.
- The goal is to create a model or algorithm that governs the system inputs to guide the system to an ideal state while reducing delay and assuring control stability.



Basic concepts in control theory

Key Terms and Concepts

- Control theory has evolved over time, categorized into:
  - **Classical Control**
  - **Modern Control**
  - **Intelligent Control**
- Today, classical and modern control theory approaches to system modeling and control are deeply intertwined and frequently integrated in practice.

Basic concepts in control theory
Key Terms and Concepts

Classical Control Theory

- Based on input-output behavior (black box approach)
- Primarily focuses on *Single-Input Single-Output* (SISO) linear, time-invariant systems
- Based on the use of mathematical models that describe the dynamics of a system using *transfer functions*.
- Focuses on the *frequency domain analysis* of system stability, response, and performance using tools such as Bode plots, Nyquist plots, and root locus.
- Also called: *frequency domain* or *transfer function-based* approach

Basic concepts in control theory
Key Terms and Concepts

Modern Control Theory

- Based on internal state variables
- Can handle *Multiple-Input Multiple-Output* (MIMO), nonlinear, time-varying, and uncertain systems
- Uses analytical approach and require more advanced mathematical tools:
  - Lyapunov functions, Hamilton-Jacobi equations, and Riccati equations
- Focuses on the *state-space representation* of the system
  - This method uses a set of first-order differential equations (state equations) to describe a system
  - This provides a more complete description by involving state variables that reveal the system's internal structure
- Also called: *state-space* or *time domain* approach

Basic concepts in control theory
Key Terms and Concepts

State, State Variables

- The *state* of a dynamic system is the smallest set of variables called *state variables* that, along with the current time, are necessary to completely determine the behavior of the system.

State Space

- The *state space* is the n-dimensional space whose axes (dimensions) are the state variables, containing all possible values for each of n state variables

State Vector

- It is a vector whose elements are state variables.
- The specific state of the system at any given time can be expressed as a state vector.

Basic concepts in control theory
Key Terms and Concepts

State space example

- Object with mass m subject to Newton's law moving along a straight trajectory

$$F(t) = m \cdot a(t)$$

- State space can be modeled with 2 state variables:
  - position and velocity or **position and momentum**

$$F(t) = m\frac{d^2x(t)}{dt^2} = m \cdot \frac{dv(t)}{dt} = \frac{dp(t)}{dt}$$

  - where $\mathrm{a}(t) = \frac{d^2x(t)}{dt^2}$ is acceleration, $\mathrm{v}(t) = \frac{dx(t)}{dt}$ is velocity, $\mathrm{x}(t)$ is position, and $p(t) = m \cdot v(t)$ is momentum

Basic concepts in control theory
Key Terms and Concepts

State space example

- System is described by a coupled set of first-order equations

$$\frac{dx(t)}{dt} = \frac{p(t)}{m} \qquad \frac{dp(t)}{dt} = F(t) \xrightarrow{as\ vectors} \frac{d}{dt}\begin{bmatrix} x(t) \\ p(t) \end{bmatrix} = \begin{bmatrix} \frac{p(t)}{m} \\ F(t) \end{bmatrix}$$

with initial condition: $[x(0) \quad p(0)]^T = \left[x(0) \quad m \cdot \frac{dx}{dt}(0)\right]^T$

- The state at a given time is the vector

$$[x(t) \quad p(t)]^T$$

Basic concepts in control theory
Key Terms and Concepts

Intelligent Control Theory

- Fuzzy Logic Control
- Adaptive Control
- Artificial Intelligence (AI)
  - Machine Learning
    - Neural Networks (Artificial Neural Networks)
      - Deep Learning Neural Networks
        - Convolutional Neural Networks
        - Recurrent Neural Networks
        - Transformer Networks
    - Reinforcement Learning

- AI's formal origins are traced to the 1956 Dartmouth Summer Research Project on Artificial Intelligence

Basic concepts in control theory

Key Terms and Concepts

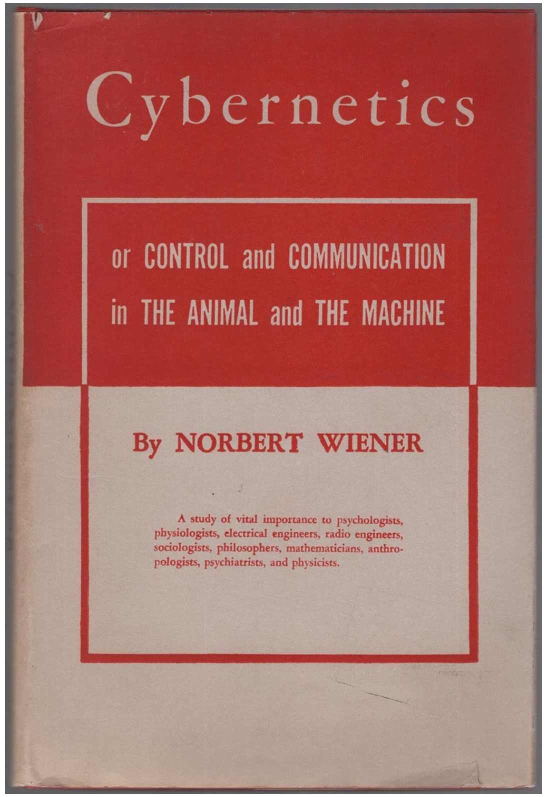
Control Engineering (Control Systems Engineering)

- Engineering discipline that applies control theory to design and develop systems with desired behaviors.
- This involves designing, analyzing, and optimizing control systems.

Basic concepts in control theory
Key Terms and Concepts

Cybernetics

- An interdisciplinary field that study universal principles of control and communication regardless of the specific nature of the system (biological, mechanical, social, etc.).
- Cybernetic concepts (like feedback and regulation) were formalized into *classical control theory*.
- Founder: Norbert Wiener (wrote a book *Cybernetics or Control and Communication in the Animal and the Machine* in 1948)



Basic concepts in control theory
Key Terms and Concepts

Automation, Automatic Control

- Area of science and technology focused on the use of technology to perform task without human intervention.
- Automation is linked to the development of almost every form of technology.
- Examples: industrial automation (robotics), home automation (home assistants), business automation (content management, document processing)

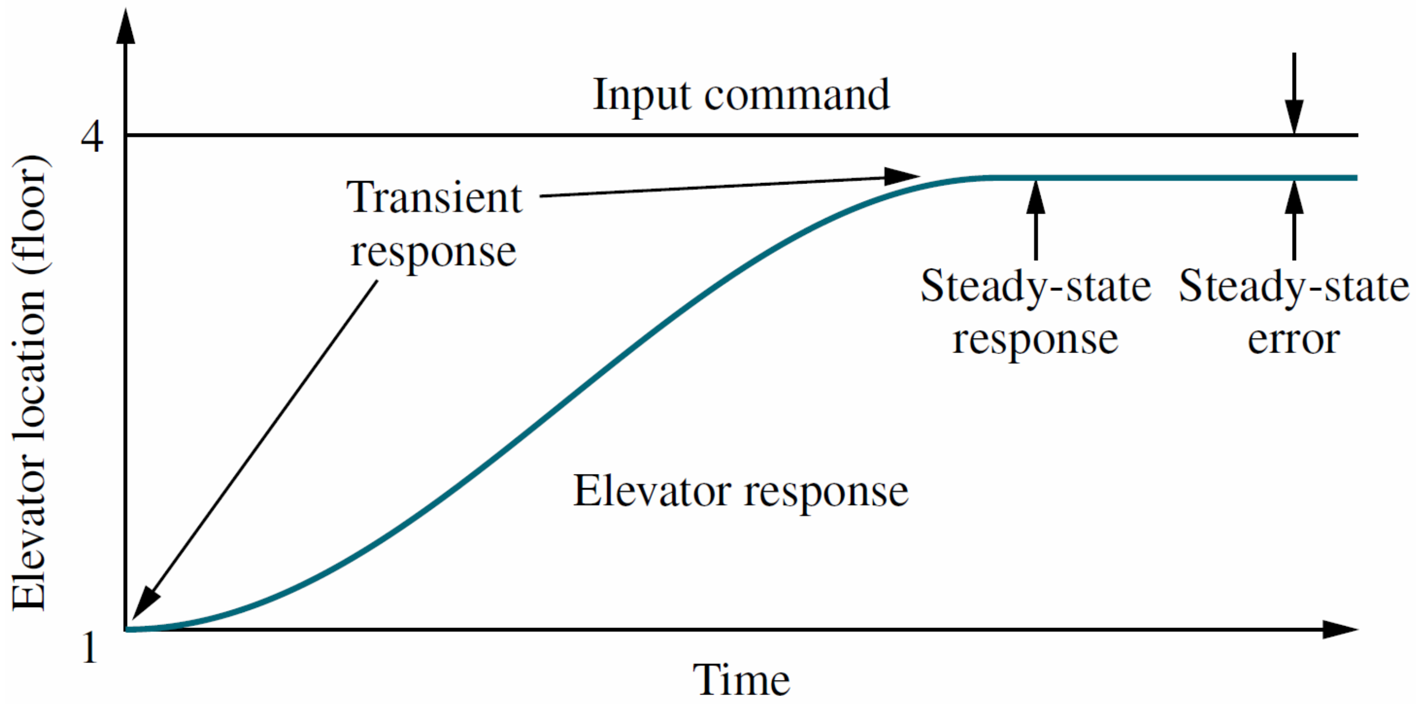
Basic concepts in control theory
Key Terms and Concepts

Actuator

- is a component that converts a control signal into a physical action or motion, effectively doing desired mechanical action in response to a command.
- The actuator is typically considered part of the plant (the physical system being controlled).
- Examples: electric motor, servo motor, stepper motor, solenoid valve, piezoelectric actuator

Basic concepts in control theory
Key Terms and Concepts

Steady-State and Transient Response



Basic concepts in control theory
Key Terms and Concepts

Steady-State and Transient Response

- When we apply an input to a system, the system doesn't instantly reach the final value.
- That reaction has two parts:
  - **Transient Response** – the initial reaction of the system as a function of time
  - **Steady-State Response** – the long-term behavior after the system has settled and stopped changing significantly
- **Steady-state error** is the difference between the desired output and the actual output of a system after it has settled (i.e., in steady state).

Course Syllabus

- **Basic concepts in control theory**
  - Key Terms and Concepts
  - **Basic Structure of a Control System**
- Basic system classification
- Basic system properties
- System Identification
- Basic types of controllers
- Control quality evaluation
- Control systems stability
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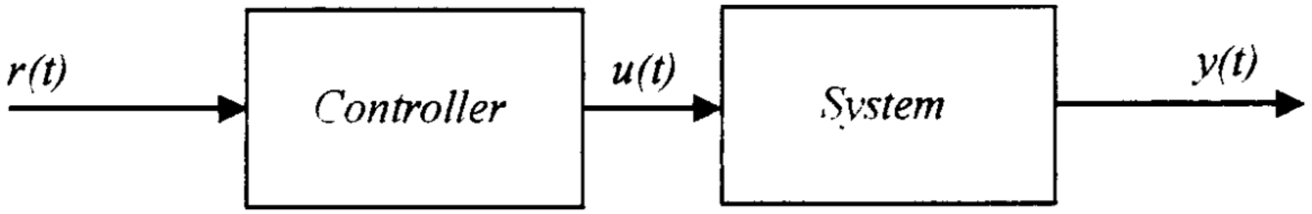
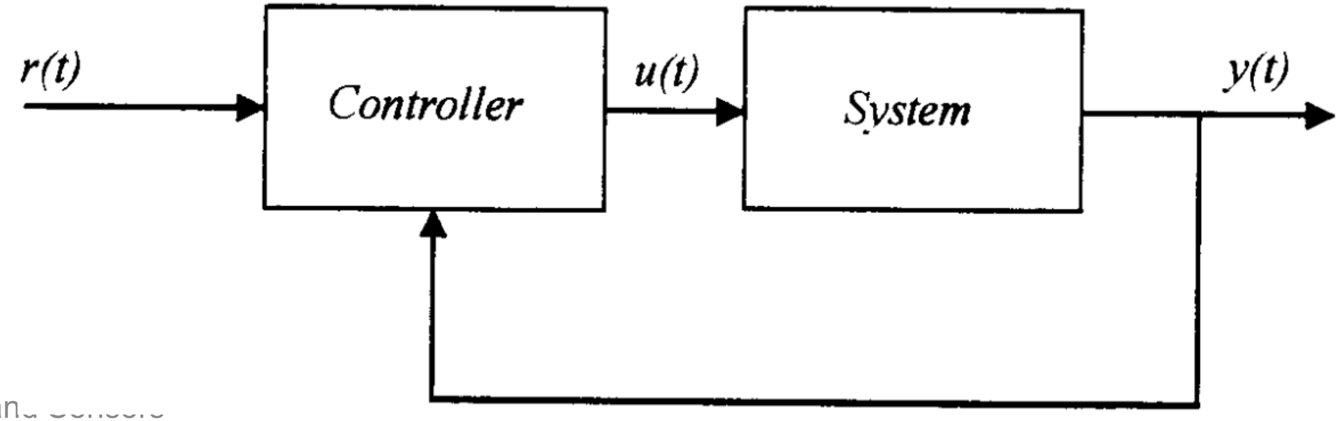
Basic concepts in control theory
Basic Structure of a Control System

- Control systems can be divided into two categories:
  - **Open-loop systems (without feedback)**



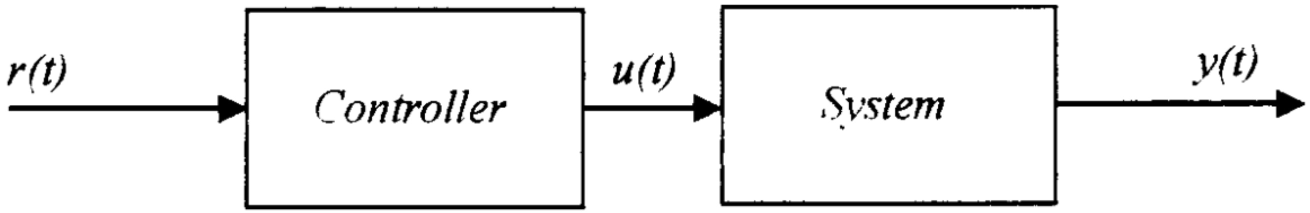
  - **Closed-loop systems (with feedback)**



Basic concepts in control theory

Basic Structure of a Control System

- **Open-loop systems (without feedback)**

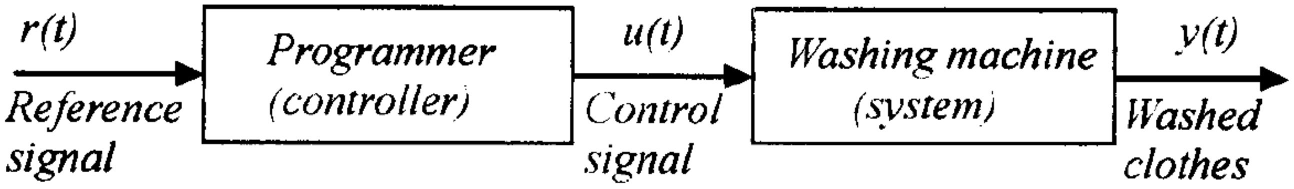


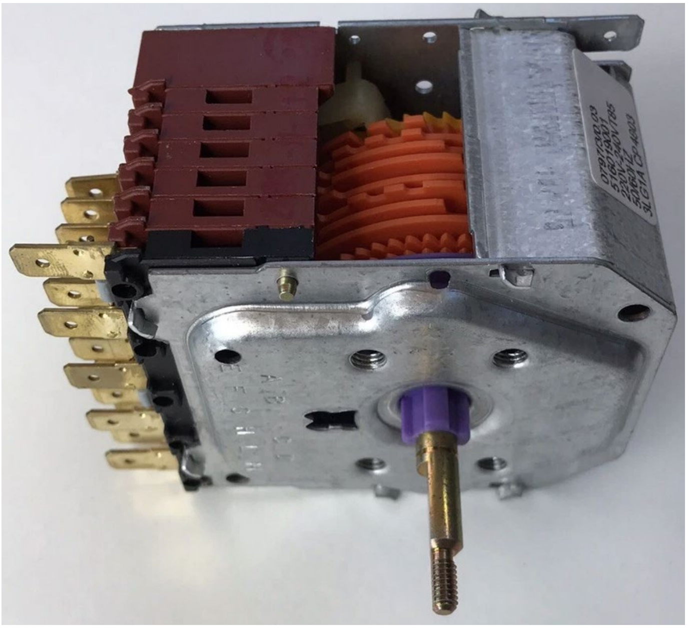
- r(t) – **Setpoint, Reference, Command signal** = the input to the Controller
- u(t) – **Control signal** = the input to the System
- y(t) – **Process** or **Controlled Variable, Desired Response**, **Output** = the output of the System

- The typical characteristic of an open-loop system is that it cannot compensate for system *model uncertainties* or any *disturbances* that add to the control signal or output.
- Disturbances are undesirable inputs (external or internal to the control system)
- Examples: Simple washing machines, Light switches, Toasters, Traffic lights

Basic concepts in control theory
Basic Structure of a Control System

- **Simple washing machine** (1970s)
  - Example of open-loop system
  - Core element: programmer/timer
  - Controls the sequence and duration of different wash cycles, including filling, washing, rinsing, and spinning.

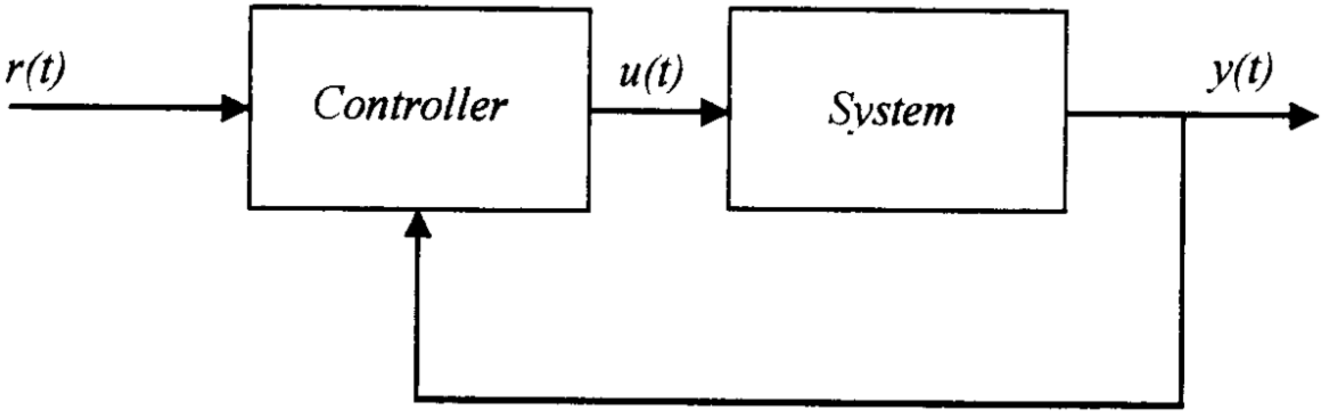




Washing Machine Timer (Programmer)

Basic concepts in control theory
Basic Structure of a Control System

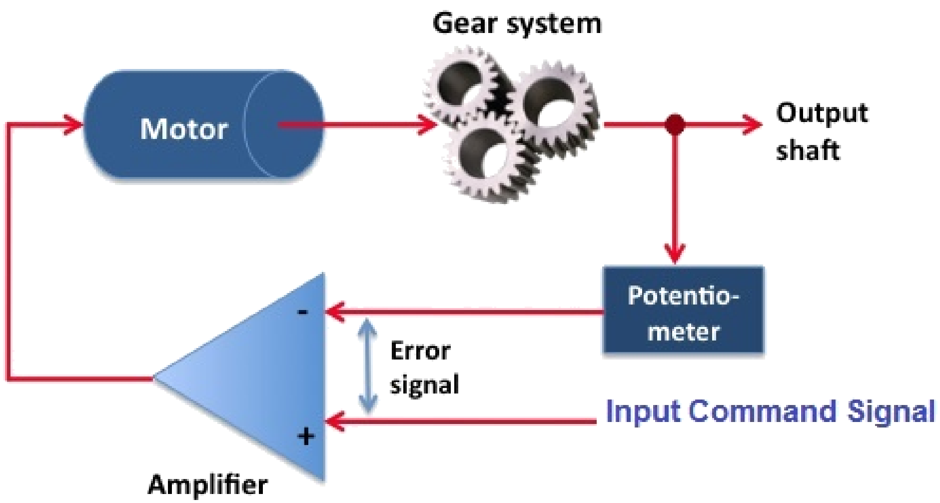
- **Closed-loop systems (with feedback)**



- A closed-loop system uses *feedback* to automatically compare the actual output with the desired value and adjust its actions to *minimize this error*.
- Key idea: The output y(t) affects the input u(t), i.e. u(t) is a function of y(t)
- Examples: Thermostat, Cruise Control in a Car, Servo Motor

Basic concepts in control theory
Basic Structure of a Control System

- **Servo Motor**
  - Example of closed-loop system
  - Uses position or speed feedback to produce motion in response to a command
  - Part of *servomechanism* – a closed-loop control system designed to accurately control position, speed, or torque of a mechanical system using feedback.



Basic concepts in control theory
Basic Structure of a Control System

- **Open-loop and Closed-loop system comparison**

	Open-loop systems	Closed-loop systems
Advantages	Simple design and implementation	High accuracy and precision
	Low cost (fewer components)	Automatically compensates for disturbances
	No need for sensors or feedback	Stable and reliable performance under varying conditions
	Fast response (no feedback delay)	Can control complex or nonlinear systems

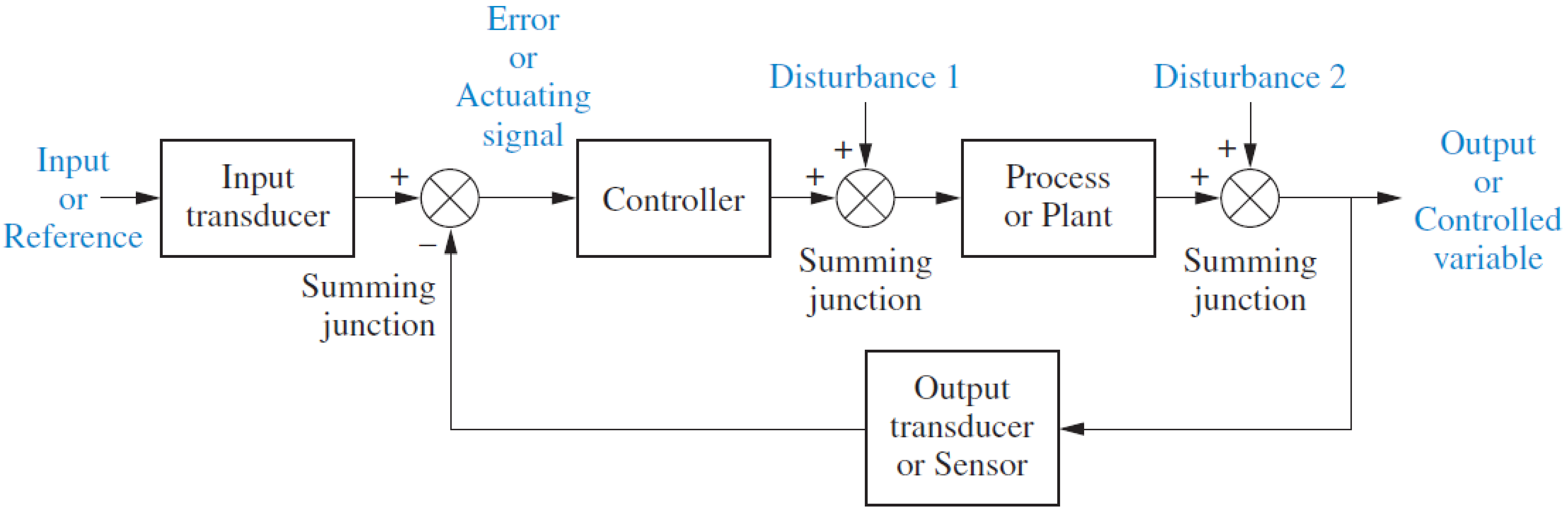
Basic concepts in control theory
Basic Structure of a Control System

- **Open-loop and Closed-loop system comparison**

	Open-loop systems	Closed-loop systems
Disadvantages	No automatic error correction	More complex design and tuning
	Poor accuracy if conditions change	Higher cost (sensors, controllers)
	Cannot handle disturbances	Risk of instability if not designed properly
	Limited to predictable, simple tasks	Slower response due to feedback loop

Basic concepts in control theory
Basic Structure of a Control System

- Closed-loop control system



Control Systems Engineering 7th Ed., p. 7

Basic concepts in control theory
Basic Structure of a Control System

- Closed-loop control system
  - A closed-loop system is a control system that uses **feedback** to compare the actual output $\boldsymbol{y(t)}$ (process/controlled variable) to the desired input $\boldsymbol{r(t)}$ (setpoint, reference/command signal) and adjusts its actions to minimize the error $\boldsymbol{e(t) = y(t) - r(t)}$.
  - **Input transducer** converts the form of the input $r(t)$ to that used by the controller.
  - **The controller** drives a plant with a control signal $u(t)$ based on the value of the error signal $e(t)$ so that this error is minimized.
  - **Disturbances** are added to the controller and process outputs via summing junctions, which yield the algebraic sum of their input signals using associated signs.
  - The output of the plant is converted by a **sensor** to the form that is used by the controller.
  - **Summing junctions** yield the algebraic sum of their input signals using associated signs.

Course Syllabus

- Basic concepts in control theory
- **Basic system classification**
- Basic system properties
- System Identification
- Basic types of controllers
- Control quality evaluation
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Basic system classification

- Linear / Nonlinear Control Systems
- Time Variant / Time Invariant Control Systems
- Causal / Noncausal Control Systems
- Deterministic / Stochastic Control Systems
- Continuous-time / Discrete-time Control Systems

- Significant portion of control theory, particularly classical control, is founded on **linear, time-invariant (LTI) systems** or those that can approximates as such.

Basic system classification
Linear Control Systems

- Necessary and sufficient condition for linearity is **superposition principle** characterized by two properties:
  - Additivity
    - If $x_1(t) \to y_1(t)$ and $x_2(t) \to y_2(t)$, then $x_1(t) + x_2(t) \to y_1(t) + y_2(t)$
  - Homogeneity (scaling)
    - If $x_1(t) \to y_1(t)$ and a is a constant, then $ax_1(t) \to ay_1(t)$

Basic system classification
Linear Control Systems

- In a linear system, when a sinusoidal input signal is applied, the output will also be sinusoidal at the **same frequency**, but with potentially **different amplitude and phase**.
- Linear systems are **mathematically well-behaved**, making them easier to analyze and design using tools from linear algebra and calculus:
  - Vectors, matrices, and linear transformations
  - Differentiation, integration, Taylor series expansion
- Linear mathematical operations:
  - Differentiation, integration, Fourier or Laplace transformation

Basic system classification

Time Invariant Control Systems

- Time-invariant systems are systems whose parameters do not change with time.
- A system is considered time-invariant when its output response to a given input does not depend on when that input is applied.
- If you shift the input signal $x(t)$ in time by T seconds, the output signal will also be shifted by the same amount, but its shape will remain the same.

If $x(t) \rightarrow y\,(t)$ then $x(t-T) \rightarrow y(t-T)$

Basic system classification
Time Variant Control Systems

- Most physical systems are time-varying due to aging, but within a certain time interval they can be considered time-invariant.
- Examples of time-variant systems:
  - missiles with varying fuel levels
  - aircraft flying through a wide range of altitudes, where the lift may change with altitude
  - robotic arm with changing payload, where the dynamics change as the robot picks up or drops objects
- Is the system time-invariant or time-variant?
  - $y(t) = 2 \cdot x\,(t) + x(t-1)$
  - $y(t) = t \cdot x\,(t) + x(t-1)$

Basic system classification
Causal Control Systems

- **Principle of Causality**
  - It's a fundamental concept in science.
  - Every event has a cause and that causes precede their effects.
- A system is **causal** if its output at any time t depends only on the present and past values of the input – not the future.
- All physical system **must be causal**.
- Example of noncausal system: $y(t) = x(t) + x(t + 1)$

Basic system classification
Deterministic Control Systems

- A **deterministic control system** is one in which the output is fully determined by the input and system dynamics – there's no randomness involved.
- Fully described by models – can use differential equations or transfer functions to model behavior exactly.
- No probabilistic behavior or random noise in system equations.

Basic system classification
Stochastic Control Systems

- **Stochastic control systems** involve random or uncertain elements, such as noise or unpredictable disturbances.
- Stochastic control methods use **probabilistic models** to describe the system's behavior and optimize the control input to achieve the desired outcome on average.
- Mathematical models use:
  - random variables, variance, standard deviation, probability density functions, stochastic differential equations

Basic system classification
Continuous-Time Control Systems

- **Continuous-Time Systems**, often referred to as analog systems, operate with input and output signals that are continuous functions of time.
- Time is treated as a continuous variable and signals are defined at every instant of time.
- We use **differential equations** to describe and design such systems.
- Many physical systems naturally exhibit continuous behavior. Their state (e.g., temperature, position, pressure) changes smoothly over time in response to inputs.
  - e.g. analog electrical circuits, motors, room heating and cooling, robotic arm

Basic system classification
Discrete-Time Control Systems

- A **discrete-time control system** is a control system where signals are processed, measured, and updated at specific time intervals, not continuously.
- Time progresses in steps and the system works with sequences of data rather than smooth curves.
- **Difference equations** are used to describe and design such systems.
- Examples:
  - digital thermostat, switching power supplies, digital motor drives, digital communication systems, robot controlled by microcontroller

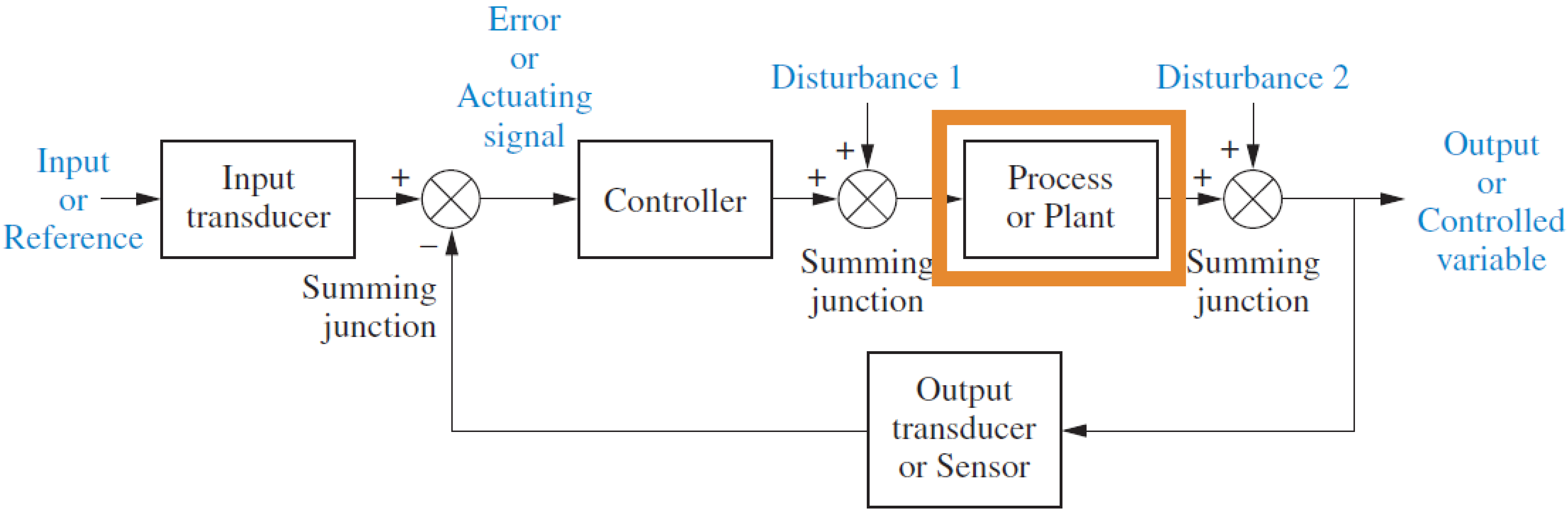
Basic system classification
Discrete-Time Control Systems

- A control system can be partly discrete and partly continuous, and it is then called a **hybrid system** or **sampled-data system**, but it is still discrete-time system.
- They require Analog-to-Digital (A/D) and Digital-to-Analog (D/A) converters to transform continuous-time signals into discrete-time signals and back.
- Control systems designed in the continuous domain are often **discretized** (e.g., using zero-order hold) to approximate the continuous plant for digital simulation or design – then the system is still discrete-time control system.

Course Syllabus

- Basic concepts in control theory
- Basic system classification
- **Basic system properties**
  - **Static Characteristics**
  - Dynamic Characteristics
- System Identification
- Basic types of controllers
- Control quality evaluation
- Control systems stability
- Controller design methods
- Digital control
- Sensors

Basic system properties



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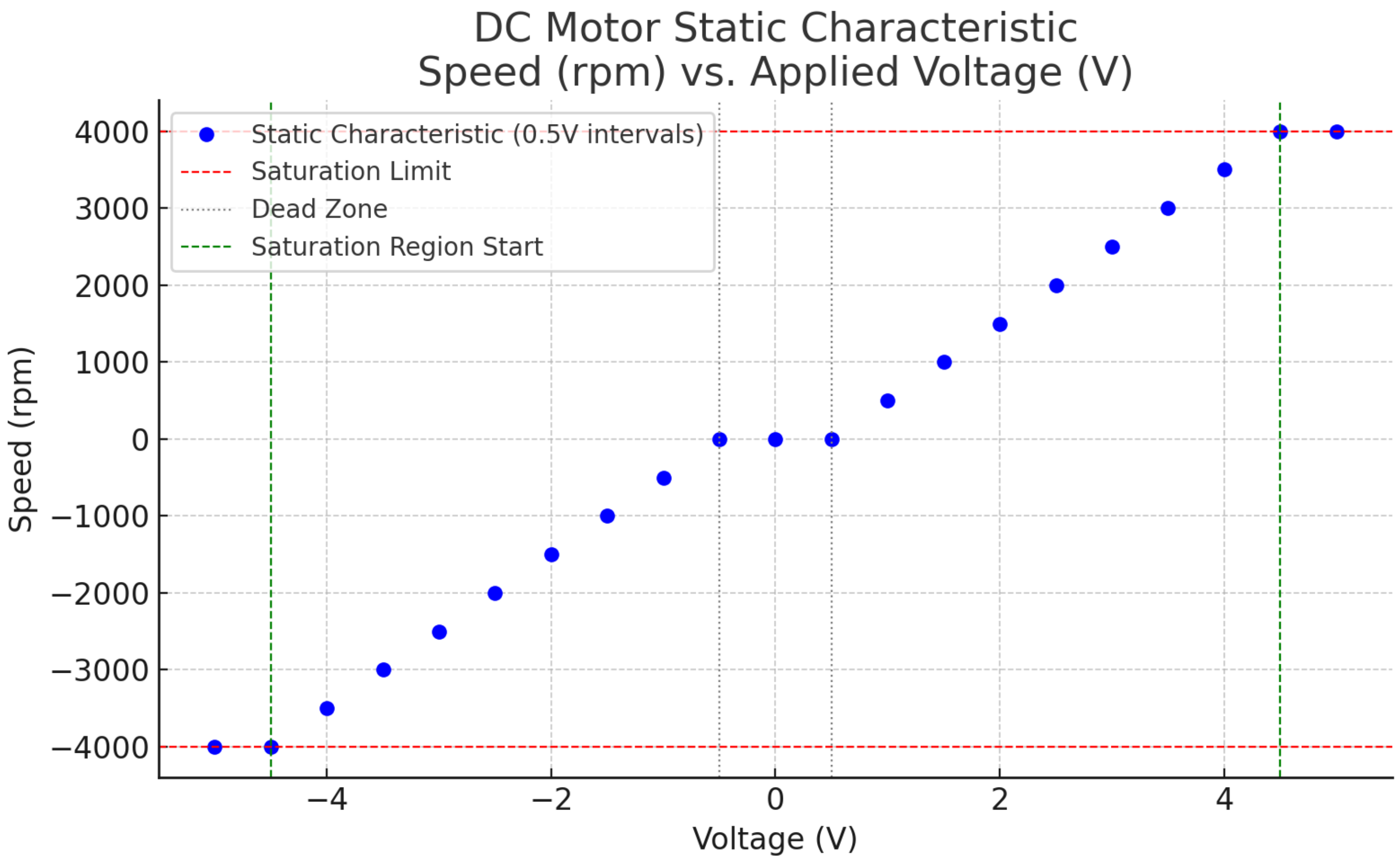
Basic system properties
Static and Dynamic Characteristics

- **Static characteristics** describe a system’s behavior under steady-state conditions – when the input and output are not changing over time.
- **Dynamic characteristics** describe how the system responds to changing inputs over time.

Basic system properties
Static Characteristics

- **Static characteristics** describe a system's behavior under steady-state conditions
- Static characteristics define the relationship between the steady-state output y and the input x with **algebraic equation** expressed as $y = f(x)$.
- Static characteristic can be:
  - **linear** $y = a \cdot x + b$ (straight line)
  - **nonlinear** $y = a \cdot x^2 + b \cdot x + c$, $y = sin(x)$, etc. (arbitrary curve)

Basic system properties
Static Characteristics

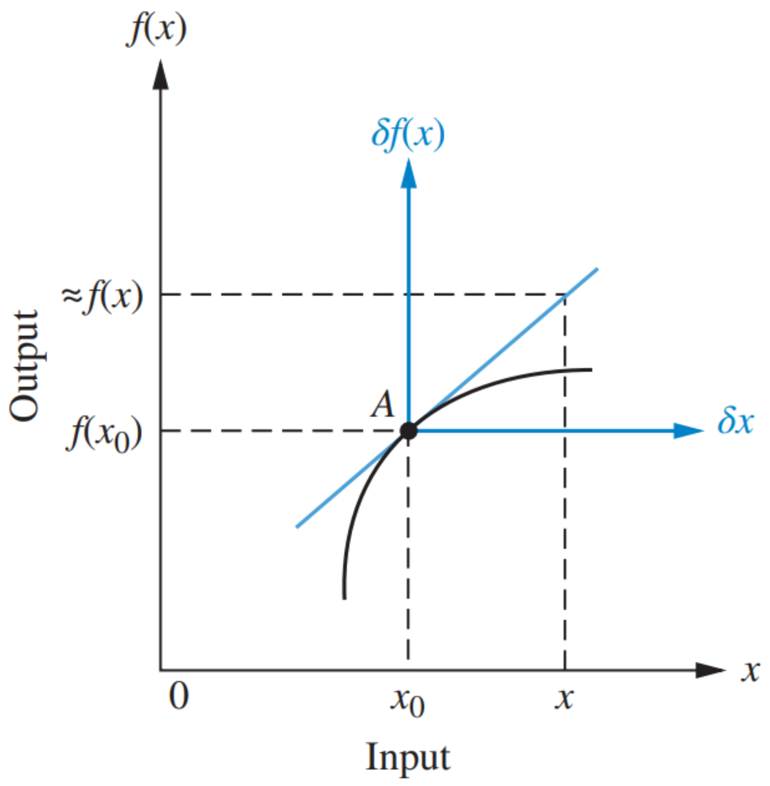


Basic system properties
Static Characteristics

- Most of the physical systems have **nonlinear static characteristics**.
- Nonlinear equations are difficult to solve, analytical solution is often impossible, must use numerical methods.
- Nonlinear equations are commonly approximated by linear equations – this is known as **linearization** around the operating point.

Basic system properties
Static Characteristics

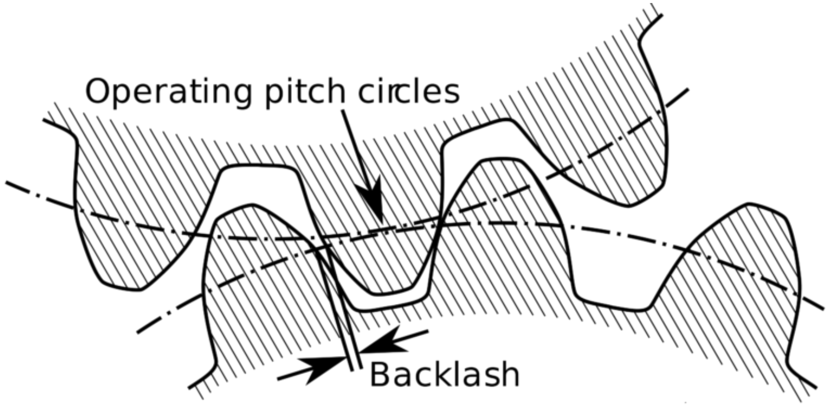
- Linearization of nonlinear static characteristic:
  - using the first order **Taylor series expansion** around the operating point x_0
  - we assume small variations around this point



$$T_1(x) = \sum_{k=0}^{1} \frac{f^{(k)}(a)}{k!}(x-a)^k = f(a) + f'(a)(x-a)$$

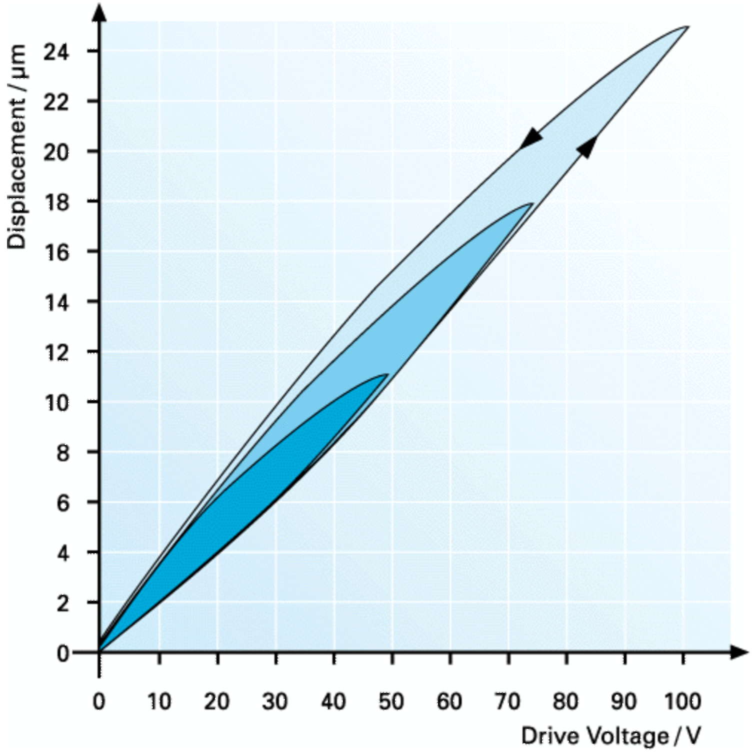
Basic system properties
Static Characteristics

- Common Nonlinearities in Static Characteristics:
  - **Dead zone** – No output for small inputs
  - **Saturation** – Output stops increasing
  - **Hysteresis** – Output depends on input history
  - **Backlash** – Slack in mechanical response



Backlash in gears

Piezo actuator



Hysteresis curves of a piezo actuator

Course Syllabus

- Basic concepts in control theory
- Basic system classification
- **Basic system properties**
  - Static Characteristics
  - **Dynamic Characteristics**
- System Identification
- Basic types of controllers
- Control quality evaluation
- Control systems stability
- Controller design methods
- Digital control
- Sensors

Basic system properties
Dynamic Characteristics

- **Dynamic characteristics** describe how the system responds to changing inputs over time.
- For continuous-time systems, dynamic characteristics are expressed using linear **Ordinary Differential Equations** (ODEs) with constant coefficients:

$$a_n y^{(n)} + a_{n-1} y^{(n-1)} + \cdots + a_1 y' + a_0 y =$$

$$b_m u^{(m)} + b_{m-1} u^{(m-1)} + \cdots + b_1 u' + b_0 u$$

- where $y(t)$ is the output, $u(t)$ is the input, a_n, b_m are real constants
- Systems modeled by these equations are known as **linear time-invariant (LTI) systems**.

Basic system properties
Dynamic Characteristics

- For causal physical system $\boldsymbol{m} \leq \boldsymbol{n}$ (input's derivatives cannot be higher than the output's derivatives).

$$a_n y^{(n)} + a_{n-1} y^{(n-1)} + \cdots + a_1 y' + a_0 y$$
$$= b_m u^{(m)} + b_{m-1} u^{(m-1)} + \cdots + b_1 u' + b_0 u$$

- To find unique solution for ODEs you need to specify **initial conditions**:
  - we need information about $y(t)$ and its $n - 1$ derivatives at the initial time, e.g. at $t = 0$:

$$y(0), y^{(1)}(0), y^{(2)}(0), \dots, y^{(n-1)}(0)$$

Basic system properties
Dynamic Characteristics

- Solution of a ODE has two components: **particular solution** $y_p(t)$ and **homogeneous solution** $y_h(t)$

$$y(t) = y_p(t) + y_h(t)$$

- **Homogeneous solution** (natural response of the system) is the solution with the input set to zero, that is $u(t) = 0$.

$$a_n y^{(n)} + a_{n-1} y^{(n-1)} + \cdots + a_1 y' + a_0 y = 0$$

- **Particular solution** (forced response of the system) is the solution of the same form as the input.

Basic system properties
Dynamic Characteristics

- Dynamic characteristics described by the ODEs are used for the description, analysis, and design of control systems.
- The process of building **mathematical models** from observed input-output data of a dynamic system is called **system identification**.
- We are interested in how the output quantity depends on the input quantity.
- System identification can be approach in various ways:
  - **Analytically** (Physical modeling)
  - **Experimentally** (Experimental modeling)

Last Lecture Overview
Introduction to Control Systems

- **Why Control Systems?**
  - Found in everyday technology: appliances, vehicles, robotics, biomedical devices, aerospace.
  - Goal: Ensure system outputs follow desired values and behaviors despite disturbances.
- **Key Concepts**
  - **System (Plant)**: Collection of components to be controlled (e.g., car engine, heating system).
  - **Dynamical Systems**: Behavior changes with time; total response = natural + forced.
  - **Control Theory**: Uses feedback to guide systems toward stability and desired outcomes.
  - **Types of Control Approaches**:
    - *Classical* (frequency-domain, transfer functions, SISO)
    - *Modern* (state-space, MIMO, nonlinear)
    - *Intelligent* (AI, adaptive, fuzzy, neural networks)

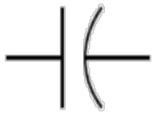
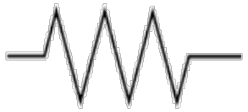
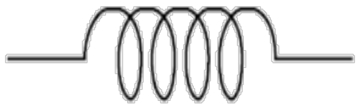
Last Lecture Overview
Control System Structures & Properties

- **Open-loop Systems** (no feedback)
  - Simple, low cost, fast but cannot handle disturbances.
  - Example: old washing machine.
- **Closed-loop Systems** (feedback)
  - Uses sensors + feedback to minimize error.
  - High accuracy, disturbance rejection, stability.
  - Example: thermostat, cruise control, servo motor.
- **System Classifications**
  - Linear vs. Nonlinear
  - Time-invariant vs. Time-variant
  - Causal vs. Non-causal
  - Deterministic vs. Stochastic
  - Continuous-time vs. Discrete-time
- **System Properties**
  - *Static*: Steady-state input-output relation (linear/nonlinear, linearization, nonlinearities like hysteresis/backlash).
  - *Dynamic*: Time-dependent behavior, described by ODEs (natural + forced response).

Basic system properties

Dynamic Characteristics – Analytical System Identification Example

- Voltage, current, and charge relationships for resistors, capacitors, and inductors

Component	Voltage-current	Current-voltage	Voltage-charge
 Capacitor	$v(t) = \frac{1}{C}\int_0^t i(\tau)d\tau$	$i(t) = C\frac{dv(t)}{dt}$	$v(t) = \frac{1}{C}q(t)$
 Resistor	$v(t) = Ri(t)$	$i(t) = \frac{1}{R}v(t)$	$v(t) = R\frac{dq(t)}{dt}$
 Inductor	$v(t) = L\frac{di(t)}{dt}$	$i(t) = \frac{1}{L}\int_0^t v(\tau)d\tau$	$v(t) = L\frac{d^2q(t)}{dt^2}$

Basic system properties

Dynamic Characteristics – Analytical System Identification Example

- Voltage, current, and charge relationships for resistors, capacitors, and inductors
  - The relationship between current and voltage in a capacitor is derived from the fundamental **definition of capacitance** $q(t) = Cv(t)$ and the definition of **electric current** $i(t) = dq(t)/dt$.
  - The resistor current-voltage relationship is **Ohm's Law**, derived from experimental observations, which states that the voltage across a resistor is directly proportional to the current flowing through it, with the resistance R being the constant of proportionality, expressed as $v(t) = Ri(t)$.
  - The fundamental relationship for an inductor $v(t) = L\frac{di(t)}{dt}$ is derived from **Faraday's Law of Induction**, which states that the induced voltage in any closed circuit is equal to the negative of the time rate of change of the magnetic flux through the circuit $v_{induced}(t) = -d\Phi/dt$.
  - The negative sign indicates that the induced voltage opposes the change in flux (**Lenz's Law**).
  - In circuit convention, we define terminal voltage across the inductor such that this minus cancels $v(t) = -v_{induced}(t)$.
  - For an inductor, the magnetic flux Φ it produces is directly proportional to the current $i(t)$ flowing through it $\Phi = Li(t)$, here, L is the **inductance**, a constant value dependent on the inductor's physical properties.
  - Combining the Laws we get the fundamental relationship for an inductor as $v(t) = -v_{induced}(t) = L\frac{di(t)}{dt}$.

Basic system properties

Dynamic Characteristics – Analytical System Identification Example

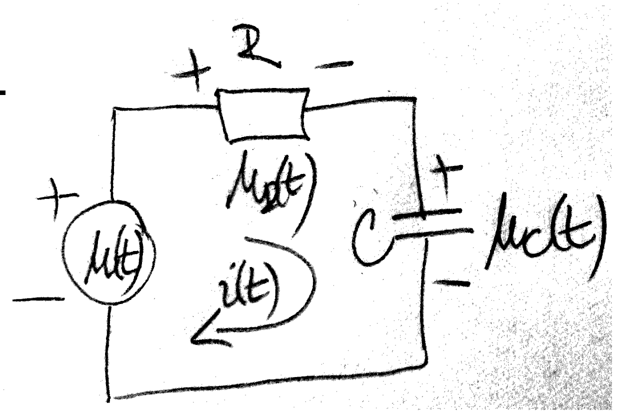
- Derivation of the differential equation of an ideal RC circuit
- Applying Kirchhoff's Voltage Law $u_R(t) + u_C(t) = u(t)$

$$Ri(t) + \frac{1}{C}\int_0^t i(\tau)d\tau = u(t)$$

- We want capacitor voltage $u_C(t)$ to be the output quantity

$$u_C(t) = \frac{1}{C}\int_0^t i(\tau)d\tau \rightarrow i(t) = C\frac{du_C(t)}{dt}$$

$$RC\frac{du_C(t)}{dt} + u_C(t) = u(t)$$



Basic system properties

Dynamic Characteristics – Analytical System Identification Example

- **Kirchhoff's Voltage Law** (KVL) – states that the algebraic sum of all the voltage differences (voltage drops) around any closed loop in a circuit is zero.
- This law is a fundamental principle in circuit analysis, based on the **conservation of energy**.
- **A closed loop** is a path in a circuit that starts and ends at the same point, without leaving the circuit.
- In a closed-loop electrical circuit, conventional **current flows** from the positive terminal to the negative terminal of the power source, forming a complete path for the flow of electric charge.
- When traversing a loop, a voltage rise (e.g., from a voltage source) is typically considered positive, while a voltage drop (e.g., across a resistor or capacitor) is considered negative.
- So, applying Kirchhoff's Voltage Law we get $u(t) - u_R(t) - u_C(t) = 0$, rearranging we get $u_R(t) + u_C(t) = u(t)$

Basic system properties

Dynamic Characteristics – Analytical System Identification Example

- Derivation of the differential equation of an ideal RC circuit, where $u_R(t)$ is the output quantity.

$$Ri(t) + \frac{1}{C}\int_0^t i(\tau)d\tau = u(t)$$

- Resistor voltage $u_R(t) = Ri(t) \rightarrow i(t) = \frac{u_R(t)}{R}$

$$u_R(t) + \frac{1}{C}\int_0^t \frac{u_R(\tau)}{R}d\tau = u(t)$$

$$\frac{du_R(t)}{dt} + \frac{1}{RC}u_R(t) = \frac{du(t)}{dt}$$

Basic system properties
Dynamic Characteristics

- **Ordinary Differential Equations** (ODEs)
- (Laplace) **Transfer Function**
  - is the ratio of the Laplace transform of a linear, time-invariant (LTI) system's output to the Laplace transform of its input, assuming zero initial conditions
- **Impulse Response**
  - is the system's output when its input is a unit impulse function (Dirac delta function), assuming zero initial conditions
- **Step Response**
  - is the system's output when subjected to a unit step input (Heaviside function)
- **Frequency Response** / Fourier Transfer Function / Frequency Transfer Function
  - is the ratio of the Fourier Transform of the output signal to the Fourier Transform of the input signal, assuming zero initial conditions

Basic system properties

Dynamic Characteristics – Fourier Transform

- Fourier Transform is a linear integral transform with kernel $e^{-j\omega t}$
- **Forward Fourier Transform**:

$$\mathcal{F}\{f(t)\} = F(j\omega) = \int_{-\infty}^{\infty} f(t)e^{-j\omega t}dt$$

- **Inverse Fourier Transform**:

$$\mathcal{F}^{-1}\{F(j\omega)\} = f(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} F(j\omega)e^{j\omega t}d\omega$$

- where $f(t)$ is time domain signal, $F(j\omega)$ is spectrum (complex-valued function of frequency), $e^{-j\omega t}$ is the kernel of the transform, ω is angular frequency in radians ($\omega = 2\pi f$, where f is the frequency in Hz)

Basic system properties
Dynamic Characteristics – Fourier Transform

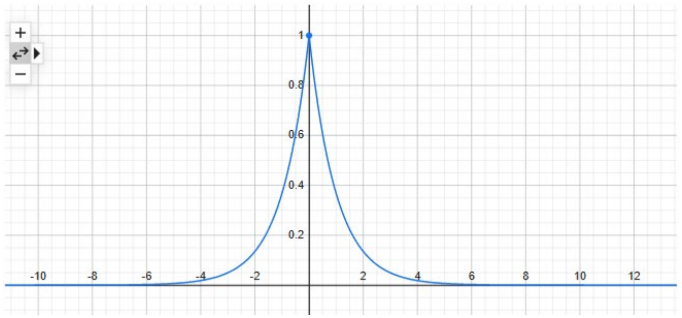
- Converts a time-domain signal into a frequency-domain representation.
- Useful for analyzing signals and systems in terms of sinusoids.
- Function $f(t)$ must satisfy some conditions in order to have a Fourier transform (Dirichlet's conditions):
  - **Absolutely integrable condition** $\int_{-\infty}^{\infty} |f(t)| dt < \infty$
  - **Piecewise continuous condition** – it must have at most a finite number of maxima, minima, and discontinuities in any finite interval.

Basic system properties

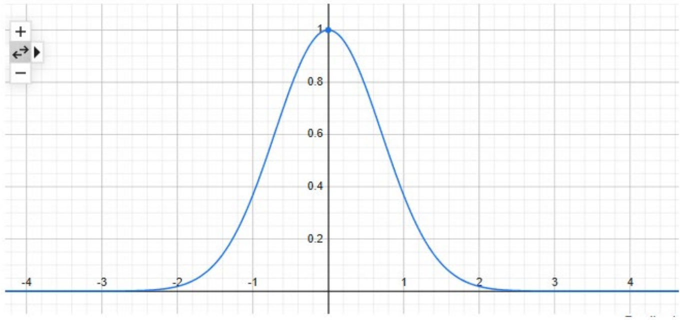
Dynamic Characteristics – Fourier Transform

- **Absolutely integrable condition** $\int_{-\infty}^{\infty} |f(t)| dt < \infty$
  - Functions that decay to zero fast enough (like exponentials, Gaussians, pulses) are absolutely integrable.

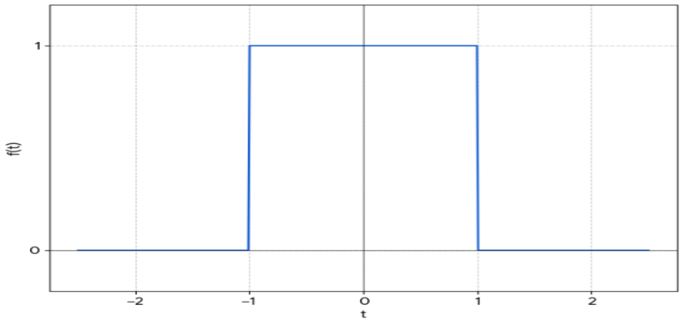
$$f(t) = e^{-|t|}$$



$$f(t) = e^{-t^2}$$



$$f(t) = \begin{cases} 1 & |t| \leq 1 \\ 0 & otherwise \end{cases}$$

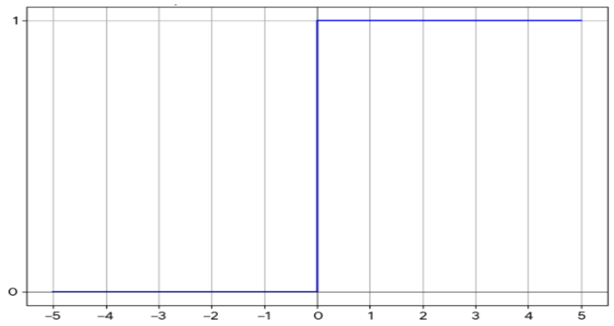


Basic system properties

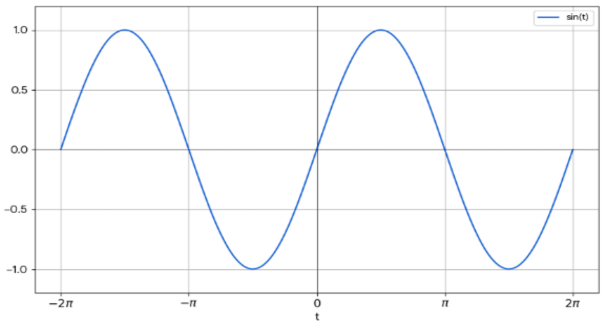
Dynamic Characteristics – Fourier Transform

- **Absolutely integrable condition** $\int_{-\infty}^{\infty} |f(t)|dt < \infty$
  - Functions that stay non-decaying (constants, sinusoids, polynomials) are not absolutely integrable.

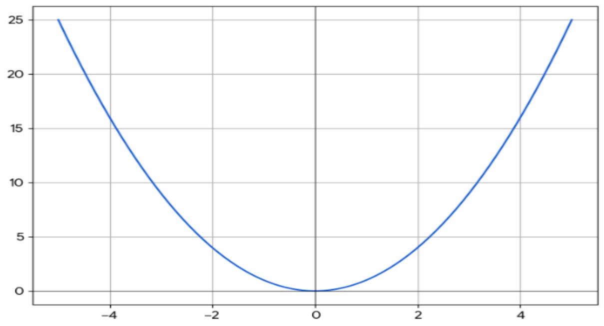
$$f(t) = \begin{cases} 0 & t < 0 \\ 1 & t \geq 0 \end{cases}$$



$$f(t) = sin(t)$$

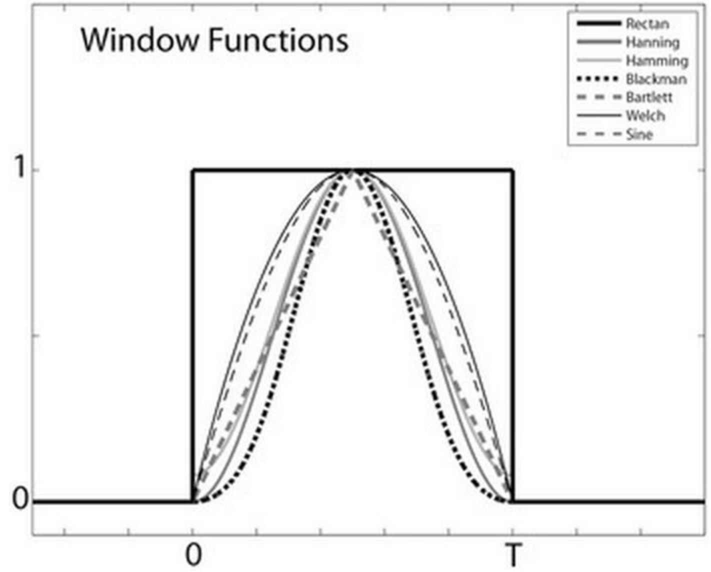


$$f(t) = t^2$$



Basic system properties
Dynamic Characteristics – Fourier Transform

- **Absolutely integrable condition** $\int_{-\infty}^{\infty}|f(t)|dt < \infty$
  - We can still use classic integral definition of the Fourier transform applied to some of not absolutely integrable functions by using **windowing**.
  - Multiplying $f(t)$ by a special *window function* that is non-zero for a finite period and zero everywhere else, will create a new, non-periodic signal that is now absolutely integrable.
  - Window functions:
    - Hanning $w(t) = 0.5 - 0.5 \cdot \cos\left(2\pi \frac{t}{T}\right), 0 \leq t \leq T$
    - Hamming $w(t) = 0.54 - 0.46 \cdot \cos\left(2\pi \frac{t}{T}\right), 0 \leq t \leq T$
    - Blackman $w(t) = 0.42 - 0.5 \cdot \cos\left(2\pi \frac{t}{T-1}\right) + 0.08 \cdot \cos\left(4\pi \frac{t}{T-1}\right), 0 \leq t \leq T$



Basic system properties
Dynamic Characteristics – Laplace Transform

- Classic integral definition of the Fourier transform:

$$\int_{-\infty}^{\infty} f(t)e^{-j\omega t}dt$$

- We introduce a dampening factor (a decaying exponential $e^{-\sigma t}$) and change lower limit of integral to 0 (assuming casual system where response begins at $t = 0$ after an input is applied)

$$\int_{0}^{\infty} f(t)e^{-\sigma t}e^{-j\omega t}dt$$

- We can group the real dampening factor σ and the imaginary frequency term $j\omega$ into a single complex variable $s = \sigma + j\omega$

$$\int_{0}^{\infty} f(t)e^{-(\sigma+j\omega)t}dt \rightarrow \int_{0}^{\infty} f(t)e^{-st}dt$$

Basic system properties
Dynamic Characteristics – Laplace Transform

- **The standard definition of the Laplace Transform**:

$$\mathcal{L}\{f(t)\} = F(s) = \int_0^\infty f(t)e^{-st}dt$$

- **Inverse Laplace Transform**:

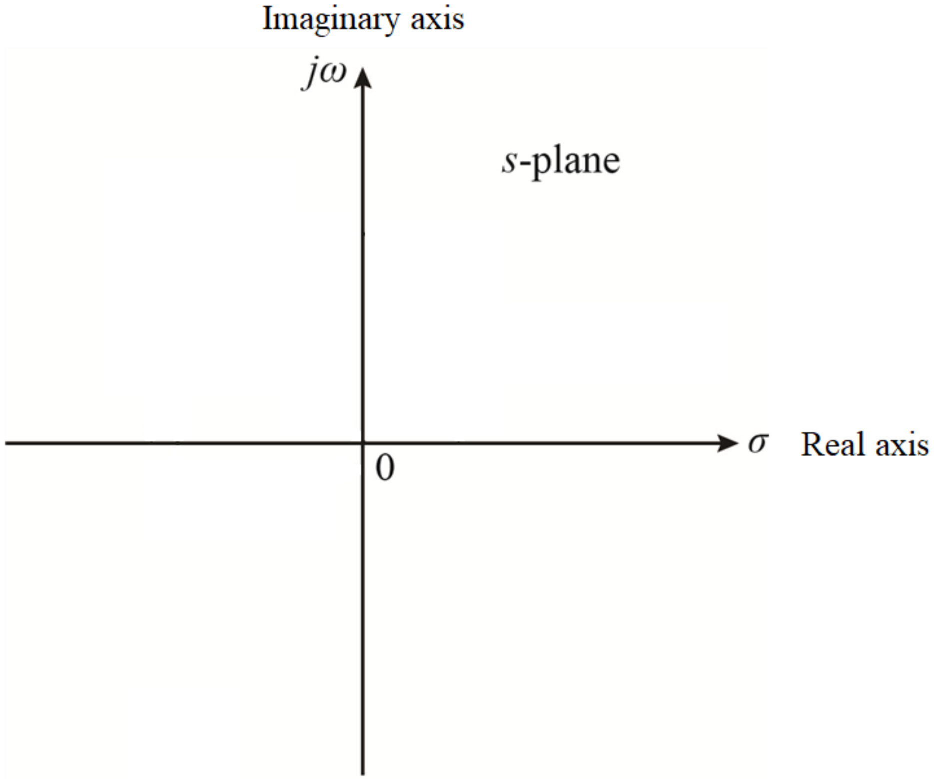
$$\mathcal{L}^{-1}\{F(s)\} = f(t) = \frac{1}{2\pi j}\int_{\sigma-j\infty}^{\sigma+j\infty} F(s)e^{st}ds$$

- where $f(t)$ is time domain signal, $F(s)$ is complex-valued function of complex frequency, $s = \sigma + j\omega$ is a complex number, often called complex frequency, ω is angular frequency, σ is a real number

Basic system properties
Dynamic Characteristics – Laplace Transform

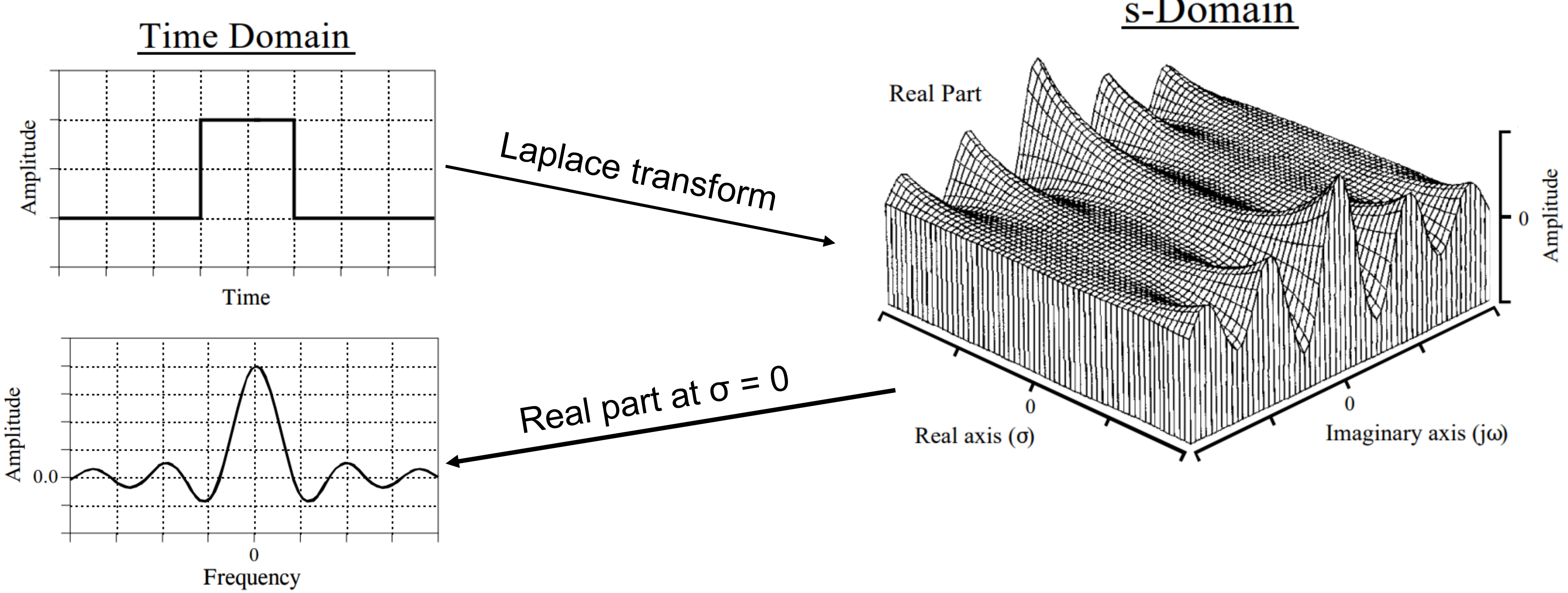
- Laplace frequency domain is just a complex plane (s-domain, s-plane)

$$s = \sigma + j\omega$$



Basic system properties
Dynamic Characteristics – Laplace Transform

- The rectangular pulse in s-domain



Basic system properties
Dynamic Characteristics – Laplace Transform

- For the integral to converge $f(t)$ must satisfy the following conditions:
  - **Exponential order condition** – there must exist constants $M > 0$, $a \in \mathbb{R}$ such that
  $$|f(t)| \leq Me^{at}, \text{ for all } t \geq 0$$
  - **Piecewise continuity on every finite interval** – $f(t)$ must not have infinitely many discontinuities in any finite interval.

Basic system properties
Dynamic Characteristics – Laplace Transform

- **Properties and Theorems of the Laplace Transform**
  - Linearity – if $F_1(s) = \mathcal{L}\{f_1(t)\}$, $F_2(s) = \mathcal{L}\{f_2(t)\}$, c_1 and c_2 are constants, then

$$\mathcal{L}\{c_1 f_1(t) + c_2 f_2(t)\} = c_1 F_1(s) + c_2 F_2(s)$$

  - Laplace transform of the derivative of a function (Differentiation theorem)

$$\mathcal{L}\left\{\frac{df(t)}{dt}\right\} = sF(s) - f(0)$$

$$\mathcal{L}\left\{\frac{d^2 f(t)}{dt^2}\right\} = s^2 F(s) - sf(0) - f^{(1)}(0)$$

Basic system properties

Dynamic Characteristics – Laplace Transform

- **Properties and Theorems of the Laplace Transform**
  - The Laplace Transform of the Integral of a Function (Integration theorem)

$$\mathcal{L}\left\{\int_0^t f(\tau)d\tau\right\} = \frac{1}{s}F(s)$$

  - Initial Value Theorem

$$\lim_{t\to 0} f(t) = \lim_{s\to\infty} sF(s)$$

  - Final Value Theorem

$$\lim_{t\to\infty} f(t) = \lim_{s\to 0} sF(s)$$

Basic system properties
Dynamic Characteristics – Laplace Transform

- **Table of Laplace Transforms**

$f(t) = \mathcal{L}^{-1}\{F(s)\}$	$F(s) = \mathcal{L}\{f(t)\}$
1. 1	$\frac{1}{s}$
2. $\mathbf{e}^{at}$	$\frac{1}{s-a}$
3. $t^n, \quad n = 1, 2, 3, \ldots$	$\frac{n!}{s^{n+1}}$
4. $t^p, p > -1$	$\frac{\Gamma(p+1)}{s^{p+1}}$
5. $\sqrt{t}$	$\frac{\sqrt{\pi}}{2s^{\frac{3}{2}}}$
6. $t^{n-\frac{1}{2}}, \quad n = 1, 2, 3, \ldots$	$\frac{1 \cdot 3 \cdot 5 \cdots (2n-1)\sqrt{\pi}}{2^n s^{n+\frac{1}{2}}}$
7. $\sin(at)$	$\frac{a}{s^2+a^2}$
8. $\cos(at)$	$\frac{s}{s^2+a^2}$
9. $t\sin(at)$	$\frac{2as}{(s^2+a^2)^2}$
10. $t\cos(at)$	$\frac{s^2-a^2}{(s^2+a^2)^2}$
11. $\sin(at) - at\cos(at)$	$\frac{2a^3}{(s^2+a^2)^2}$
12. $\sin(at) + at\cos(at)$	$\frac{2as^2}{(s^2+a^2)^2}$
13. $\cos(at) - at\sin(at)$	$\frac{s(s^2-a^2)}{(s^2+a^2)^2}$
14. $\cos(at) + at\sin(at)$	$\frac{s(s^2+3a^2)}{(s^2+a^2)^2}$
15. $\sin(at+b)$	$\frac{s\sin(b) + a\cos(b)}{s^2+a^2}$
16. $\cos(at+b)$	$\frac{s\cos(b) - a\sin(b)}{s^2+a^2}$

Basic system properties

Dynamic Characteristics – Laplace Transform

- **Laplace Transform Example**
  - Differential equation of an RC circuit

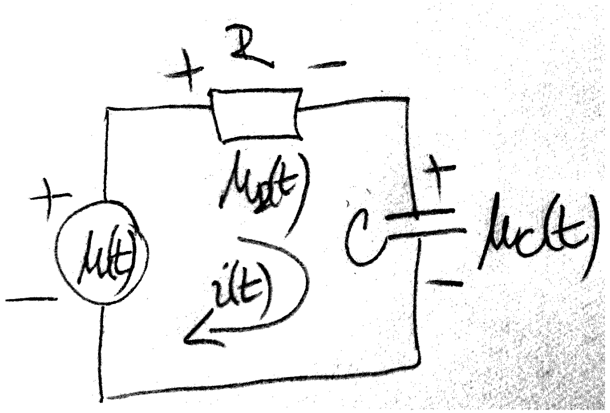
$$RC\frac{du_C(t)}{dt} + u_C(t) = u(t)$$

  - Apply Laplace transform, using differentiation theorem ($u_C(0) = 0$)

$$RCsU_C(s) + U_C(s) = U(s)$$

$$(RCs + 1)U_C(s) = U(s)$$

$$U_C(s) = \frac{1}{RCs + 1}U(s)$$



Basic system properties
Dynamic Characteristics – Laplace Transform

- **Laplace Transform Example**
  - Apply a step input $u(t) = U_0 h(t)$, where U_0 is a step voltage, $h(t)$ is a Heaviside function and assuming $\mathcal{L}\{h(t)\} = 1/s$

$$U_C(s) = \frac{1}{RCs + 1} U(s) = \frac{U_0}{s(RCs + 1)}$$

  - Do a partial fraction decomposition

$$U_C(s) = \frac{U_0}{s(RCs + 1)} = \frac{A}{s} + \frac{B}{RCs + 1} = \frac{U_0}{s} + \frac{-RCU_0}{RCs + 1} = U_0 \frac{1}{s} - U_0 \frac{1}{s - (-\frac{1}{RC})}$$

  - Apply inverse Laplace transform assuming $\mathcal{L}\{e^{at}\} = \frac{1}{s-a}$ and $\mathcal{L}\{h(t)\} = 1/s$

$$u_C(t) = U_0 - U_0 e^{-\frac{1}{RC}t}$$

Basic system properties
Dynamic Characteristics – Laplace Transform

- **Laplace Transform Example**
  - Step response of RC circuit with $U_0 = 5\,V$, $R = 1\,k\Omega$, $C = 1\,mF$
  - What will be the value of the response at time $t = 0\,s$? Use the Initial Value Theorem

$$\lim_{t\to 0} f(t) = \lim_{s\to\infty} sF(s) = \lim_{s\to\infty} s\frac{U_0}{s(RCs+1)} = 0$$

  - What will be the steady state value of the response (at time $t \to \infty$) ? Use the Final Value Theorem

$$\lim_{t\to\infty} f(t) = \lim_{s\to 0} sF(s) = \lim_{s\to 0} s\frac{U_0}{s(RCs+1)} = U_0$$

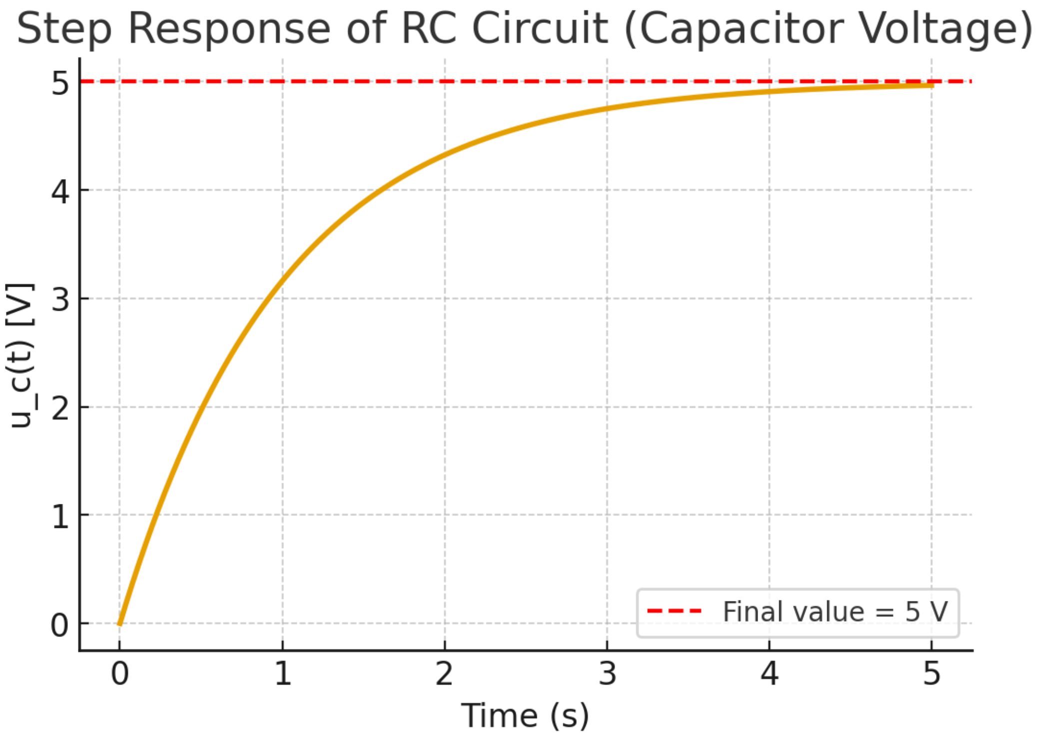
Basic system properties

Dynamic Characteristics – Laplace Transform

- **Laplace Transform Example**
  - Step response of RC circuit with $U_0 = 5\,V$, $R = 1\,k\Omega$, $C = 1\,mF$

$$u_C(t) = U_0 - U_0 e^{-\frac{1}{RC}t}$$

$$u_C(t) = 5 - 5e^{-t}$$



Basic system properties
Dynamic Characteristics – Laplace Transform

- **Laplace Transform MATLAB Example**
  - Inverse Laplace transform of $U_C(s) = \frac{U_0}{s(RCs+1)}$

```
clear all; % clears the workspace
syms s R C U_0; % definition of symbolic variables
U_C = U_0/s/(R*C*s+1) % definition of the Laplace transform of RC circuit ODE
h = ilaplace(U_C) % inverse Laplace transform
pretty(h) % display prettier result
```

  - Display the step response of an RC circuit with $U_0 = 5\,V$, $R = 1\,k\Omega$, $C = 1\,mF$

```
vars = [R C U_0]; % definition of symbolic variables for substitution
values = [1000 0.001 5]; % numeric values for symbolic variables
h = subs(h, vars,values) % substitute symbolic variables by the numeric values
fplot(h,[0,5]) % plot the response for t = 0..5s
```

Course Syllabus

- Basic concepts in control theory
- Basic system classification
- **Basic system properties**
  - Static Characteristics
  - **Dynamic Characteristics**
    - Ordinary Differential Equations (ODEs)
    - **Transfer Function**
    - Step Response
    - Impulse Response
    - Frequency Response
- System Identification
- Basic types of controllers
- Control quality evaluation
- Control systems stability
- Controller design methods
- Digital control
- Sensors

Basic system properties
Dynamic Characteristics – Transfer Function

- Nth order, linear, ordinary differential equation:

$$a_n y^{(n)} + a_{n-1} y^{(n-1)} + \dots + a_1 y' + a_0 y =$$

$$b_m u^{(m)} + b_{m-1} u^{(m-1)} + \dots + b_1 u' + b_0 u$$

- where $y(t)$ is the output, $u(t)$ is the input, a_n, b_m are real constants
- Taking the Laplace transform of both sides, assuming zero initial conditions:

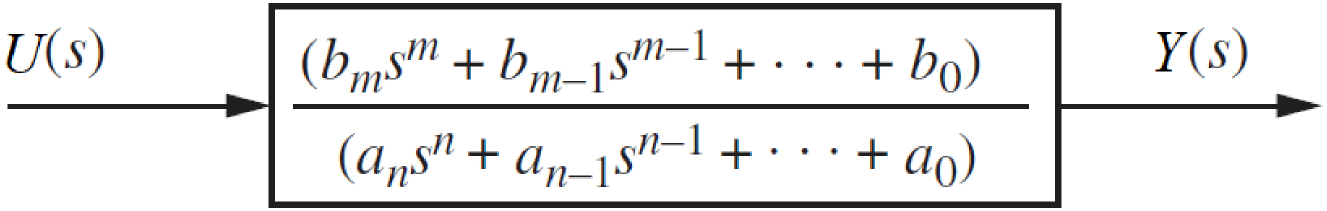
$$(a_n s^n + a_{n-1} s^{n-1} + \dots + a_1 s + a_0) \cdot Y(s) =$$
$$(b_m s^m + b_{m-1} s^{m-1} + \dots + b_1 s + b_0) \cdot U(s)$$

Basic system properties
Dynamic Characteristics – Transfer Function

- (Laplace) **Transfer Function** is the ratio of the Laplace transform of a linear, time-invariant (LTI) system's output to the Laplace transform of its input, assuming zero initial conditions

$$G(s) = \frac{Y(s)}{U(s)} = \frac{b_m s^m + b_{m-1} s^{m-1} + \cdots + b_1 s + b_0}{a_n s^n + a_{n-1} s^{n-1} + \cdots + a_1 s + a_0}$$

- It is often represented as a block diagram



Basic system properties

Dynamic Characteristics – Transfer Function

- **Transfer Function Example – RC Circuit**
  - The Laplace transform of an RC Circuit ODE is

$$U_C(s) = \frac{1}{RCs + 1} U(s)$$

  - The transfer function is

$$G(s) = \frac{U_C(s)}{U(s)} = \frac{1}{RCs + 1}$$

Last Lecture Overview
Dynamic System Modeling & Transforms

- **Analytical System Identification**
  - Derived governing equations for electrical elements:
    - Resistor: $v(t) = Ri(t)$ (Ohm's Law)
    - Capacitor: $i(t) = C\, dv(t)/dt$
    - Inductor: $v(t) = L\, di/dt$
  - Kirchhoff's Voltage Law: Sum of voltages in a closed loop = 0.
- **RC-Circuit Modeling**
  - $RC\frac{du_C(t)}{dt} + u_C(t) = u(t) \rightarrow$ 1st order ODE.
  - Step-input solution: $u_C(t) = U_0 - U_0 e^{-\frac{t}{RC}}$
  - Illustrated initial ($0\ V$) and final (U_0) values via Initial/Final Value Theorems.
- MATLAB Example: Used *ilaplace()* and *fplot()* to derive and visualize the step response.
- **Key Dynamic Responses**
  - **Impulse response**: Output for unit impulse input.
  - **Step response**: Output for unit step input.
  - **Frequency response**: Ratio of output/input in frequency domain.

Last Lecture Overview

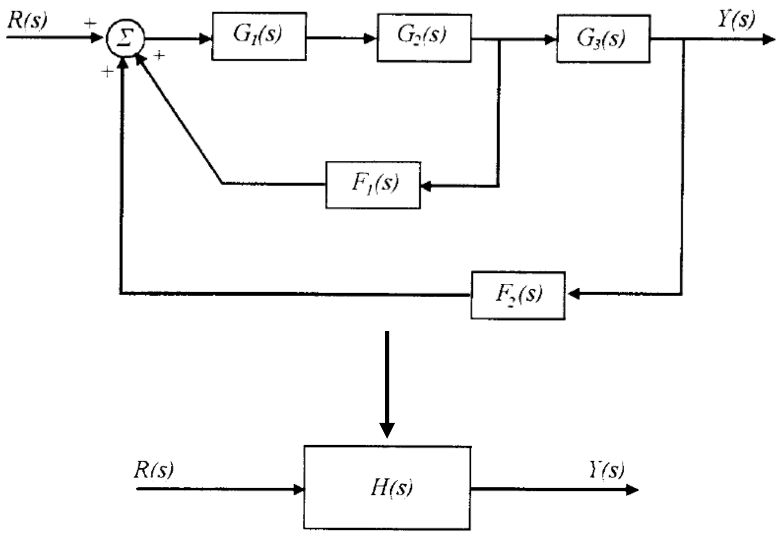
Fourier & Laplace Transforms and Transfer Functions

- **Fourier Transform**
  - Converts time signals ↔ frequency domain.
  - Forward: $\mathcal{F}\{f(t)\} = F(j\omega) = \int_{-\infty}^{\infty} f(t)e^{-j\omega t}dt$
  - Requires *absolutely integrable*, *piecewise continuous* signals.
  - **Windowing** (Hanning, Hamming, Blackman) makes non-decaying signals integrable.
- **Laplace Transform**
  - Extends Fourier by adding decay term $e^{-\sigma t}$, $s = \sigma + j\omega$
  - Forward: $\mathcal{L}\{f(t)\} = F(s) = \int_{0}^{\infty} f(t)e^{-st}dt$
  - Properties: Linearity, Differentiation/Integration theorems, Initial & Final Value theorems.
  - **Transfer Function**: $G(s) = \frac{Y(s)}{U(s)}$

    Core representation of linear time-invariant (LTI) systems in the s-domain.

Basic system properties
Dynamic Characteristics – Block Algebra

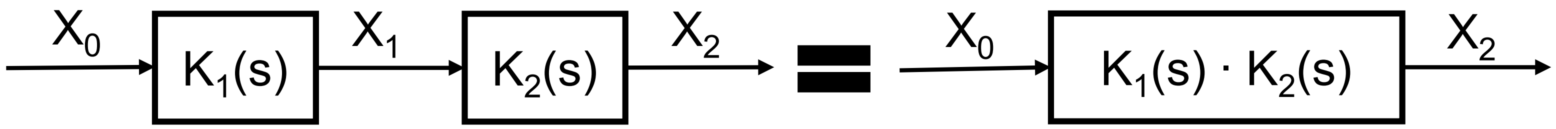
- **Block algebra** is a set of reduction rules used to reduce the complex diagram to a single block representing the overall system’s transfer function.
- Fundamental reductions rules:
  - **Blocks in series**
  - **Blocks in parallel**
  - **Feedback loops**



Modern Control Engineering 2002, p. 103,104

Basic system properties
Dynamic Characteristics – Block Algebra

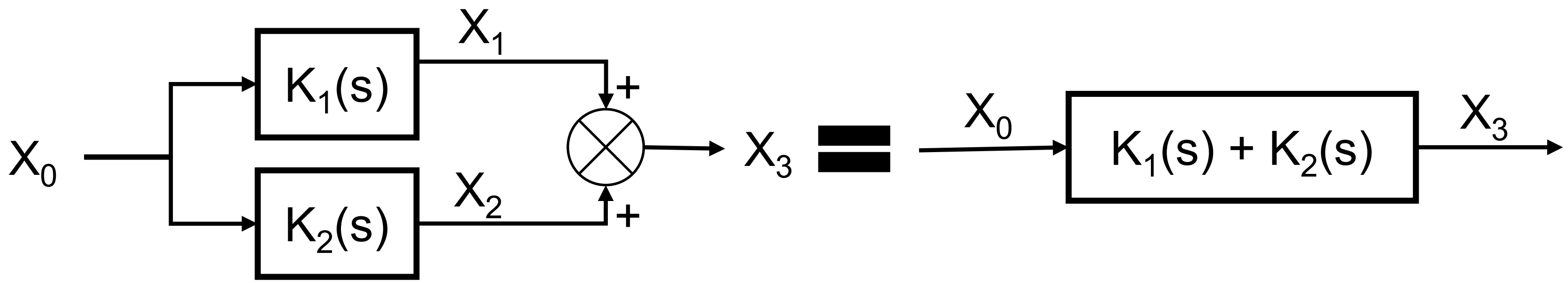
- **Blocks in series (in cascade)**



- $X_1(s) = K_1(s) \cdot X_0(s)$
- $X_2(s) = K_2(s) \cdot X_1(s)$
- Overall transfer function $X_2(s)/X_0(s) = K_1(s) \cdot K_2(s)$

Basic system properties
Dynamic Characteristics – Block Algebra

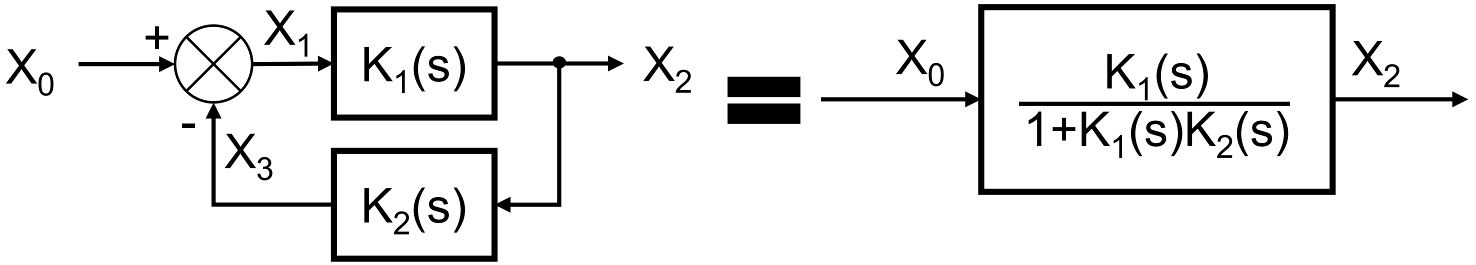
- **Blocks in parallel**



- $X_1(s) = K_1(s) \cdot X_0(s)$
- $X_2(s) = K_2(s) \cdot X_0(s)$
- $X_3(s) = X_1(s) + X_2(s)$
- Overall transfer function $X_3(s)/X_0(s) = K_1(s) + K_2(s)$

Basic system properties
Dynamic Characteristics – Block Algebra

- **Feedback loops**



- $X_1(s) = X_0(s) - X_3(s)$
- $X_2(s) = K_1(s) \cdot X_1(s)$
- $X_3(s) = K_2(s) \cdot X_2(s)$
- Overall transfer function $X_2(s)/X_0(s) = K_1(s)/(1 + K_1(s)K_2(s))$

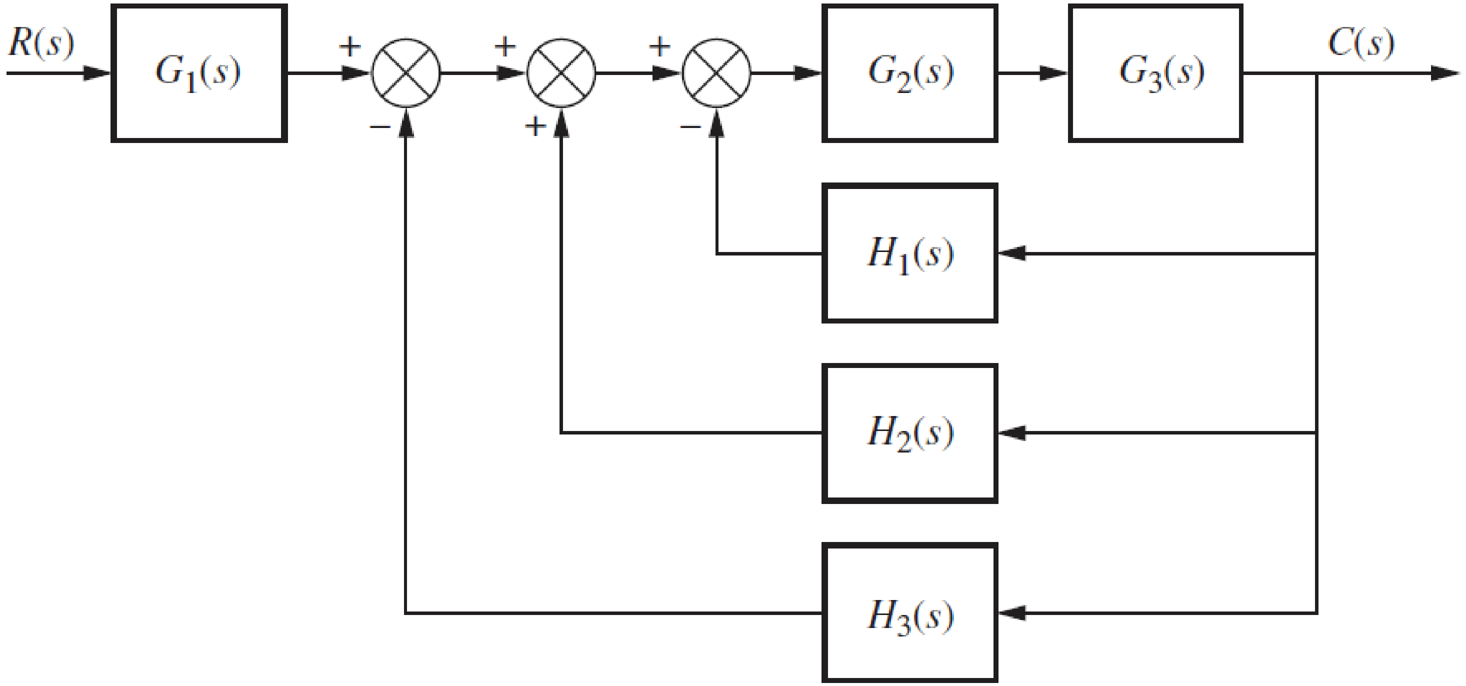
$K_1(s)$ **forward path transfer function**

$K_2(s)$ **feedback path transfer function**

$K_1(s)K_2(s)$ **loop gain**

Basic system properties
Dynamic Characteristics – Block Algebra

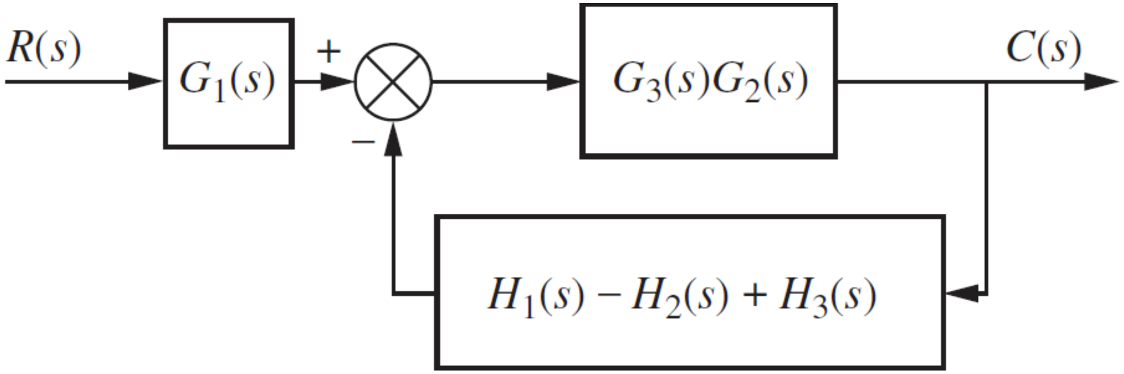
- **Block Diagram Reduction Example**



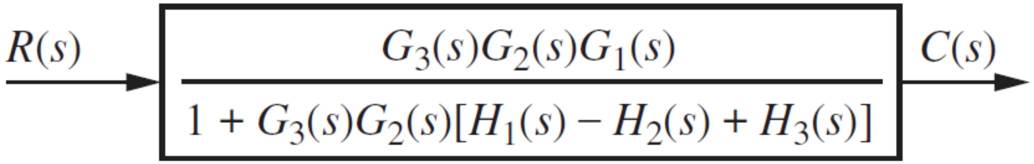
Control Systems Engineering 7th Ed., p. 243

Basic system properties
Dynamic Characteristics – Block Algebra

- **Block Diagram Reduction Example**



(b)



(c)

Control Systems Engineering 7th Ed., p. 243

Basic system properties

Dynamic Characteristics – Poles and zeros

- **Characteristic polynomial** – is the denominator of a transfer function

$$U(s) \rightarrow \boxed{\frac{b_m s^m + \cdots + b_1 s + b_0}{\boldsymbol{a_n s^n + \cdots + a_1 s + a_0}}} \rightarrow Y(s)$$

- **Characteristic equation** – is formed by setting the characteristic polynomial to zero
- The roots of the characteristic equation are the **system's poles**
  - These roots determine the stability of the system
  - They dictate the system's transient response characteristics
  - The system's order is equal to the total number of poles

$$U(s) \rightarrow \boxed{\frac{b_m}{a_n}\frac{(s - z_m)\cdots(s - z_1)}{(s - p_n)\cdots(s - p_1)}} \rightarrow Y(s)$$

- Roots of the numerator polynomial are called **system's zeros**
  - They influence transient behavior

Basic system properties
Dynamic Characteristics – Poles and zeros

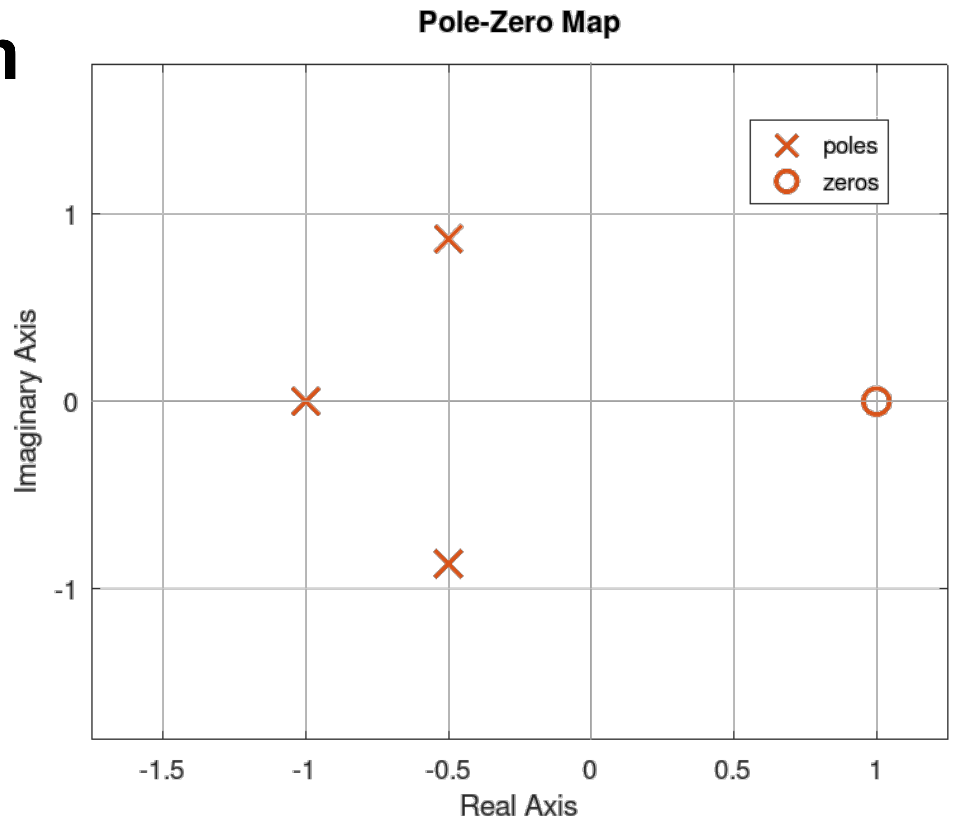
- If we factor the numerator and denominator polynomials we get the following form of the transfer function:

$$G(s) = \frac{b_m s^m + b_{m-1} s^{m-1} + \cdots + b_1 s + b_0}{a_n s^n + a_{n-1} s^{n-1} + \cdots + a_1 s + a_0} = \frac{b_m}{a_n} \frac{(s - z_m) \cdots (s - z_1)}{(s - p_n) \cdots (s - p_1)}$$

- where z_m are system's zeros and p_n are system's poles
- Poles and zeros can be **real**, **purely imaginary** or **complex conjugate.**

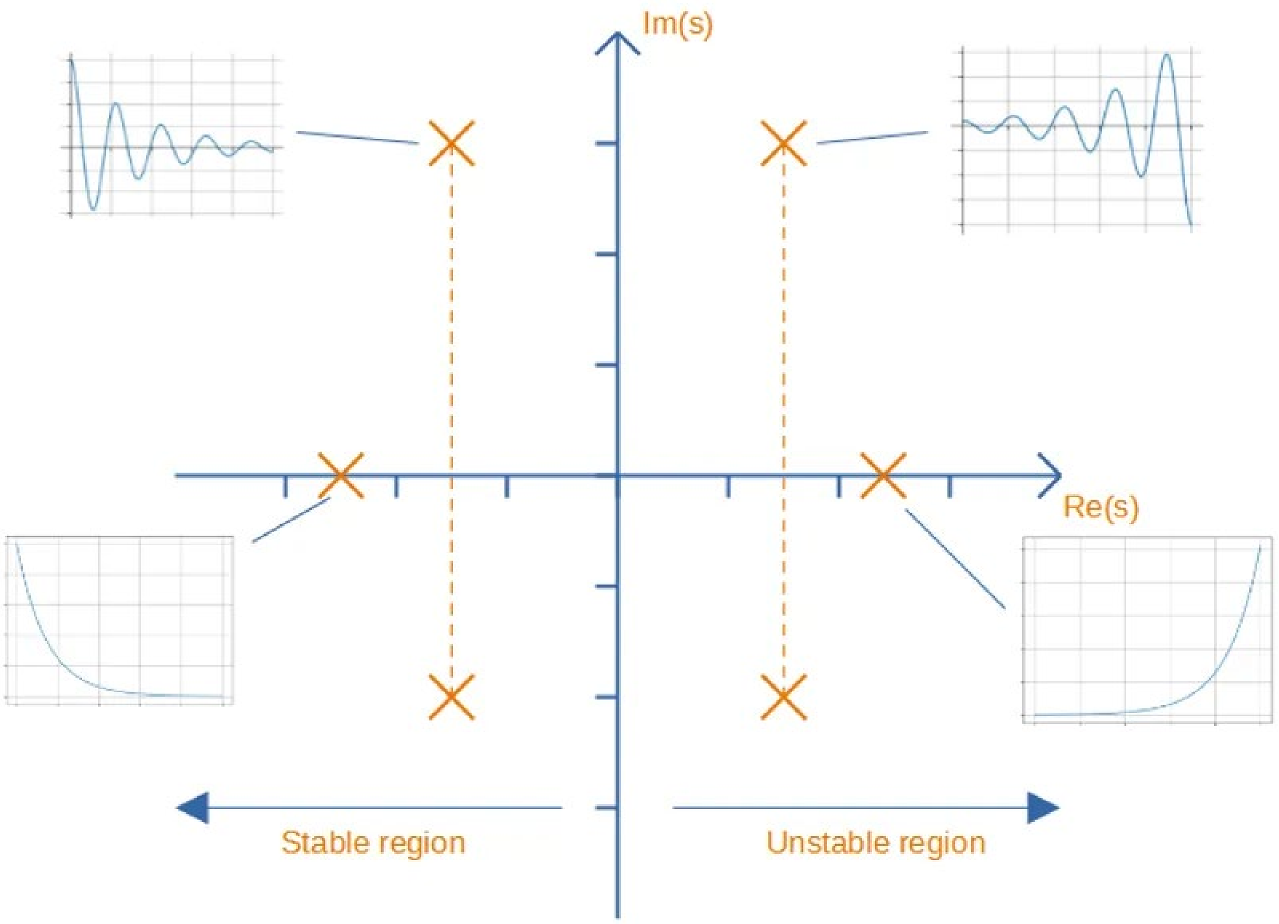
Basic system properties
Dynamic Characteristics – Poles and zeros

- **Pole-zero plot / map / diagram**



Basic system properties
Dynamic Characteristics – Poles and zeros

- Left half-plane is **stable region**
- Right half-plane is **unstable region**
- Imaginary axis is **the boundary**

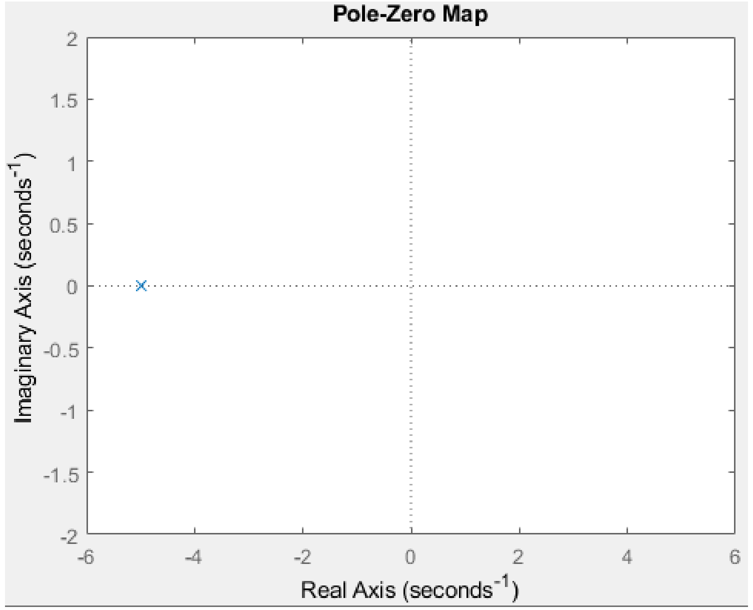


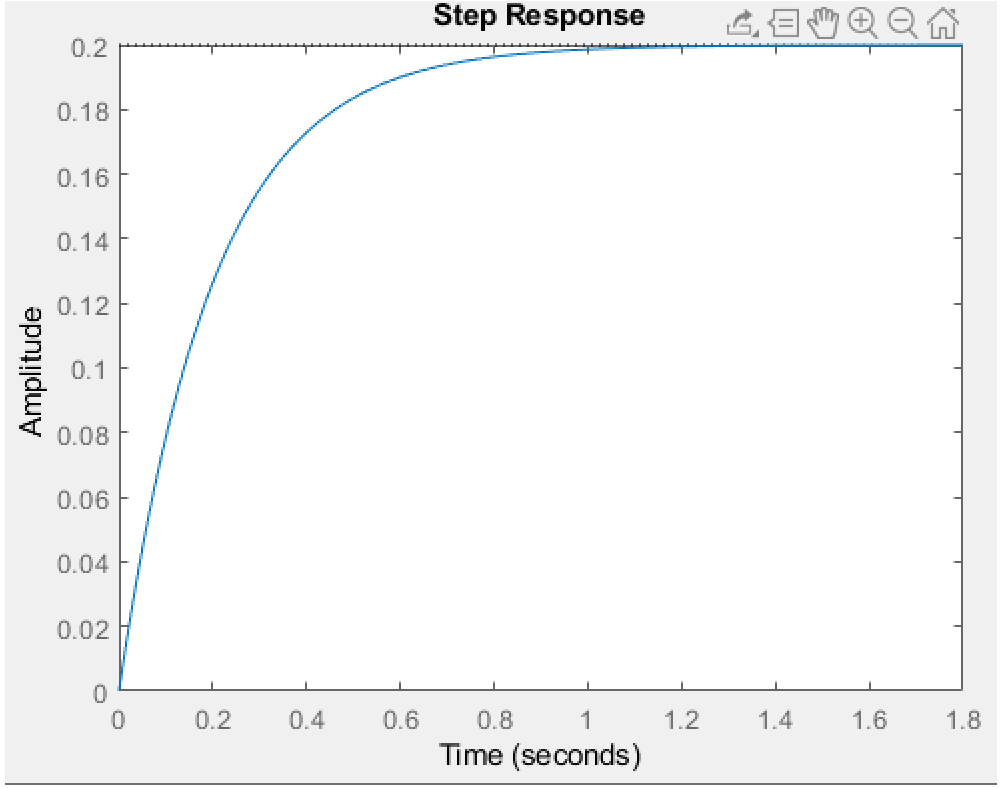
Basic system properties

Dynamic Characteristics – Poles and zeros

- **Real poles cause aperiodic system response**

$G(s) = \frac{1}{s+5}$, one pole $p = -5$





Basic system properties
Dynamic Characteristics – Poles and zeros

- **Real poles cause aperiodic system response**

$$G(s) = \frac{1}{s + 5}$$

- MATLAB Code to generate pole-zero map and step response

```
clear all; % clears the workspace
s = tf('s'); % definition of symbolic variable
G = 1 / (s + 5); % definition of the transfer function
pzmap(G); % plot pole-zero map
figure; % create new window for the next plot
step(G); % plot the step response
```

Basic system properties
Dynamic Characteristics – Poles and zeros

- Negative real poles are reciprocals of **the time constants** of a system

$$T_i = -\frac{1}{p_i}$$

- What is the time constant of an RC circuit with the transfer function ?

$$G(s) = \frac{1}{RCs + 1}$$

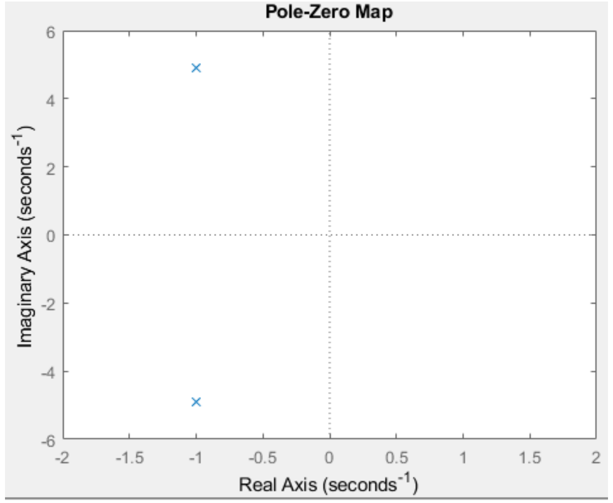
Basic system properties
Dynamic Characteristics – Poles and zeros

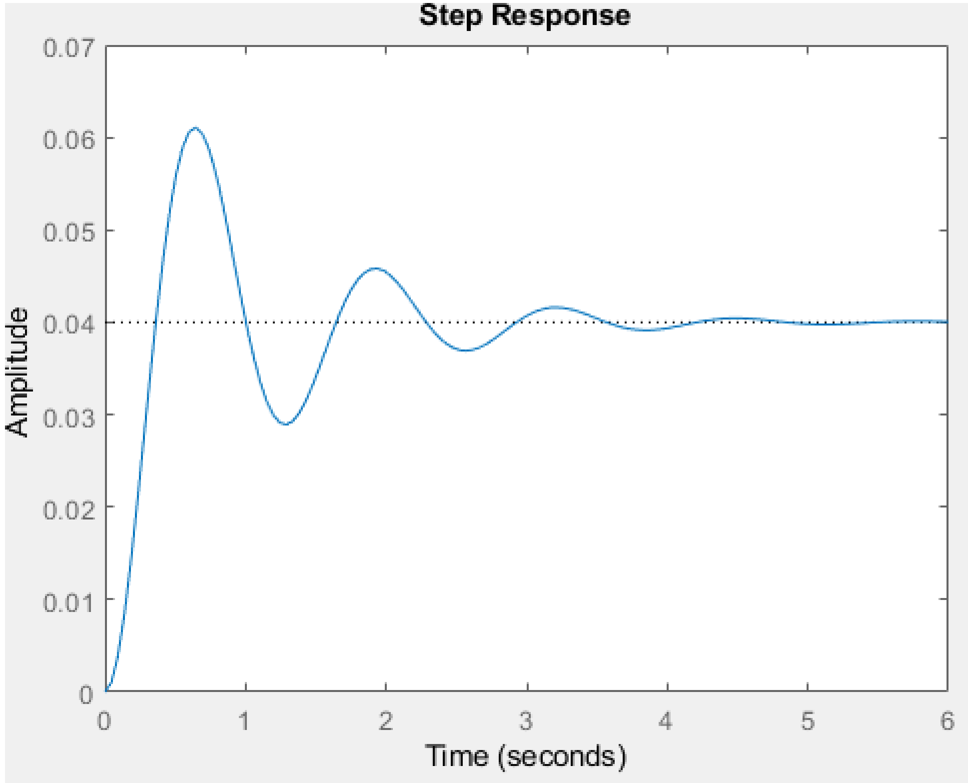
- **Complex conjugate poles cause oscillatory response**

$$G(s) = \frac{1}{s^2 + 2s + 25}$$

Complex conjugate poles

$$p = -1 \pm j4.9$$





Basic system properties
Dynamic Characteristics – Poles and zeros

- **Complex conjugate poles cause oscillatory response**

$$G(s) = \frac{1}{s^2 + 2s + 25}$$

- MATLAB Code to generate pole-zero map and step response

```
clear all; % clears the workspace
s = tf('s'); % definition of symbolic variable
G = 1 / (s^2 + 2*s + 25); % definition of the transfer function
pzmap(G); % plot pole-zero map
figure; % create new window for the next plot
step(G); % plot the step response
```

Basic system properties
Dynamic Characteristics – Poles and zeros

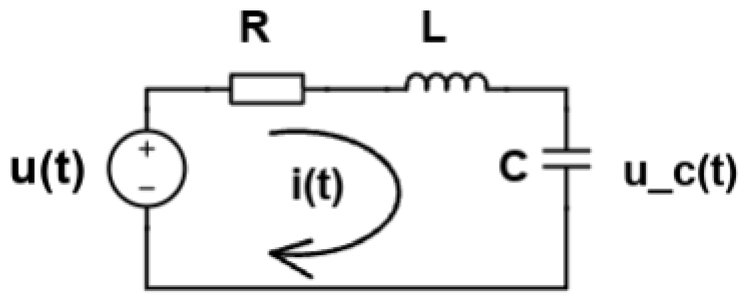
- **Ideal series RLC circuit example**
- Applying Kirchhoff's Voltage Law $u_R(t) + u_L(t) + u_C(t) = u(t)$

$$Ri(t) + L\frac{di(t)}{dt} + \frac{1}{C}\int_0^t i(\tau)d\tau = u(t)$$

- Capacitor voltage $u_C(t)$ is the output quantity

$$u_C(t) = \frac{1}{C}\int_0^t i(\tau)d\tau \rightarrow i(t) = C\frac{du_C(t)}{dt}$$

$$RC\frac{du_C(t)}{dt} + LC\frac{d^2u_C(t)}{dt^2} + u_C(t) = u(t)$$



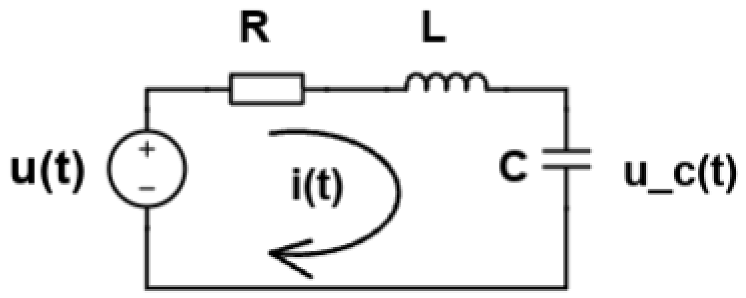
Basic system properties
Dynamic Characteristics – Poles and zeros

- **Ideal series RLC circuit example**
- Apply Laplace transform, using differentiation theorem with zero initial conditions

$$RCsU_C(s) + LCs^2U_C(s) + U_C(s) = U(s)$$

$$(RCs + LCs^2 + 1)U_C(s) = U(s)$$

$$G(s) = \frac{U_C(s)}{U(s)} = \frac{1}{LCs^2 + RCs + 1}$$



Basic system properties
Dynamic Characteristics – Poles and zeros

- **Ideal series RLC circuit example**
- RLC circuit transfer function

$$G(s) = \frac{1}{LCs^2 + RCs + 1}$$

- The transfer function of a **standard second-order control system**

$$G(s) = \frac{k\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2} = \frac{k\omega_n^2}{(s + \zeta\omega_n - j\omega_d)(s + \zeta\omega_n + j\omega_d)}$$

where ω_n is a natural frequency, ζ is a damping ratio, $\omega_d = \omega_n\sqrt{1 - \zeta^2}$ is damped natural frequency, and k is a system gain

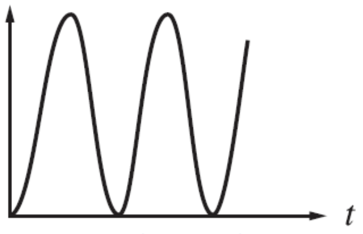
Basic system properties
Dynamic Characteristics – Poles and zeros

- **Natural frequency $\boldsymbol{\omega_n}$** (undamped frequency) – the angular frequency at which the system would oscillate if there were no damping
- **Damping ratio/factor/coefficient $\boldsymbol{\zeta}$** – a dimensionless number that determines how quickly oscillations in a system's response decay
  - *undamped* system if $\zeta = 0$
  - *underdamped* system if $0 < \zeta < 1$
  - *critically damped* if $\zeta = 1$
  - *overdamped* system if $\zeta > 1$
- **Damped natural frequency $\boldsymbol{\omega_d}$** – the angular frequency at which the system oscillate in the presence of damping

Basic system properties

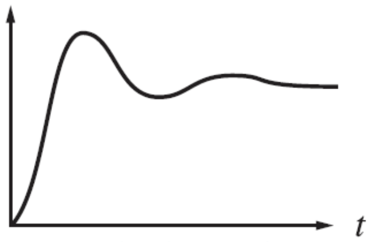
Dynamic Characteristics – Poles and zeros

- Second-order response as a function of damping ratio



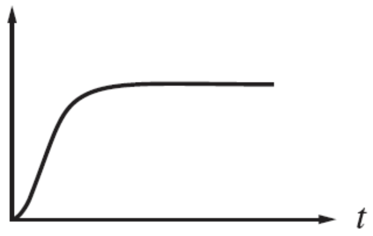
Undamped

$\zeta = 0$



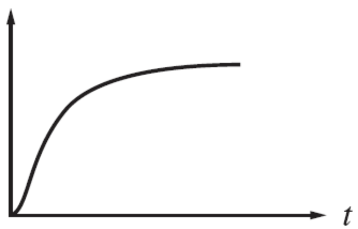
Underdamped

$0 < \zeta < 1$



Critically damped

$\zeta = 1$



Overdamped

$\zeta > 1$

Basic system properties

Dynamic Characteristics – Poles and zeros

- **Ideal series RLC circuit example**

$$G(s) = \frac{1}{LCs^2 + RCs + 1} = \frac{1/LC}{s^2 + R/Ls + 1/LC} = \frac{k\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

- Comparing the transfer functions of an RLC circuit and a standard second-order control system we get the following parameters:
  - Natural frequency $\omega_n = \frac{1}{\sqrt{LC}}$
  - Damping ratio $\zeta = \frac{R}{2}\sqrt{\frac{C}{L}}$
  - System gain $k = 1$

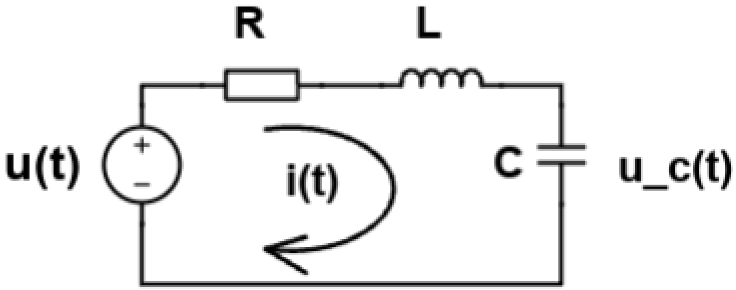
Basic system properties

Dynamic Characteristics – Poles and zeros

- **Ideal series RLC circuit example**

$$G(s) = \frac{1}{LCs^2 + RCs + 1} = \frac{1}{(s - p_1)(s - p_2)}, p_{1,2} = \frac{-RC \pm \sqrt{(RC)^2 - 4LC}}{2LC}$$

- Now substitute specific values for RLC constants
  a) R = 100 Ohm, L = 10 H, C = 200 μF
  b) R = 500 Ohm, L = 10 H, C = 200 μF

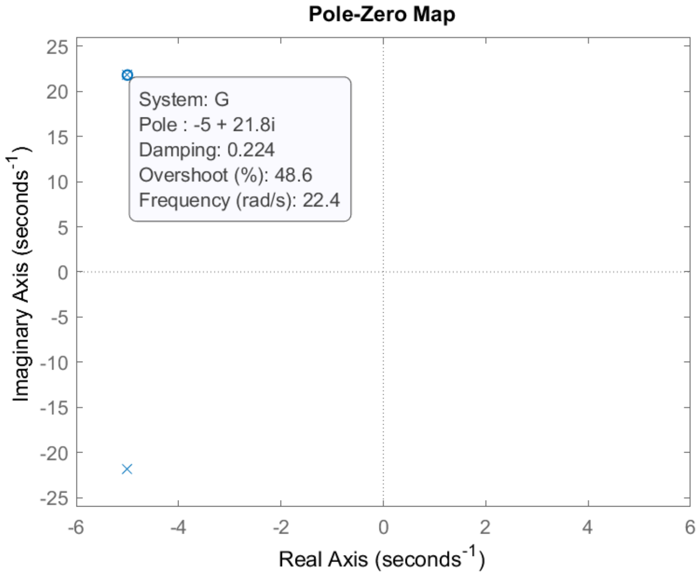


Basic system properties
Dynamic Characteristics – Poles and zeros

- **Ideal series RLC circuit (R = 100 Ohm, L = 10 H, C = 200 μF)**

$$G(s) = \frac{1}{LCs^2 + RCs + 1} = \frac{\frac{1}{LC}}{s^2 + \frac{R}{L}s + \frac{1}{LC}} = \frac{500}{s^2 + 10s + 500}$$

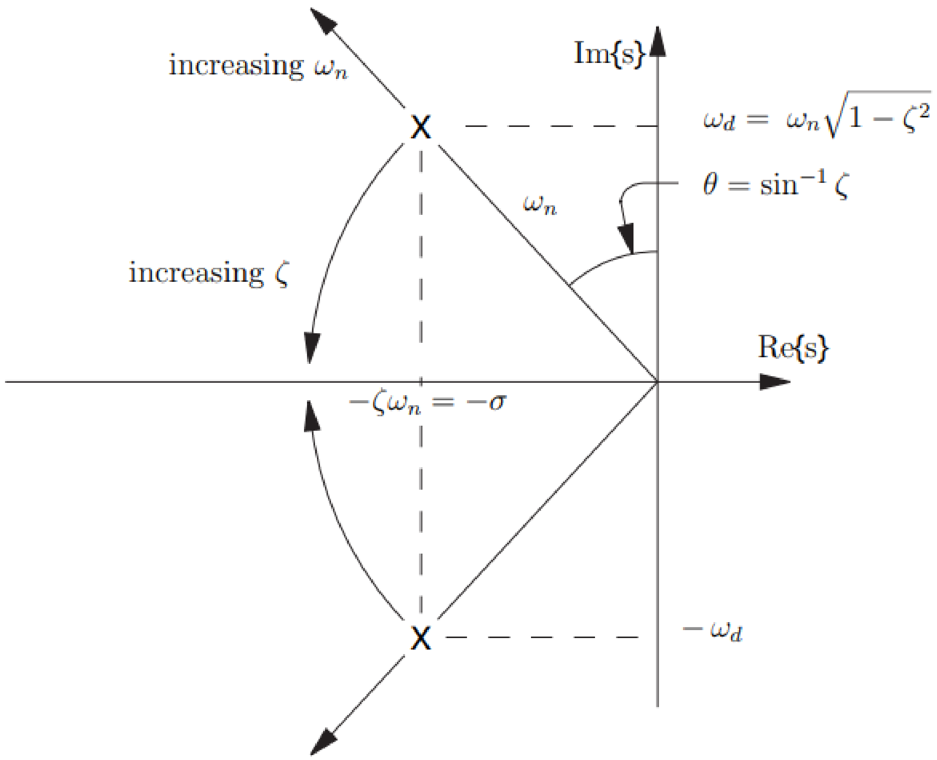
$$= \frac{500}{(s + 5 - j21.8)(s + 5 + j21.8)}$$

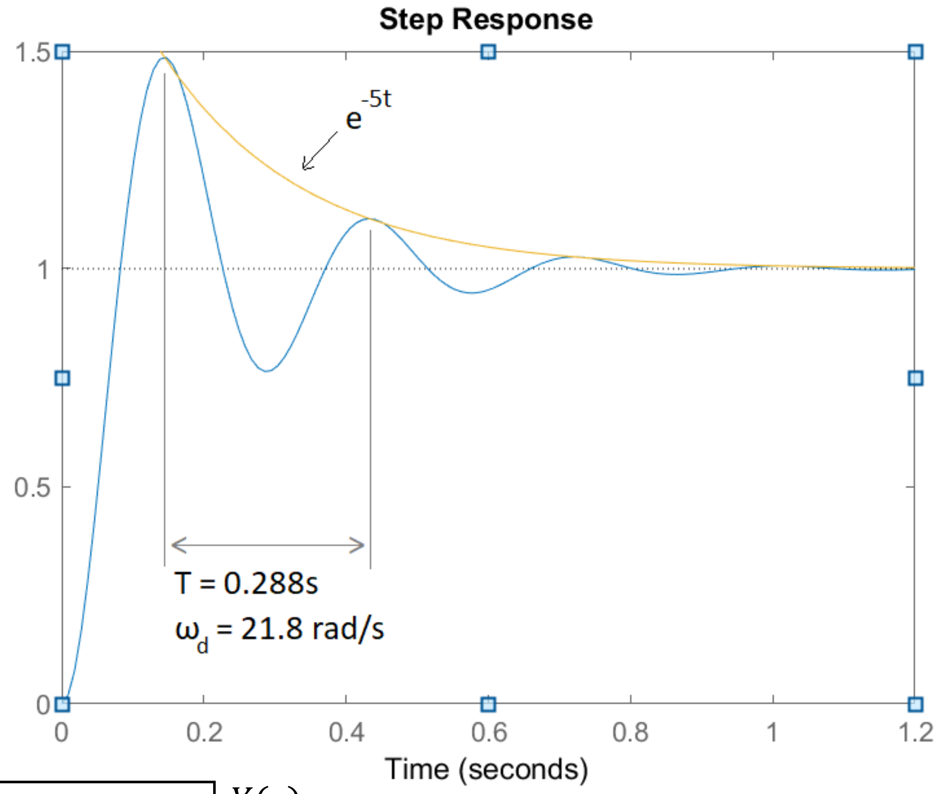


- Damping ratio is $\zeta = \frac{R}{2}\sqrt{\frac{C}{L}} = 0.2236$
- Natural frequency $\omega_n = \frac{1}{\sqrt{LC}} = 22.36\, rad/s$
- Damped natural frequency $\omega_d = \omega_n\sqrt{1 - \zeta^2} = 21.8\, rad/s$

Basic system properties
Dynamic Characteristics – Poles and zeros

- **Ideal series RLC circuit (R = 100 Ohm, L = 10 H, C = 200 µF)**





$$U(s) \longrightarrow \boxed{\dfrac{500}{(s+5-j21.8)(s+5+j21.8)}} \longrightarrow Y(s)$$

Basic system properties
Dynamic Characteristics – Poles and zeros

- **Ideal series RLC circuit (R = 100 Ohm, L = 10 H, C = 200 µF)**
- MATLAB Code to generate pole-zero map and step response

```
clear all; % clears the workspace
s = tf('s'); % definition of symbolic variable
L = 10; C = 200e-6; R = 100;
G = 1/(L*C*s^2 + R*C*s + 1); % definition of the transfer function
pzmap(G); % plot pole-zero map
figure; % create new window for the next plot
step(G); % plot the step response
hold on; % place the next plot in the same figure
syms t; % define symbolic variable t
fplot(exp(-R/2/L*t)+1); % plot exponential decay function
```

Basic system properties
Dynamic Characteristics – Poles and zeros

- **Ideal series RLC circuit (R = 500 Ohm, L = 10 H, C = 200 μF)**

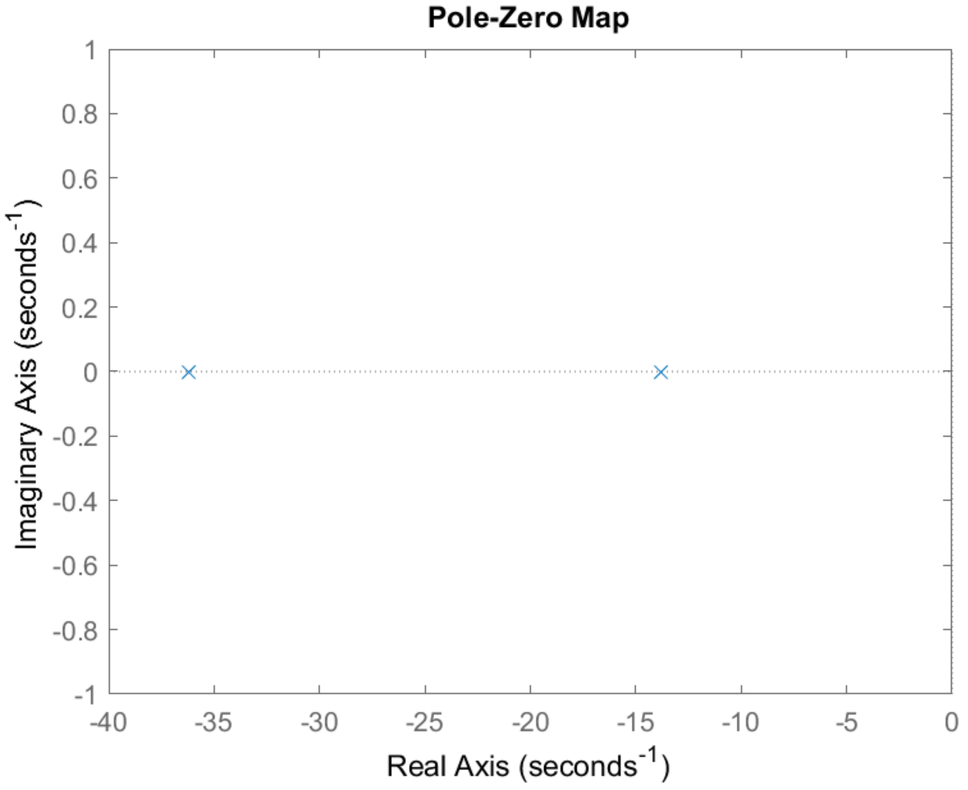
$$G(s) = \frac{1}{LCs^2 + RCs + 1} = \frac{\frac{1}{LC}}{s^2 + \frac{R}{L}s + \frac{1}{LC}} = \frac{500}{s^2 + 50s + 500} =$$

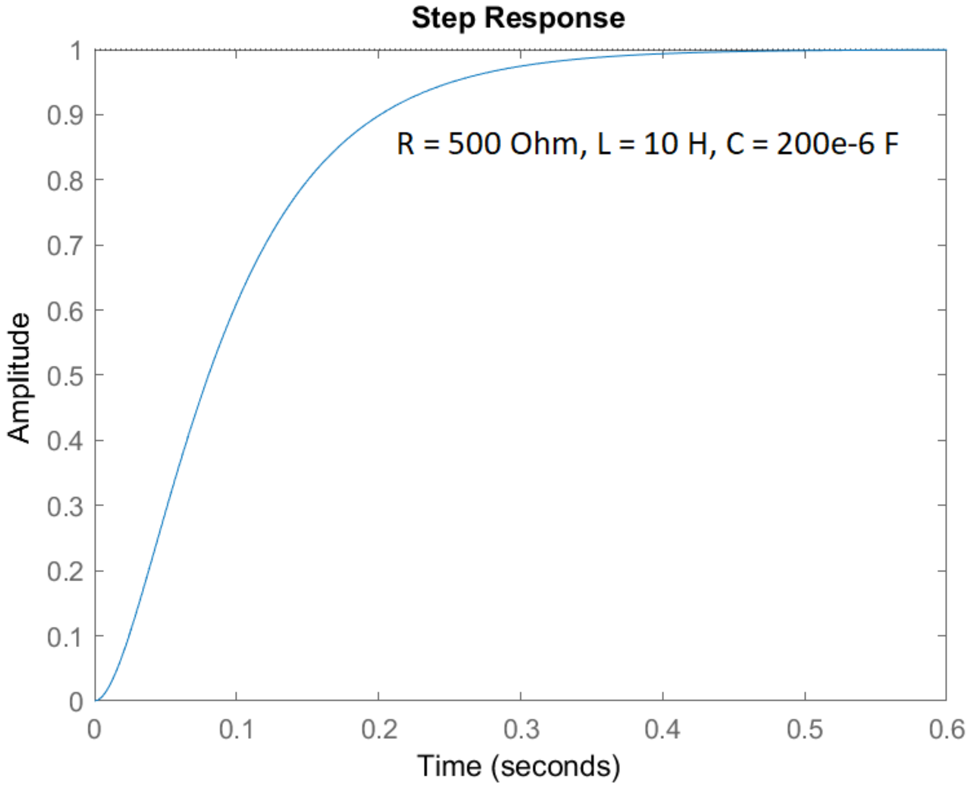
$$\frac{500}{(s + 36.2)(s + 13.8)}$$

- Damping ratio is $\zeta = \frac{R}{2}\sqrt{\frac{C}{L}} = 1.12$

Basic system properties
Dynamic Characteristics – Poles and zeros

- **Ideal series RLC circuit (R = 500 Ohm, L = 10 H, C = 200 µF)**





Basic system properties
Dynamic Characteristics – Poles and zeros

- **Ideal series RLC circuit (R = 500 Ohm, L = 10 H, C = 200 µF)**
- MATLAB Code to generate pole-zero map and step response

```
clear all; % clears the workspace
s = tf('s'); % definition of symbolic variable
L = 10; C = 200e-6; R = 500;
G = 1/(L*C*s^2 + R*C*s + 1); % definition of the transfer function
pzmap(G); % plot pole-zero map
figure; % create new window for the next plot
step(G); % plot the step response
```

Course Syllabus

- Basic concepts in control theory
- Basic system classification
- **Basic system properties**
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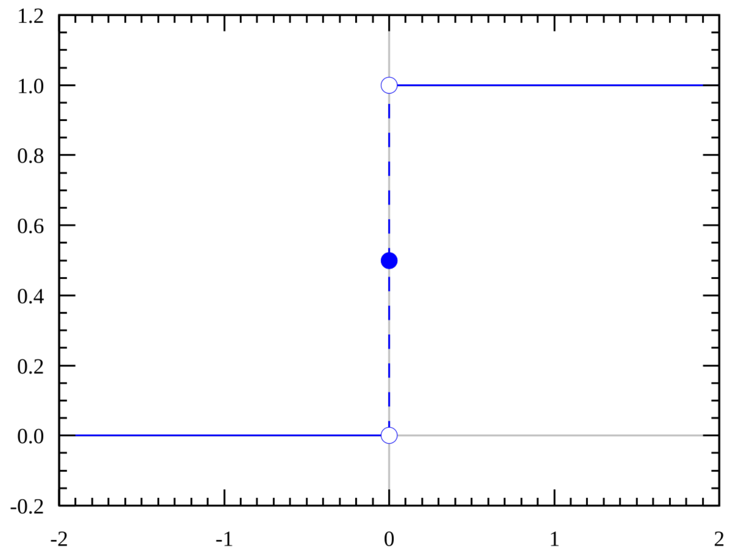
- Sensors

Basic system properties
Dynamic Characteristics – Step Response

- **Step response** is the output of a system when subjected to a step input, like *unit step function* (Heaviside function)

$$u(t) = \begin{cases} 1 & t > 0 \\ 0 & t < 0 \end{cases}$$

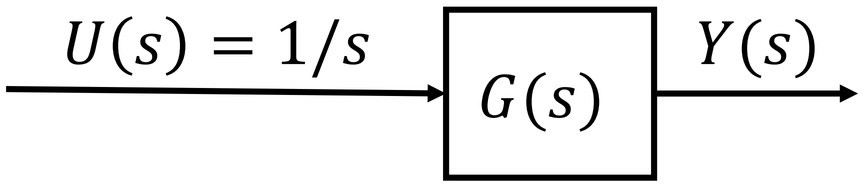


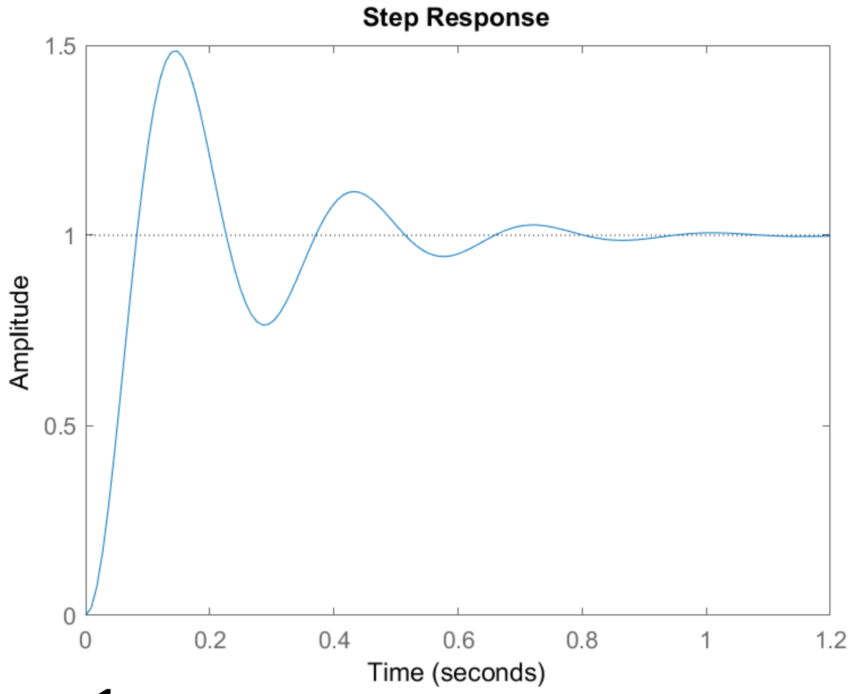
- Laplace transform of the step function

$$\mathcal{L}\{u(t)\} = \frac{1}{s}$$

Basic system properties

Dynamic Characteristics – Step Response





- From the transfer function definition

$$G(s) = \frac{Y(s)}{U(s)} \rightarrow Y(s) = G(s)U(s)$$

- Analytical step response function

$$Y(s) = G(s)\frac{1}{s} \rightarrow y(t) = h(t) = \mathcal{L}^{-1}\left\{G(s)\frac{1}{s}\right\}$$

Basic system properties
Dynamic Characteristics – Step Response

- **Step response of an RLC circuit** (R = 100 Ohm, L = 10 H, C = 200 µF)

$$U(s) = 1/s \longrightarrow \boxed{\frac{500}{s^2 + 10s + 500}} \longrightarrow Y(s)$$

- Analytical step response function

$$y(t) = h(t) = \mathcal{L}^{-1}\left\{G(s)\frac{1}{s}\right\} = \mathcal{L}^{-1}\left\{\frac{500}{s^2 + 10s + 500}\frac{1}{s}\right\}$$

$$= 1 - e^{-5t}\left[cos\left(5\sqrt{19}t\right) + \frac{\sqrt{19}}{19}sin(5\sqrt{19}t)\right]$$

Basic system properties

Dynamic Characteristics – Step Response

- **Step response of an RLC circuit** (R = 100 Ohm, L = 10 H, C = 200 μF)

$$U(s) = 1/s \longrightarrow \boxed{\frac{500}{s^2 + 10s + 500}} \longrightarrow Y(s)$$

- MATLAB Code to calculate the inverse Laplace transform

```
clear all; % clears the workspace
syms s; % definition of symbolic variable
L = 10; C = 200e-6; R = 100;
G = 1/(L*C*s^2 + R*C*s + 1); % definition of the transfer function
h = ilaplace(G/s); % calculate inverse Laplace transform
pretty(h); % prints symbolic expression in more readable format
```

Course Syllabus

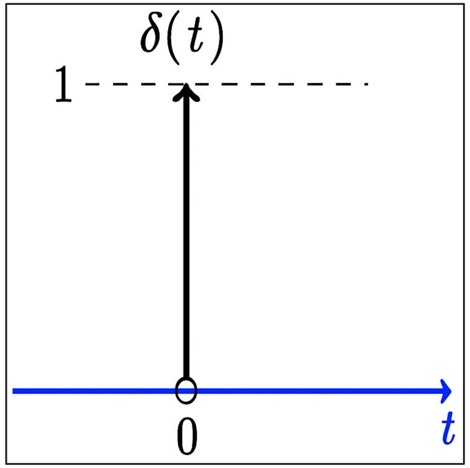
- Basic concepts in control theory
- Basic system classification
- **Basic system properties**
  - Static Characteristics
  - **Dynamic Characteristics**
    - Ordinary Differential Equations (ODEs)
    - Transfer Function
    - Step Response
    - **Impulse Response**
    - Frequency Response
- System Identification
- Basic types of controllers
- Control quality evaluation
- Control systems stability
- Controller design methods
- Digital control
- Sensors

Basic system properties
Dynamic Characteristics – Impulse Response

- **Impulse response** is the output of a system when subjected to an impulse input function (Dirac delta function)

$$\delta(t) = \begin{cases} 0 & t \neq 0 \\ \infty & t = 0 \end{cases} \text{ such that } \int_{-\infty}^{\infty} \delta(t)dt = 1$$

- In practical implementations Dirac delta is approximated with inputs that have **small duration** (small enough compared to the system's dynamics) and **finite amplitude**.



Basic system properties
Dynamic Characteristics – Impulse Response

- **Laplace transform** of the impulse input function

$$\mathcal{L}\{\delta(t)\} = 1$$

- Relation to **Heaviside step function** $u(t)$

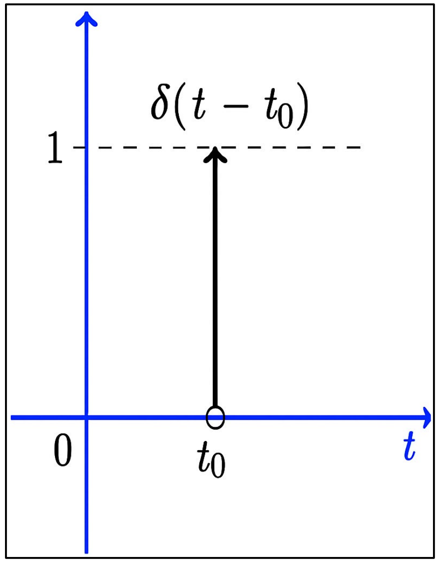
$$\delta(t) = \frac{du(t)}{dt} \text{ and } u(t) = \int_{-\infty}^{t} \delta(\tau)d\tau$$

- **Time shifted Dirac delta function**

$$\delta(t - t_0) = \begin{cases} 0 & t \neq t_0 \\ \infty & t = t_0 \end{cases} \text{ such that } \int_{-\infty}^{\infty} \delta\,(t - t_0)dt = 1$$

- **Laplace transform** of the time-shifted impulse function

$$\mathcal{L}\{\delta(t - t_0)\} = e^{-st_0}$$

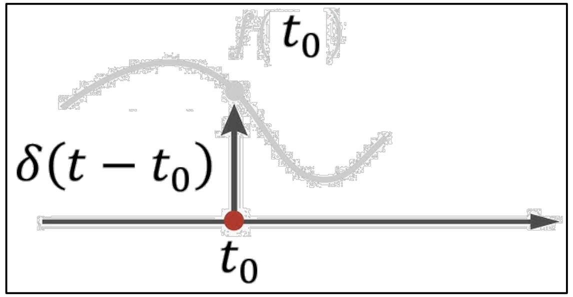


Basic system properties

Dynamic Characteristics – Impulse Response

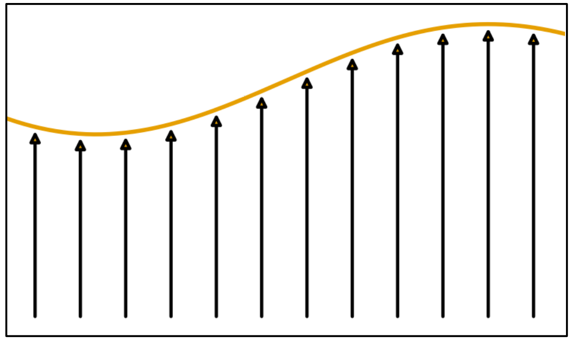
- **The sifting (sampling) property** of the Dirac delta function (extracting the function's value at a specific point t_0)

$$\int_{-\infty}^{\infty} f(t)\delta(t-t_0)dt = f(t_0)$$

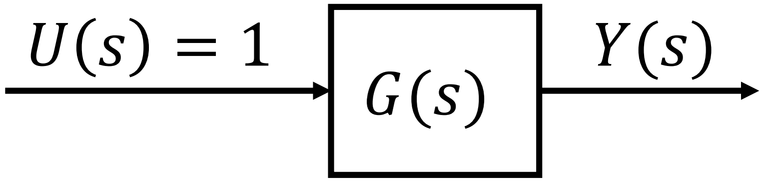


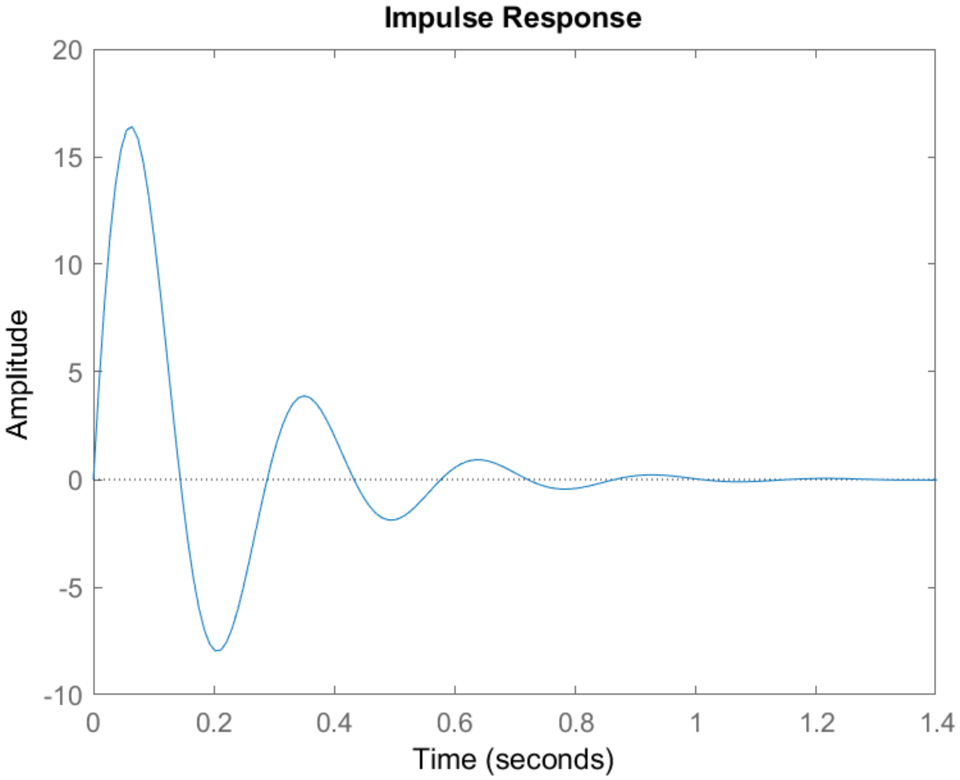
- Representing a signal as a **sum of weighted and shifted Dirac deltas** is directly related to how sampling is modeled in signal processing

$$f(t) = \int_{-\infty}^{\infty} f(\tau)\delta(\tau-t)d\tau$$



Basic system properties
Dynamic Characteristics – Impulse Response





- From the transfer function definition

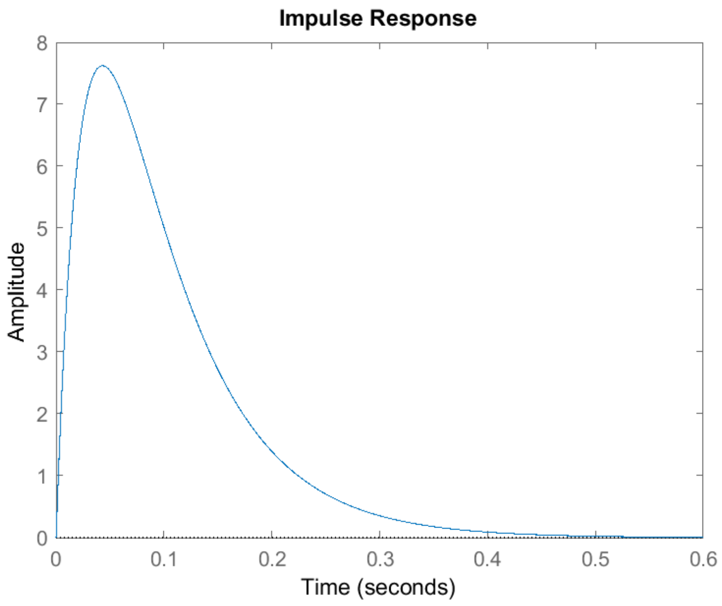
$$G(s) = \frac{Y(s)}{U(s)} \rightarrow Y(s) = G(s)U(s)$$

- **Analytical impulse response function**

$$Y(s) = G(s)1 \rightarrow y(t) = g(t) = \mathcal{L}^{-1}\{G(s)\}$$

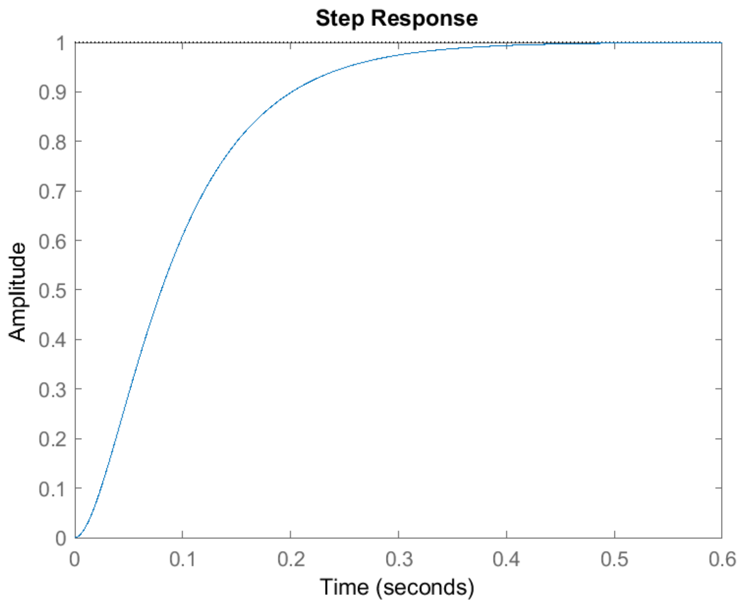
Basic system properties
Dynamic Characteristics – Impulse Response

- **Relation to the step response $h(t)$**
  - If you differentiate step response you get impulse response and vice versa, if you integrate impulse response you get step response.



$$h(t)=\int_0^t g(\tau)d\tau \quad \longrightarrow$$

$$\longleftarrow \quad g(t)=\frac{dh(t)}{dt}$$



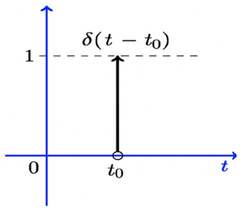
Basic system properties
Dynamic Characteristics – Impulse Response

- MATLAB Code to show the relation between step and impulse response

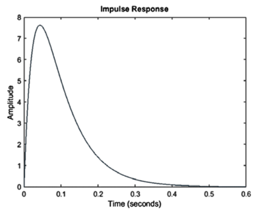
```
clear all; % clears the workspace
s = tf('s'); % definition of symbolic variable
L = 10; C = 200e-6; R = 500; % parameters of RLC circuit
G = 1/(L*C*s^2 + R*C*s + 1); % definition of the transfer function
[g t] = impulse(G); % calculate impulse response from the transfer function and save values into
    g,t variables
impulse(G); % plot impulse response
figure; % opens new, empty figure
plot(t,cumtrapz(t,g)); % plot the integral of the impulse response into newly opened figure
hold on; % next plot will be shown in the same figure
[h t] = step(G); % calculate step response
plot(t,h,'r'); % plot step response (red) over the integral of the impulse response (blue)
```

Basic system properties
Dynamic Characteristics – Impulse Response

- **Impulse response of an RLC circuit** (R = 100 Ohm, L = 10 H, C = 200 µF)



$$U(s) = 1 \longrightarrow \boxed{\frac{500}{s^2 + 10s + 500}} \longrightarrow Y(s)$$



- Analytical step response function

$$y(t) = g(t) = \mathcal{L}^{-1}\{G(s)1\} = \mathcal{L}^{-1}\left\{\frac{500}{s^2 + 10s + 500}\right\}$$

$$= \frac{100\sqrt{19}}{19} e^{-5t} sin(5\sqrt{19}t)$$

Basic system properties
Dynamic Characteristics – Impulse Response

- **Impulse response of an RLC circuit** (R = 100 Ohm, L = 10 H, C = 200 μF)

$$U(s) = 1 \longrightarrow \boxed{\frac{500}{s^2 + 10s + 500}} \longrightarrow Y(s)$$

- MATLAB Code to calculate the inverse Laplace transform

```
clear all; % clears the workspace
syms s; % definition of symbolic variable
L = 10; C = 200e-6; R = 100;
G = 1/(L*C*s^2 + R*C*s + 1); % definition of the transfer function
g = ilaplace(G); % calculate inverse Laplace transform
pretty(g); % prints symbolic expression in more readable format
fplot(g,[0 1]); % draw plot of an impulse function
```

Basic system properties
Dynamic Characteristics – Impulse Response

- **Convolution**
  - Convolution is a very powerful technique that can be used to calculate the response of a system to an *arbitrary input* by using the *impulse response* of a system.
  - Continuous time convolution is an operation on two continuous time signals defined by the integral

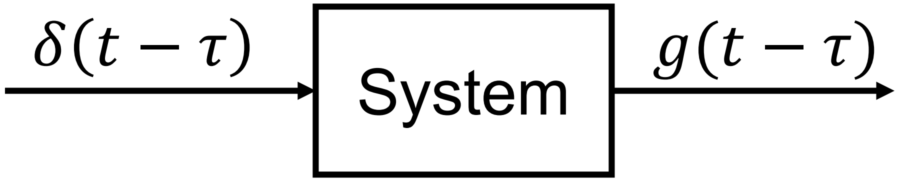
$$(f * g)(t) = \int_{-\infty}^{\infty} f(\tau)g(t-\tau)d\tau$$

  - Informally this notation $f(t) * g(t)$ also denotes convolution

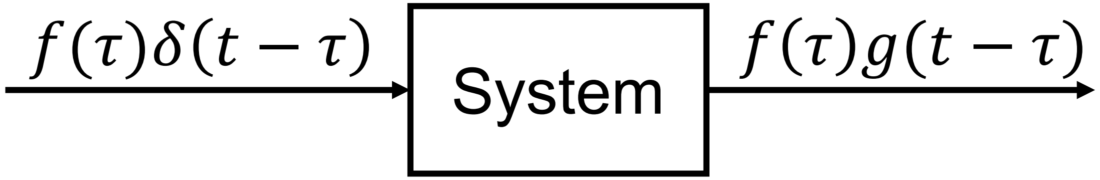
Basic system properties

Dynamic Characteristics – Impulse Response

- **Convolution**
  - Consider an LTI system whose input is time-shifted Dirac delta function then its output is time-shifted impulse response.



  - Let $f(t)$ be any input whose value at $t=\tau$ is $f(\tau)$, then because of linearity, if we scale the input by any factor, the output will be scaled by the same factor.



Basic system properties

Dynamic Characteristics – Impulse Response

- **Convolution**
  - Now we integrate both sides over all values of τ

$$\int_{-\infty}^{\infty} f(\tau)\delta(t-\tau)d\tau \longrightarrow \boxed{\text{System}} \longrightarrow \int_{-\infty}^{\infty} f(\tau)g(t-\tau)d\tau$$

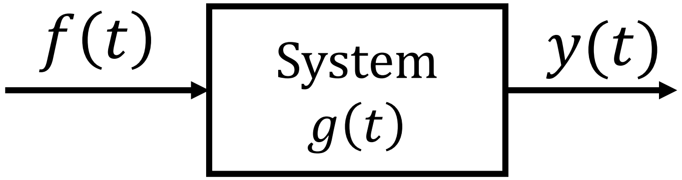
  - Assuming the Dirac delta is even function $\delta(t-\tau)=\delta(\tau-t)$, at the input we have a signal $f(t)$ represented as a sum of weighted and shifted Dirac deltas and output is the convolution integral.

$$f(t)=\int_{-\infty}^{\infty} f(\tau)\delta(\tau-t)d\tau \longrightarrow \boxed{\text{System}} \longrightarrow \int_{-\infty}^{\infty} f(\tau)g(t-\tau)d\tau=(f*g)(t)$$

Basic system properties

Dynamic Characteristics – Impulse Response

- **Convolution**
  - The convolution integral states that the system is entirely characterized by its response to a Dirac delta function, i.e. its impulse response $g(t)$



$$y(t) = (f * g)(t) = \int_{-\infty}^{\infty} f(\tau)g(t-\tau)d\tau$$

  - The **convolution is commutative**

$$(f * g)(t) = (g * f)(t) = \int_{-\infty}^{\infty} g(\tau)f(t-\tau)d\tau$$

  - Convolution animation https://lpsa.swarthmore.edu/Convolution/CI.html

Basic system properties

Dynamic Characteristics – Impulse Response

- **Convolution theorem**
  - The convolution theorem states that the Laplace transform of the convolution of two functions is equal to the product of their individual Laplace transforms.

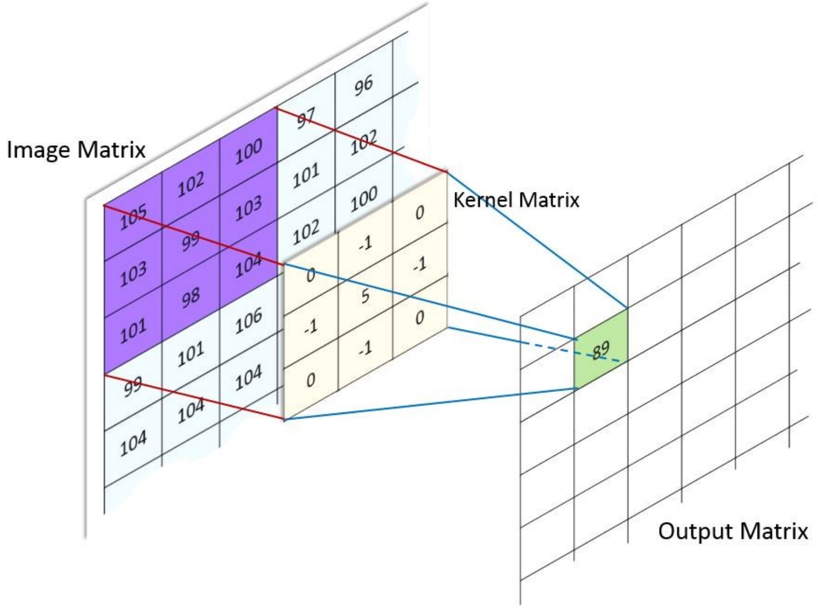
$$\mathcal{L}\{(f * g)(t)\} = F(s)G(s)$$

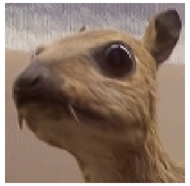
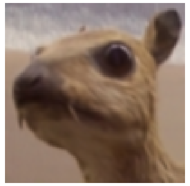
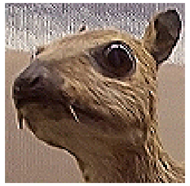
$$(f * g)(t) = \mathcal{L}^{-1}\{F(s)G(s)\}$$

Basic system properties

Dynamic Characteristics – Impulse Response

- **Convolution Example – Convolution filters**
  - Fundamental building blocks in image processing and computer vision.



Original	*Gaussian Blur*	*Sharpen*	*Edge Detection*
$\begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$	$\frac{1}{16}\begin{bmatrix} 1 & 2 & 1 \\ 2 & 4 & 2 \\ 1 & 2 & 1 \end{bmatrix}$	$\begin{bmatrix} 0 & -1 & 0 \\ -1 & 5 & -1 \\ 0 & -1 & 0 \end{bmatrix}$	$\begin{bmatrix} -1 & -1 & -1 \\ -1 & 8 & -1 \\ -1 & -1 & -1 \end{bmatrix}$
			

Basic system properties
Dynamic Characteristics – Impulse Response

- **Convolution Example – Convolutional Neural Networks**
  - A type of deep learning artificial neural network, primarily used for image analysis and pattern recognition by identifying features from images.