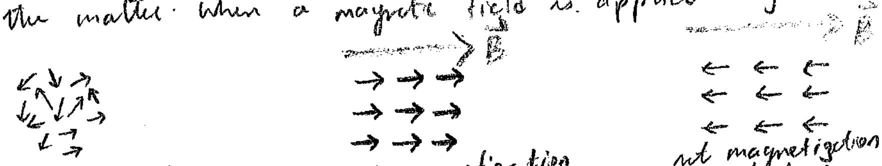
Magnetic Fields in Matter

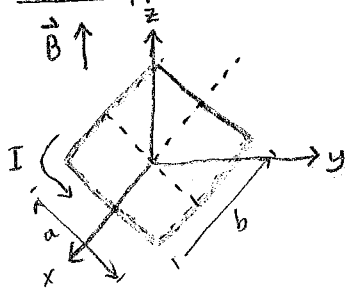
- All magnetic phenomena is due to electric charges in motion. In atomic scale we find tiny currents: e^- orbiting around the nucleus, or spinning around themselves(!!) (This point of view doesn't fit modern atom theory) we better think that the objects like protons and electrons have some essential magnetic property (exactly in the same way they have charge)
- In practice these magnetic properties of the atomic particles are so small that we can treat them as magnetic dipoles. Normally, they cancel each other because of their random orientation in the matter. When a magnetic field is applied they become magnetically polarized, or magnetized.

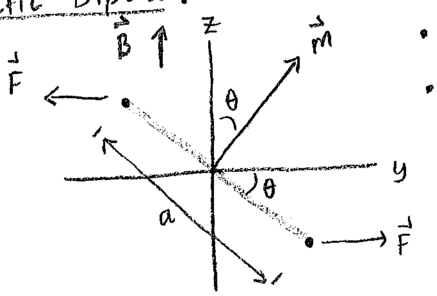


no net magnetization | net magnetization **PARAMAGNETISM** | net magnetization opposite to $\vec{B}$ **DIAMAGNETISM**

FERROMAGNETISM: They retain their magnetization even after the external field is removed

Torque Applied on Magnetic Dipoles:





- The forces on side "a" cancel each other.
- The forces on side "b" creates a torque.

$$\vec{N} = a F \sin\theta\, \hat{x}$$

$$\vec{F}_{mag} = I\int d\vec{l}\times\vec{B} \Rightarrow F = IbB$$

$$\vec{N} = IabB\sin\theta\,\hat{x} = mB\sin\theta\,\hat{x}$$

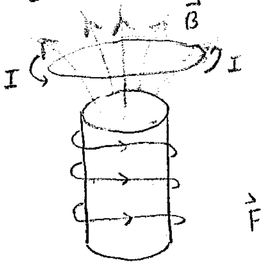
$$\boxed{\vec{N} = \vec{m}\times\vec{B}}$$ m: magnetic dipole moment

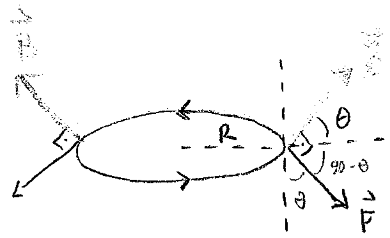
$$\vec{N} = \vec{m}\times\vec{B} \xleftrightarrow{\text{analogy}} \vec{N} = \vec{p}\times\vec{E}$$

- In both cases, torque is in a direction to line the dipole up parallel to the field.
- It is this torque causing paramagnetism: Every e^- constitutes a magnetic dipole. But electrons are always lock together in an atom with opposing magnetic dipole. Therefore paramagnetism normally occurs in atoms or molecules with an odd # of e^-.
- In a uniform field the net force on a current loop is zero. ($\vec{B}$ is constant; can be written out of the integral)

$$\vec{F} = I\oint(d\vec{l}\times\vec{B}) = I\left(\oint d\vec{l}\right)\times\vec{B} = 0$$

- In a non uniform field





$$\vec{F} = I\oint d\vec{l}\times\vec{B}$$

$$|\vec{F}| = I\,2\pi R\,B\cos\theta$$

~~$I\pi R^2$~~

$I\pi R^2 B$

- Similar to the electric dipole, for an infinitesimal loop with $\vec{m}$ in a non-uniform field $\vec{B}$

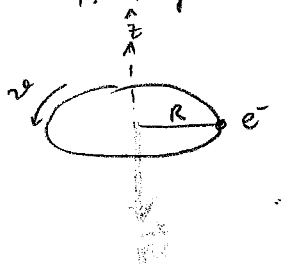
$$\boxed{\vec{F} = \vec{\nabla}(\vec{m}\cdot\vec{B})}$$ For magnetic dipole → (Prob. 6.4)

$$\boxed{\vec{F} = \vec{\nabla}(\vec{p}\cdot\vec{E})}$$ For electric dipole

Effect of a Magnetic Field on Atomic Orbits

- "Electrons not only spin; they also revolve around nucleus" → We need to be extra careful while using these kind of arguments.
- Let us assume that the orbit is circular

★ An orbital motion with a single e^- does not constitute a steady current, but since the period is very short we can write



$$I = \frac{e}{T} = \frac{ev}{2\pi R} \xrightarrow{|\vec{m}| = I\pi R^2} \vec{m} = -\frac{1}{2}\frac{ev}{\cancel{\pi} R}\cancel{\pi}R^{\cancel{2}}\hat{z} = \underbrace{\ominus}_{-\text{ charge of } e^-}\frac{1}{2}evR\hat{z} \quad : \text{orbital dipole moment}$$

⇒ Like other magnetic dipole moments, this one is also subjected to a torque $\vec{m}\times\vec{B}$ when the atom is placed in a magnetic field.

→ However, it is a lot harder to tilt entire orbit than it is the spin. Therefore the orbital contribution to paramagnetism is small.

★ However, there is a more significant effect on the orbital motion:

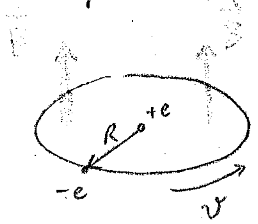
⇝ The e^- speeds up or slows down, depending on the orientation of $\vec{B}$

- If there is no $\vec{B}$: $F = ma$ → m_e: mass of e^-

$$(*) \quad \frac{1}{4\pi\epsilon_0}\frac{e^2}{R^2} = m_e\frac{v^2}{R}$$

there will be an additional force $-e(\vec{v}\times\vec{B})$:

- In presence of $\vec{B}$,



$$(**) \quad \frac{1}{4\pi\epsilon_0}\frac{e^2}{R^2} + e\bar{v}B = m_e\frac{\bar{v}^2}{R}$$

$\bar{v}$: new speed of e^- after we turn on $\vec{B}$.

It is added to this force because $-e(\vec{v}\times\vec{B})$ is also toward the center.

- $\bar{v} > v$: If we subtract $(*)$ from $(**)$:

$$e\bar{v}B = m_e\frac{\bar{v}^2}{R} - \frac{m_e}{R}v^2 = \frac{m_e}{R}\left[\bar{v}^2 - v^2\right]$$

$$= \frac{m_e}{R}(\bar{v}+v)(\bar{v}-v)$$

$e\bar{v}B = \frac{m_e}{R}(\bar{v}+v)\Delta v$

$\bar{v}\left(1+\frac{v}{\bar{v}}\right)\Delta v$

$e\bar{v}B = 2\bar{v}\Delta v\frac{m_e}{R}$

- If we assume $\Delta v \equiv \bar{v} - v$ is small

$$\frac{eRB}{m_e} = \frac{(\bar{v}+v)}{\bar{v}}\Delta v = \left(1 + \underbrace{\frac{v}{\bar{v}}}_{\sim 1}\right)\Delta v \Rightarrow \boxed{\Delta v \simeq \frac{eRB}{2m_e}}$$

⇒ When $\vec{B}$ is turned on e^- speeds up.

★ Previously, we showed that $\vec{B}$ does no work, and incapable of speeding the particle up. However, here we miss the fact that we are turning on the $\vec{B}$; so actually we are changing $\vec{B}$. We will see later that changing $\vec{B}$ induces some $\vec{E}$, and it is this $\vec{E}$ which actually speeds up the e^-.

- Change in orbital speed means a change in the dipole moment

$$\Delta \vec{m} = -\frac{1}{2} e \Delta v R \hat{z} = -\frac{1}{2} e \frac{eRB}{2m_e} R \hat{z} = -\frac{e^2 R^2}{4m_e} \vec{B} \rightarrow \boxed{\Delta \vec{m} = -\frac{e^2 R^2}{4 m_e} \vec{B}}$$

☆ Note that the change in $\vec{m}$ is opposite to the direction of $\vec{B}$
(An e^- circling the other way would have a dipole moment pointing upward but such an orbit would be slowed down by the field, so change in $\vec{m}$ is still opposite to $\vec{B}$) ~ think of $-e(\vec{v} \times \vec{B})$ ~

- Normally, e^- orbits are randomly oriented and orbital magnetic moments cancel out. But, in presence of $\vec{B}$, each atom pick a little extra $\vec{m}$, and these increments are anti-parallel to $\vec{B}$. This mechanism is called diamagnetism.

☆ Diamagnetism is a universal phenomenon, affecting all atoms. However, it is much weaker than paramagnetism. It is usually observed mainly in atoms with even # of e^-, where paramagnetism is usually absent.

<u>Magnetization</u>

- In presence of $\vec{B}$, matter becomes magnetized. There are 2 mechanisms behind it
  i) <u>Paramagnetism</u>: Dipoles associated with the spins of unpaired e^- experience a torque, lining them up parallel to the field.
  ii) <u>Diamagnetism</u>: The orbital speed of the e^- altered when $\vec{B}$ is introduced such that the change in their $\vec{m}$ is in a direction opposite to $\vec{B}$.

- Whatever the cause, we will describe the state of magnetic polarization by the vector quantity.

$$\boxed{\vec{M} : \text{magnetic dipole moment per unit volume.}} \qquad \frac{I\,m^2}{m^3}$$

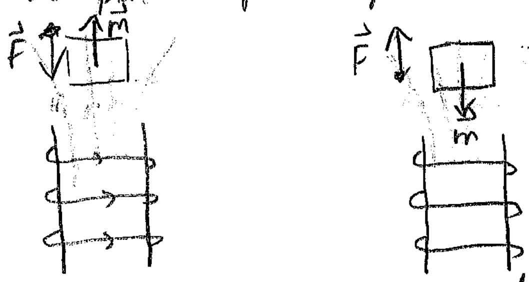
We will call it "magnetization". $\xleftrightarrow{\text{analogous}}$ $\vec{P}$.

- We aren't interested in how the magnetization got there (paramagnetism, diamagnetism, ferromagnetism) We will take $\vec{M}$ as given

- Note that we can magnetize ordinary material like glass and wood. We usually have the tendency that only some known materials like iron, nickel and cobalt are affected by the magnetic field.

- Yet, in practice, it is impossible to see the reaction of a piece of wood when we put a bar magnet next to it. The reason is paramagnetism and diamagnetism are extremely weak. They can only be detected very accurate experiments.

- When we place a paramagnetic material over a solenoid



$\rightsquigarrow$ These forces are extremely weak. So, we need very accurate experiments to detect them.

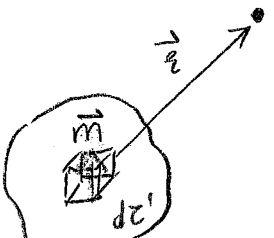
Paramagnets are ~~repelled~~ attracted

Diamagnets are ~~attracted~~ repelled

The Field of a Magnetized Object

Suppose we have a piece of magnetized material : $\vec{M}$ is given. what $\vec{B}$ does this object produce?

$$\vec{A}(\vec{r}) = \frac{\mu_0}{4\pi} \frac{\vec{m} \times \hat{\mathscr{r}}}{\mathscr{r}^2}$$: vector potential of a single dipole.



Therefore, each volume element carries a dipole moment $\vec{M}\, d\tau'$:

$$\boxed{\vec{A}(\vec{r}) = \frac{\mu_0}{4\pi} \int \frac{\vec{M}(\vec{r}\,') \times \hat{\mathscr{r}}}{\mathscr{r}^2}\, d\tau'}$$; total vector potential.

- Therefore, we can obtain $\vec{B} = \vec{\nabla} \times \vec{A}$. However as in the electrical case, it is useful to express above integral in a different way:

$\rightsquigarrow$ we know the identity $\vec{\nabla}' \frac{1}{\mathscr{r}} = \frac{\hat{\mathscr{r}}}{\mathscr{r}^2}$

$$\Rightarrow \vec{A}(\vec{r}) = \frac{\mu_0}{4\pi} \int \left[\vec{M}(\vec{r}\,') \times \left(\vec{\nabla}' \frac{1}{\mathscr{r}} \right) \right] d\tau'$$

$\rightsquigarrow$ Consider the product rule $\vec{\nabla} \times (f\vec{A}) = f(\vec{\nabla} \times \vec{A}) - \vec{A} \times (\vec{\nabla} f)$

$\vec{A} \leftrightarrow \vec{M}$ and $f \leftrightarrow \frac{1}{\mathscr{r}}$

$$\Rightarrow \vec{\nabla}' \times \left(\frac{1}{\mathscr{r}} \vec{M} \right) = \frac{1}{\mathscr{r}} (\vec{\nabla}' \times \vec{M}) - \vec{M} \times \left(\vec{\nabla}' \frac{1}{\mathscr{r}} \right)$$

$$\Rightarrow \vec{M}(\vec{r}\,') \times \left(\vec{\nabla}' \frac{1}{\mathscr{r}} \right) = \frac{1}{\mathscr{r}} \left[\vec{\nabla}' \times \vec{M}(\vec{r}\,') \right] - \vec{\nabla}' \times \left[\frac{\vec{M}(\vec{r}\,')}{\mathscr{r}} \right]$$

$$\vec{A}(\vec{r}) = \frac{\mu_0}{4\pi} \int \frac{1}{\mathscr{r}} \left[\vec{\nabla}' \times \vec{M}(\vec{r}\,') \right] d\tau' - \int \vec{\nabla}' \times \left[\frac{\vec{M}(\vec{r}\,')}{\mathscr{r}} \right] d\tau'$$

(Hw) Prove that $\int \vec{\nabla}' \times \left[\frac{\vec{M}(\vec{r}\,')}{\mathscr{r}} \right] d\tau' = - \oint \frac{1}{\mathscr{r}} \left[\vec{M}(\vec{r}\,') \times d\vec{a}\,' \right]$ $\rightsquigarrow$ See. Prob. 1.60 It is related with div. theorem.

Therefore,

$$\vec{A}(\vec{r}) = \underbrace{\frac{\mu_0}{4\pi}\int \frac{1}{\mathscr{r}}\left[\vec{\nabla}' \times \vec{M}(\vec{r}')\right] d\tau'}_{\text{Volume integral}} + \underbrace{\frac{\mu_0}{4\pi}\oint \frac{1}{\mathscr{r}}\left[\vec{M}(\vec{r}') \times d\vec{a}'\right]}_{\text{surface integral.}}$$

Remember:

$$\vec{A}(\vec{r}) = \frac{\mu_0}{4\pi}\int \frac{\vec{J}(\vec{r}')}{\mathscr{r}} d\tau' \quad \text{and} \quad \vec{A}(\vec{r}) = \frac{\mu_0}{4\pi}\int \frac{\vec{K}(\vec{r}')}{\mathscr{r}} da'$$

From the analogy between above equations:

$$\boxed{\vec{J}_b(\vec{r}') \equiv \vec{\nabla}' \times \vec{M}(\vec{r}')} \quad \text{and} \quad \boxed{\vec{K}_b(\vec{r}') \equiv \vec{M}(\vec{r}') \times \hat{n}}$$

bound volume current

bound surface current.

where $\hat{n}$ is the normal unit vector

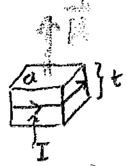
with these definitions:

$$\boxed{\vec{A}(\vec{r}) = \frac{\mu_0}{4\pi}\int_{\mathcal{V}} \frac{\vec{J}_b(\vec{r}')}{\mathscr{r}} d\tau' + \frac{\mu_0}{4\pi}\oint_{S} \frac{\vec{K}_b(\vec{r}')}{\mathscr{r}} da'}$$

★ Therefore, the above equation tells us that the potential of a magnetized object is the same as would be produced by a volume current $\vec{J}_b = \vec{\nabla} \times \vec{M}$ through the material, plus a surface current $K_b := \vec{M} \times \hat{n}$ on the boundary.

→ Instead of integrating the contributions of all infinitesimal dipoles, we can first find the bound currents and then find the field they produce in the same way we would calculate any other volume or surface currents.

→ The above situation is very similar to $\rho_b = -\vec{\nabla} \cdot \vec{P}$ and $\sigma_b = \vec{P} \cdot \hat{n}$ in electrostatics.



$M = \frac{m}{at}$: dipole moment per unit volume.

$m = Ia$

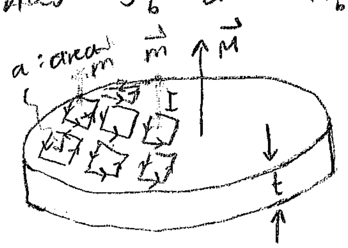
$M = \frac{I\cancel{a}}{\cancel{a}t} \Rightarrow I = Mt$

$K_b = \frac{I}{t} = M \rightsquigarrow \boxed{\vec{K}_b = \vec{M} \times \hat{n}}$

• No current on the top or bottom surface of the disc since $\vec{M} \parallel \hat{n}$

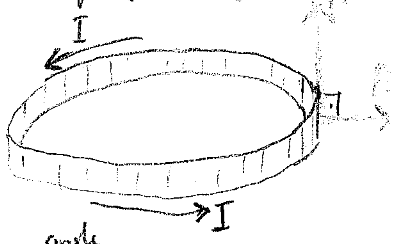
Physical Interpretation of Bound Currents

How $\vec{J}_b$ and $\vec{K}_b$ arise physically?



A disc of magnetized material.

★ All internal currents cancel: Everytime one is going to the right, an opposite is going to left.



A single ribbon of current I

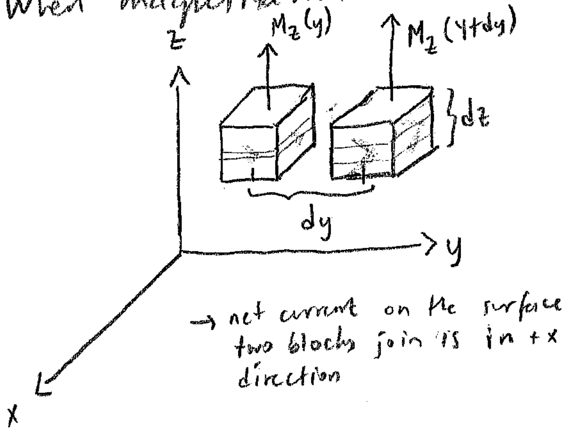
However → ★ At the edge no opposite currents to do cancelling.

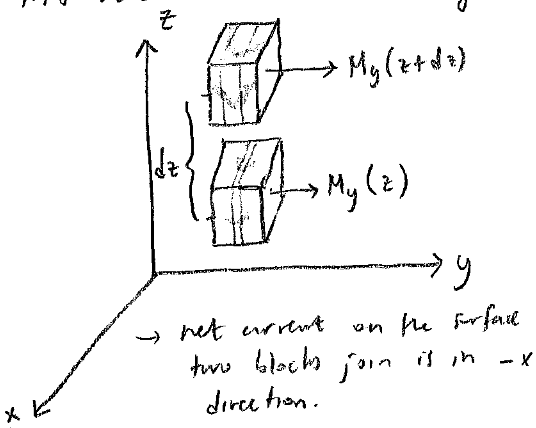
★ Left system is equivalent to the right system

☆ No single charge makes the whole trip; each charge moves only in a tiny little loop within a single atom. Nevertheless, the net effect is a macroscopic current flowing over the surface of the magnetized object.

☆ The term "bound" reminds us of that every charge is attached to a particular atom, but they still collectively create a genuine surface current producing a magnetic field.

• When magnetization is nonuniform the internal currents no longer cancel.





$$I_x = \left[M_z(y+dy) - M_z(y)\right]dz$$

$$= \frac{\partial M_z}{\partial y}\,dy\,dz \rightsquigarrow \frac{I_x}{dy\,dz} = \frac{\partial M_z}{\partial y}$$

$$\Rightarrow (J_b)_x = \frac{\partial M_z}{\partial y}$$

Therefore a nonuniform magnetization in y direction would similarly contribute an amount $-\frac{\partial M_y}{\partial z}$

$$\Rightarrow (J_b)_x = \frac{\partial M_z}{\partial y} - \frac{\partial M_y}{\partial z}$$

$\Rightarrow$ In general: $\boxed{\vec{J}_b = \vec{\nabla} \times \vec{M}}$

Remember continuity equation: $\vec{\nabla}\cdot\vec{J} = -\partial\rho/\partial t$

• For a steady current $-\partial\rho/\partial t = 0 \rightarrow \vec{\nabla}\cdot\vec{J} = 0$ otherwise some charge would pile up in the wire.

• This is also applied to $\vec{J}_b$:

$$\vec{\nabla}\cdot\vec{J}_b = 0 \quad \text{because } \vec{\nabla}\cdot(\vec{\nabla}\times\vec{M}) = 0$$

The Magnetic Field Inside Matter:

• Like $\vec{E}$, actual microscopic magnetic field fluctuates inside matter abruptly from point to point.

• when we speak of magnetic field inside matter we mean macroscopic field; the average over regions large enough to contain many atoms. The $\vec{M}$ is smoothed out in the same way.

Ampère's Law in Magnetized Materials and Auxilary field $\vec{H}$:

We will put everything together: the field attributable to bound currents plus the field due to everything else.
We will call the latter one free current

★ The free current might flow through wires imbedded in the magnetized substance or if it is a conductor, through the material itself.

~~Ele~~ Equatorial Linear velocity for electron

$\vec{K} = \sigma \vec{v}$

$\mu_e \cong -9.28 \times 10^{-24} \frac{J}{T}$

$\vec{v} = \omega R \sin\theta \, \hat{\phi}$

$\omega = 2\pi f \rightarrow \omega = \frac{2\pi}{T} \rightarrow T = \frac{2\pi}{\omega}$

m

$V = \frac{2\pi R \sin\theta}{\frac{2\pi}{\omega}} = R\omega \sin\theta$

The ~~mag~~ $\vec{m}$ for a uniformly magnetized sphere will be exactly the same with the $\vec{m}$ of a spinning spherical shell.

$$\frac{4}{3}\pi R^3 \sigma R \omega = M \frac{4}{3}\pi R^3$$

m

$\frac{\cancel{A} m^2}{s \cancel{m} \phi}$ ✓

$$m = \frac{4}{3}\pi R^4 \sigma \omega$$

$$\omega = \frac{3m}{4\pi R^4 \sigma}$$

$V = R\omega \sin\theta$

$V_{eq} = R\omega$

$$V_{eq} = \frac{3}{4} \frac{m}{\pi R^3 \sigma}$$

$\sigma = \frac{e}{4\pi R^2}$

$3 \times 10^8 \frac{m}{s}$

$$\frac{3 \times 9.28 \times 10^{-24}}{1.4 \times 10^{-15} \times 1.6 \times 10^{-19}} = \frac{3}{\cancel{4}} \frac{m}{\cancel{\pi} R^{\cancel{3}} \frac{e}{\cancel{4\pi} \cancel{R^2}}}$$

$$\frac{30 \times 10^{10}}{2.25} \frac{m}{s}$$

13.3×10^{10}

$$V_{eq} = \frac{3m}{Re}$$

66'

• The total current can be written as

$$\boxed{\vec{J} = \vec{J}_b + \vec{J}_f}$$

• We simply separated the current into two parts. $\vec{J}_f$ is there because someone connected the wire to a battery (it involves the actual transport of charge). $\vec{J}_b$ is there because of magnetization (it results from many aligned atomic dipoles.)

Ampère's law : $\vec{\nabla}\times\vec{B} = \mu_0\vec{J} = \mu_0(\vec{J}_b+\vec{J}_f)$

$$\Rightarrow \frac{1}{\mu_0}(\vec{\nabla}\times\vec{B}) = \vec{J}_b+\vec{J}_f = \vec{\nabla}\times\vec{M}+\vec{J}_f$$

$$\Rightarrow \frac{1}{\mu_0}(\vec{\nabla}\times\vec{B}) - \vec{\nabla}\times\vec{M} = \vec{J}_f$$

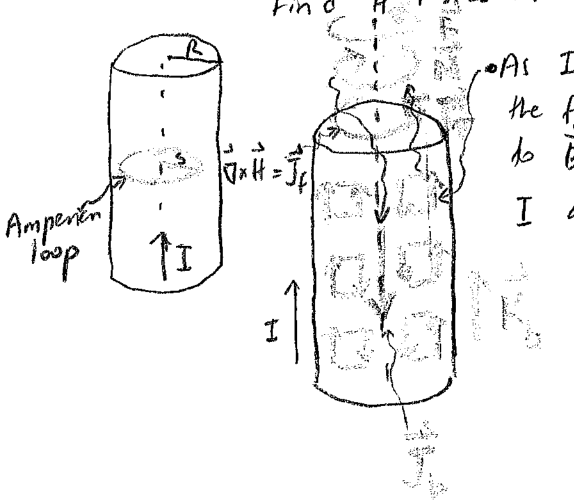
$$\Rightarrow \vec{\nabla}\times\left(\frac{1}{\mu_0}\vec{B}-\vec{M}\right) = \vec{J}_f \qquad \boxed{\vec{H} \equiv \frac{1}{\mu_0}\vec{B}-\vec{M}}$$

$$\Rightarrow \boxed{\vec{\nabla}\times\vec{H} = \vec{J}_f} \xrightarrow[\text{form}]{\text{in integral}} \boxed{\oint \vec{H}\cdot d\vec{l} = I_{fenc}}$$

I_{fenc} : total free current passing through the Amperian loop.

☆ $\vec{H} \xleftrightarrow{\text{analogous}} \vec{D}$: $\vec{H}$ permits us to express Ampère's Law in terms of the free current alone – and free current is what we control directly. Bound current is created when the material gets magnetized; we cannot turn them on and off independently.

Example 6.2) A long copper of radius R carries a uniformly distributed (free) current I. Find $\vec{H}$ inside an outside the rod. (Given: copper is weakly diamagnetic)



• As I flow through the copper the dipoles line up opposite to the field. This corresponds to a circular magnetization opposite to $\vec{B}$. And this $\vec{M}$ creates a bound current $\vec{J}_b$ antiparallel to I and a surface current parallel to I.

• we don't have enough information to calculate how great $\vec{J}_b$ and K_b. However we can calculate $\vec{H}$ easily: <u>Inside the wire:</u> $(s \leq R)$

$$\oint \vec{H}\cdot d\vec{l} = I_{fenc}.$$

$$H\,2\pi \cancel{s} = I_{fenc} = I\,\frac{\cancel{\pi} s^{\cancel{2}}}{\cancel{\pi}R^2}$$

$$\Rightarrow \boxed{\vec{H} = \frac{I}{2\pi R^2}\,s\,\hat{\phi} \quad (s\leq R)}$$

☆ We cannot know what is $\vec{B}$ inside the wire, since we have no way of knowing $\vec{M}$ at this stage

<u>Outside the wire</u> $(s>R)$

$$H\,2\pi s = I$$

$$\boxed{\vec{H} = \frac{I}{2\pi s}\,\hat{\phi}}$$

when $s>R$, $\vec{M}=0$

$$\vec{H} = \frac{1}{\mu_0}\vec{B} - \cancel{\vec{M}}^{\,0}$$

$$\boxed{\vec{B} = \mu_0\vec{H} = \frac{\mu_0 I}{2\pi s}\,\hat{\phi}}$$

the same as for a nonmagnetized wire.

- In practice (in a laboratory) experimentalists usually consider $\vec{H}$ because the current is the thing they read on the dial. It directly determines $\vec{H}$. However $\vec{D}$ isn't in that fashion in the lab because if you want to set up an $\vec{E}$ you don't put some amount of free charge on the plates of a parallel plate capacitor. Instead you connect it to a battery of known voltage. It is the V that they read on their dial, and it determines $\vec{E}$.

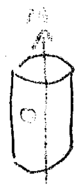
A Deceptive Anology

$\mu_0 \vec{H}$ $\quad$ $\vec{J}_f$

- $\vec{\nabla} \times \vec{H} = \vec{J}_f$ looks like $\vec{\nabla} \times \vec{B} = \mu_0 \vec{J}$ But they are essentially different.
- $\vec{\nabla} \times \vec{B} = \mu_0 \vec{J}$ is not enough to determine $\vec{B}$, we should also know what $\vec{\nabla} \cdot \vec{B}$ is. In magnetostatics $\vec{\nabla} \cdot \vec{B} = 0$ and we usually forget about this fact and simply think that $\vec{\nabla} \times \vec{B} = \mu_0 \vec{J}$ is enough to obtain $\vec{B}$.
- $\vec{\nabla} \cdot \vec{H} \neq 0$ in general.

From $\vec{H} \equiv \frac{1}{\mu_0} \vec{B} - \vec{M} \longrightarrow \vec{\nabla} \cdot \vec{H} = -\vec{\nabla} \cdot \vec{M}$ Therefore if $\vec{\nabla} \cdot \vec{M}$ vanishes then the analogy between $\vec{B}$ and $\mu_0 \vec{H}$ would be reasonable.

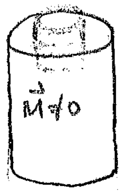
- Consider a cylinder-shaped bar magnet carrying a permanent uniform magnetization parallel to its axis. No free current anywhere and if we näively apply $\oint \vec{H} \cdot d\vec{\ell} = I_{f,enc} \rightarrow \vec{H} = 0$



$$\left.\begin{aligned} &\Rightarrow \vec{B} = \mu_0 \vec{M} \text{ inside the magnet} \\ &\Rightarrow \vec{B} = 0 \text{ outside the magnet.} \end{aligned}\right\} \text{This is no sense.}$$

- It is true that $\vec{\nabla} \times \vec{H} = 0$ everywhere because $\vec{J}_f = 0$. However, $\vec{\nabla} \cdot \vec{H} = -\vec{\nabla} \cdot \vec{M} \neq 0$
- $\vec{\nabla} \cdot \vec{M} \neq 0$ because magnetic field lines do not form a closed loop inside the magnet so that there is a source of magnetization inside the magnet.

Vacuum $\vec{M} = 0$



Therefore $\oint \vec{M} \cdot d\vec{a} \neq 0$

$\int (\vec{\nabla} \cdot \vec{M})\, d\tau = \oint \vec{M} \cdot d\vec{a} \rightarrow \vec{\nabla} \cdot \vec{M} \neq 0$

Magnetic Susceptibility and Permeability

In para/diamagnetic materials the $\vec{M}$ is sustained by the field. When $\vec{B}$ is removed $\vec{M}$ disappears. For most of the materials magnetization is proportional to the field, provided the field is not too strong. To make some analogy with the electrical case, we can express

$$\vec{M} = \frac{1}{\mu_0} \chi_m \vec{B}$$

But commonly it is expressed in terms of $\vec{H}$ instead of $\vec{B}$:

$$\boxed{\vec{M} = \chi_m \vec{H}} \qquad \chi_m : \text{magnetic susceptibility.}$$

- χ_m is a dimensionless quantity; it varies from one substance to another.
  - $\rightarrow \chi_m > 0$ paramagnets
  - $\rightarrow \chi_m < 0$ diamagnets

Diamagnetic	χ_c
Gold	-3.4×10^{-5}
Copper	-9.7×10^{-6}

paramagnetic	χ_c
Aluminium	2.1×10^{-5}
oxygen	1.9×10^{-6}

- Materials that obey $\vec{M} = \chi_m \vec{H}$ are called linear media.

$$\vec{H} = \frac{1}{\mu_0}\vec{B} - \vec{M} \Rightarrow (\vec{H} + \vec{M})\mu_0 = \vec{B} \Rightarrow (\vec{H} + \chi_m \vec{H})\mu_0 = \vec{B} \rightarrow \boxed{\vec{B} = \mu_0 (1+\chi_m)\vec{H}}$$

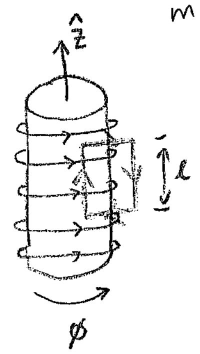
- Thus $\vec{B}$ is also proportional to $\vec{H}$

$$\boxed{\vec{B} = \mu \vec{H}} \quad \text{where } \mu \equiv \mu_0(1+\chi_m) \qquad \mu : \text{permeability of the material.}$$

- In vacuum, there is no matter to magnetize $\Rightarrow \chi_m = 0 \Rightarrow \mu = \mu_0$

$\Rightarrow \mu_0$: permeability of free space.

Example 6.3) An infinite solenoid (n turns per unit length, current I) is filled with linear material of susceptibility χ_m. Find the magnetic field inside the solenoid:



We cannot compute $\vec{B}$ directly with these givens because $\vec{B}$ is due to bound currents. However, we can calculate $\vec{H}$ from free current alone.

$$\oint \vec{H} \cdot d\vec{l} = I_{fenc}$$

$$H\cancel{l} = n\cancel{l} I$$

$$\boxed{\vec{H} = nI\hat{z}}$$

Since $\vec{B} = \mu_0(1+\chi_m)\vec{H}$

$$= \mu_0(1+\chi_m) n I \hat{z}$$

- If the medium is paramagnetic $\vec{B}$ is slightly increased. If it is diamagnetic $\vec{B}$ is slightly decreased.

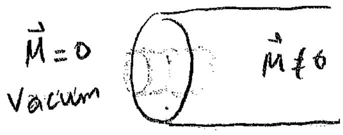
$$\vec{K}_b = \vec{M} \times \hat{n}$$
$$= \chi_m (\vec{H} \times \hat{n})$$
$$= \chi_m n I (\hat{z} \times \hat{n})$$
$$= \chi_m n I \hat{\phi}$$

- $\chi_m > 0$ surface current is in the same dir. with I
- $\chi_m < 0$ $\vec{K}_b$ is in the opposite direction with I.

• One last comment about the analogy between $\vec{H}$ and $\vec{B}$:

Since $\left.\begin{array}{l} \vec{M} = \chi_m \vec{H} \\ \vec{B} = \mu \vec{H} \end{array}\right\}$ $\vec{M}$ & $\vec{H}$ are proportional to $\vec{B}$. So does it mean that their divergence will vanish like $\vec{B}$'s?

→ No! Because of the same reason we discussed before.



$\oint \vec{M} \cdot d\vec{a} \neq 0 \Rightarrow \vec{\nabla} \cdot \vec{M} \neq 0$

Ferromagnetism

• In linear medium alignment of atomic dipoles is maintained by an external magnetic field.

• Ferromagnets do not require to sustain the magnetization