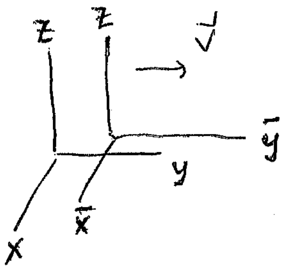
Magnetism as a relativistic phenomenon



★ Same laws apply in any inertial reference frame.

$F = ma$

Galilean transformations: $\bar{x} = x - vt$

$\bar{y} = y$

$\bar{z} = z$

$\bar{t} = t$ ⇝ (we will ~~talk~~ consider this in detail about this later)

$F = m\ddot{x}$ ⇝ $\bar{F} = m\ddot{\bar{x}} = \bar{F} = m\ddot{x}$ ~~Same~~ classical mechanics obeys the principle of relativity.

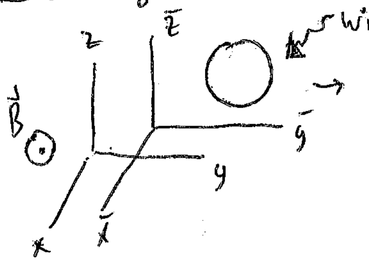
(?) Does the principle of relativity apply to the laws of electrodynamics?

Apparently No: Moving charge creates a magnetic field whereas charge at rest does not.

Remember Lorentz Law: $\vec{F}_{mag} = q\,\vec{v}\times\vec{B}$ — v dependence! explicit reference to the velocity.

▼ This kind of reminds us ~~that~~ "the existence of a unique stationary reference frame" with respect to all velocities are to be measured

Interesting coincidence:



$\vec{B}$ is in x direction as it is created by a magnet in x, y, z coordinates.

From x, y, z:

As the loop rides through the magnetic field motional emf is established, and acc. to flux rule

$$\mathcal{E} = -\frac{d\Phi}{dt}$$

▼ And there will be a "magnetic force"

From $\bar{x}, \bar{y}, \bar{z}$:

If someone in $\bar{x}, \bar{y}, \bar{z}$ naively applied the laws of edynamics, they would predict "no magnetic force" ?

Yet, as the magnet keeps going the magnetic field in $\bar{x}, \bar{y}, \bar{z}$ system would change and it induces an electric field by Faraday's law. The resulting electric force would generate an emf in the loop given by:

$$\mathcal{E} = -\frac{d\bar{\Phi}}{dt}$$

Faraday's law and flux rule predict exactly the same emf, people in both ref. frames will get the right answer. even though their physical interpretation of the process is completely opposite.

"1905 - A. Einstein - On the electrodynamics of Moving bodies"

▼ Electromagnetic phenomena like mechanical ones must obey the principle of relativity.

* Scientist until Einstein widely believed that e and m field are created and propagate in a medium called <u>ether</u>. From this point of view, there was no problem about the previous issue: ground observer was right and the other was wrong. (with a naive observation)

⇒ However, we need to be careful and sure that ground observer is at rest. All in all, the whole problem until Einstein to find that rest ether frame.

★ No experiment could prove the existance of ether (Michelson and Morley experiment) No "ether wind" was detected by measuring the speed of light in opposite directions on earth.

Einstein, inspired by theoretical (maybe experimental) hints, proposed two famous postulates.

maybe or correlated. i) The principle of relativity: The laws of physics apply in all inertial reference system

ii) The universal speed of light: The speed of light in vacuum is the same for all inertial observers, regardless of the motion of the source.

(?) What kind of results these 2 assumption will bring us? Before that lets comment on timing issue in inertial ref. frames.

<u>Measuring distance, time and ~~synchorization~~ synchronization of clocks.</u>

Event: any change happening around us: departure of a train, a flash of ~~lightning~~ lightning, turning on a flashlight

What does it mean to accept the existance of a ref frame?

→ Measuring distance (Accepting the existence of a coordinate system actually means that space is divided into premeasured equally spaced parcels)

→ [illegible]: After an event happens at a given location it takes some time for the light to reach us. ~~Fer simply~~ By Instead of calculating the real time of an event we simply put a time tag to the event just as we put a location tag.

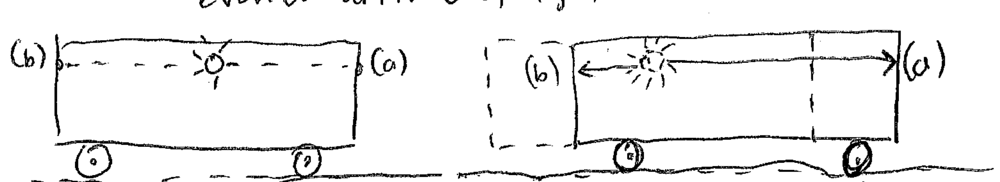
→ ~~It requires us to t~~ we can think about this in terms of synchronization of clocks.

<u>Geometry of relativity</u>: (Some thought experiments)

i) The relativity of ~~simultanity~~ simultainity:

★ Two events that are simultaneous in one inertial system are not in general simultaneous in another.

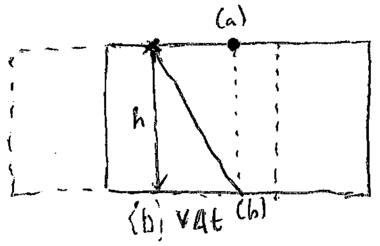
event: arrival of light to a certain point



In car ref frame light reaches points (a) and (b) at the same time

In ground ref frame (b) happens before (a).

(ii) Time dilation



A light ray that leaves the bulb and strikes the floor of the car directly below.

Q: How long does it take the light to make this trip?

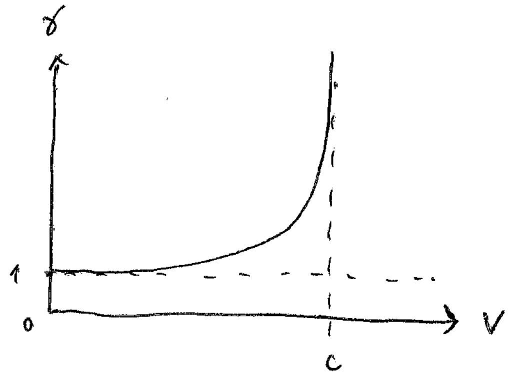
- An observer, inside the train: $\Delta\bar{t} = \frac{h}{c}$
- An observer, on the ground: $\Delta t = \frac{\sqrt{h^2+(v\Delta t)^2}}{c}$

$$\Delta t^2 = \frac{h^2+(v\Delta t)^2}{c^2} \rightarrow \Delta t^2 - \frac{v^2\Delta t^2}{c^2} = \frac{h^2}{c^2} \rightarrow \Delta t^2\left(1-\frac{v^2}{c^2}\right) = \frac{h^2}{c^2} \rightarrow \boxed{\Delta t = \frac{h}{c}\frac{1}{\sqrt{1-v^2/c^2}}}$$

$$\Rightarrow \boxed{\Delta\bar{t} = \sqrt{1-\frac{v^2}{c^2}}\,\Delta t}$$

Let us define the scaling factor:

$$\gamma \equiv \frac{1}{\sqrt{1-\frac{v^2}{c^2}}} \quad \Rightarrow \boxed{\Delta\bar{t} = \frac{\Delta t}{\gamma}}$$



* Time elapsed between the same two events
  (a) light leaves bulb.
  (b) light strikes center of floor.

* The interval recorded on train clock is shorter by the factor γ

Conclusion: Moving clocks run slow.
(Ground clock will measure more time for each passing clock tick on the train)

Ex: If the car goes with $V = 0.5c$, what would ground observer measure for each second passed in the train?

$$1s = \sqrt{1-\frac{(0.5c)^2}{c^2}}\,\Delta t = \sqrt{1-\frac{1}{4}\frac{c^2}{c^2}} = \frac{\sqrt{3}}{2}\Delta t \rightarrow \Delta t = \frac{2}{\sqrt{3}} \cong 1.155s.$$

Is the time dilation inconsistent with the principle of relativity?

- car observer says the train clock runs slow.
- From car's point of view can with equal justice claim that ground clock runs slow.

(?) Who is right? Both. Clocks that are properly synchronized in one system will not be synchronized when observed from another system.

Ex: The twin paradox: On her 21st bday, an astronaut takes off in a rocket ship at a speed of $\frac{12}{13}c$. After 5 years elapsed on her watch, she turns around and heads back at the same speed to rejoin her twin brother, who stayed home. Question: How old is each twin at their reunion?

From earth view:

$$\gamma = \frac{1}{\sqrt{1-\left(\frac{12}{13}\right)^2}} = \frac{13}{5}$$

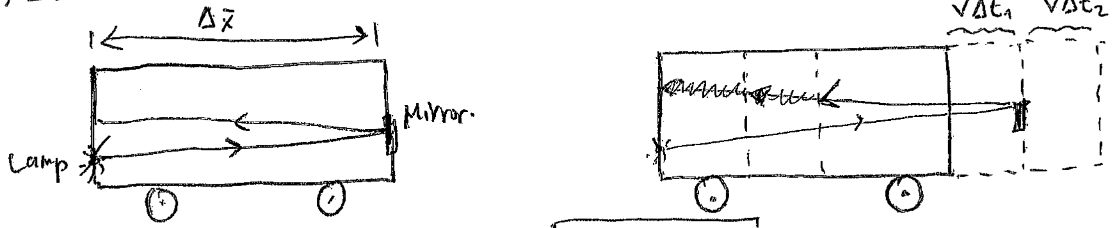
$$\Delta t = \gamma\Delta\bar{t} = \frac{13}{5}\times 10 = 26 \text{ : in earth frame}$$

Twin on earth $= 21+26 = 47$

Twin on the rocket $= 21+10 = 31$

★ The so-called paradox arises when we tell this story from the point of view of travelling twin. → Travelling twin isn't in an inertial ref frame. The twins aren't equivalent. The second point of view is simply wrong.

(iii) Lorentz contraction : How long does the signal take to complete the round trip?



• To an observer in the train $\boxed{\Delta \bar{t} = 2\dfrac{\Delta \bar{x}}{c}}$ (*)

$\Delta \bar{x}$: measurements made in the train.

• To an observer on the ground

$$\Delta t_1 = \frac{\Delta x + V\Delta t_1}{c} \qquad \Delta t_2 = \frac{\Delta x - V\Delta t_2}{c}$$

$$\Delta t_1 - \frac{V\Delta t_1}{c} = \frac{\Delta x}{c} \qquad \Delta t_2 + \frac{V\Delta t_2}{c} = \frac{\Delta x}{c}$$

$$\Delta t_1\left(1-\frac{V}{c}\right) = \frac{\Delta x}{c} \qquad \Delta t_2\left(1+\frac{V}{c}\right) = \frac{\Delta x}{c}$$

$$\Delta t_1 = \frac{\Delta x}{\cancel{c}}\frac{\cancel{c}}{c-V} \qquad \Delta t_2 = \frac{\Delta x}{\cancel{c}}\frac{\cancel{c}}{c+V}$$

$$\Delta t_1 = \frac{\Delta x}{c-V} \qquad \Delta t_2 = \frac{\Delta x}{c+V}$$

So the round-trip time is :

$$\Delta t = \Delta t_1 + \Delta t_2 = \frac{\Delta x}{c\left(1-\frac{V}{c}\right)} + \frac{\Delta x}{c\left(1+\frac{V}{c}\right)} = \frac{\Delta x}{c}\left[\frac{1+\cancel{V/c} + 1-\cancel{V/c}}{1-\frac{V^2}{c^2}}\right] = 2\frac{\Delta x}{c}\frac{1}{(1-V^2/c^2)}$$

$$\boxed{\Delta t = 2\frac{\Delta x}{c}\frac{1}{(1-V^2/c^2)}} \text{ (**)} \qquad \gamma = \frac{1}{\sqrt{1-V^2/c^2}} \rightarrow \gamma^2 = \frac{1}{1-V^2/c^2}$$

By using (*) and (**) : (and by using $\frac{\Delta \bar{t}}{\Delta t} = \frac{1}{\gamma}$)

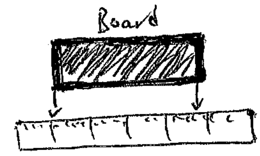
$$\frac{\Delta \bar{t}}{\Delta t} = \frac{\Delta \bar{x}}{\Delta x}\left(1-\frac{V^2}{c^2}\right) \rightarrow \frac{1}{\cancel{\gamma}} = \frac{\Delta \bar{x}}{\Delta x}\underbrace{\left(1-\frac{V^2}{c^2}\right)}_{\gamma^2} \rightarrow \Delta \bar{x} = \gamma \Delta x$$

• Of course, observer in the train doesn't think her car is shortened, her meter sticks are contracted by that same amount.

• From the train observer's point of view, it is objects on the ground are shortened.

A; Train observer → B is shortened
B; Ground observer → A is shortened.
} Who is right ? : Both!!

• Assume we want to find ℓ :



Board is still : Record readings at each end, and substract them to get the length of the board.

If the board **is moving:** we need to read two ends at the same instant time. If we don't the board will move in the course of measurement, and obviously you'll get the wrong answer.

The observer on the ground will claim that he measured both ends at the same time. But the person watching him from the train will complain that he first read the front end, then waited ~~&~~ for a moment before reading the back end.

<u>Ex</u>: The barn and ladder paradox. (The notion of a "rigid" object loses its meaning in relativity, for when it changes its speed, different parts do not in general accelerate simultaneously — in this way the material streches or shrinks to reach the length appropriate to its new velocity

★ Dimensions perpendicular to the velocity are not contracted. We also assumed while deriving time dilation formula that the height of the train was the same. ~~Let us justify this with a thought exp. from Taylor and Wheeler:~~ The reason is simple: No movement in that direction :)

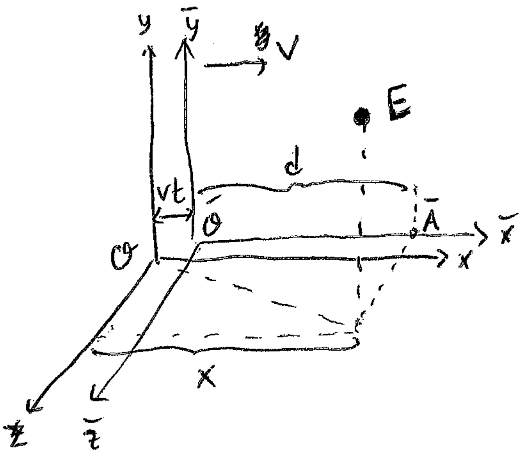
The Lorentz Transformations

Any physical process consists of one or more events.

<u>Event</u>: Something that takes place at a specific location (x, y, z) and precise time (t)

We know $\underbrace{\cancel{E}(x, y, z, t)}_{\text{coordinates in one inertial system } S}$ of a particular event and want to calculate $\underbrace{\cancel{E}(\bar{x}, \bar{y}, \bar{z}, \bar{t})}_{\text{coordinates of the same event in some other inertial system } \bar{S}}$

• We need a "dictionary" for translating from the language of S to the language of $\bar{S}$.



$\bar{S}$ slides along x axis at speed V. If we start the clock $t=0$ at the moment the origins O and $\bar{O}$ coincide then at time t, $\bar{O}$ will be a distance vt from O and hence

$$x = d + vt$$

Before 1900s anyone would have said immediately that

$$d = \bar{x}$$

• Thus, the ~~[illegible]~~ the required dictionary (or transformations)

$$\left.\begin{array}{ll}(i) & \bar{x} = x - vt \\ (ii) & \bar{y} = y \\ (iii) & \bar{z} = z \\ (iv) & \bar{t} = t\end{array}\right\}\text{ Galilean Transformation.}$$

without any doubt before Einstein, everyone assumed the flow of time was the same for all observers.

• In the context of special relativity. (ii) and (iii) are okay but (i) and (iv) should be replaced with the relativistic ones.

d: Distance from $\bar{O}$ to $\vec{A}$ as measured in S

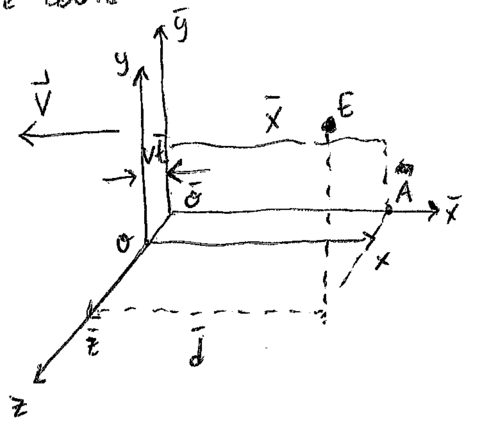
$\bar{x}$: Distance from $\bar{O}$ to $\bar{A}$ as measured in $\bar{S}$

$\bar{A}$: is the point on $\bar{x}$ axis which is even with E when the event occurs.

• Because $\bar{O}$ and $\bar{A}$ are at rest in $\bar{S}$, $\bar{x}$ is the "moving stick" which appears ~~to~~ contracted to S:

$$d = \frac{\bar{x}}{\gamma} \longrightarrow x = \underbrace{d}_{\frac{\bar{x}}{\gamma}} + vt \Rightarrow \boxed{(x - vt)\gamma = \bar{x}}$$

• We could have run the same argument from the point of view of $\bar{S}$



$$\bar{x} = \bar{d} - v\bar{t}$$

$\bar{d}$: is the distance from O to A at $\bar{S}$ system

A: is that point on x axis which is even with E when the event occurs.

$$\bar{d} = \frac{x}{\gamma} \rightarrow \bar{x} = \frac{x}{\gamma} - v\bar{t}$$

$$\bar{x} + v\bar{t} = \frac{x}{\gamma} \rightarrow \boxed{x = \gamma(\bar{x} + v\bar{t})}$$

For the symmetry of the situation dictates that the formula for x, in terms of $\bar{x}$ and $\bar{t}$ should be identical to the formula for $\bar{x}$ in terms of x and t except for a switch in the sign of v.

What about time?

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$$\cancel{\Delta\bar{t} = \sqrt{1 - \frac{v^2}{c^2}}\,\Delta t} \rightarrow \cancel{\Delta\bar{t} = \frac{\Delta t}{\gamma}} \qquad \gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$$

$$\cancel{\bar{t} = \frac{t}{\gamma}} \quad \text{Time dilation}$$

$$\bar{x} = \gamma(x - vt)$$

$$x = \gamma(\bar{x} + v\bar{t}) \rightarrow x = \gamma\left[\gamma(x - vt) + v\bar{t}\right]$$

$$x = \gamma^2(x - vt) + \gamma v\bar{t} \rightarrow x - \gamma^2(x - vt) = \gamma v\bar{t}$$

$$\bar{t} = \frac{x}{\gamma v} - \frac{\gamma^{\cancel{2}}(x - vt)}{\cancel{\gamma} v} = \gamma\left(\frac{x}{\gamma^2 v} - \frac{x}{v} + t\right)$$

$$= \gamma\left(\frac{x}{v\,\frac{1}{1-\frac{v^2}{c^2}}} - \frac{x}{v} + t\right)$$

$$= \gamma\left(\frac{x\left(1-\frac{v^2}{c^2}\right)}{v} - \frac{x}{v} + t\right)$$

$$= \gamma\left(\frac{x - \frac{xv^2}{c^2}}{v} - \frac{x}{v} + t\right) = \gamma\left(\cancel{\frac{x}{v}} - \frac{xv^{\cancel{2}}}{\cancel{v}c^2} - \cancel{\frac{x}{v}} + t\right)$$

$$= \gamma\left(t - \frac{v}{c^2}x\right)$$

<u>Lorentz transformations</u> → If I am S, this is what I think that is happens in $\bar{S}$.

$$\boxed{\begin{aligned} \bar{x} &= \gamma(x - vt) \\ \bar{y} &= y \\ \bar{z} &= z \\ \bar{t} &= \gamma\left(t - \frac{v}{c^2}x\right) \end{aligned}} \qquad \boxed{\begin{aligned} x &= \gamma(\bar{x} + v\bar{t}) \\ y &= \bar{y} \\ z &= \bar{z} \\ t &= \gamma\left(\bar{t} + \frac{v}{c^2}\bar{x}\right) \end{aligned}}$$

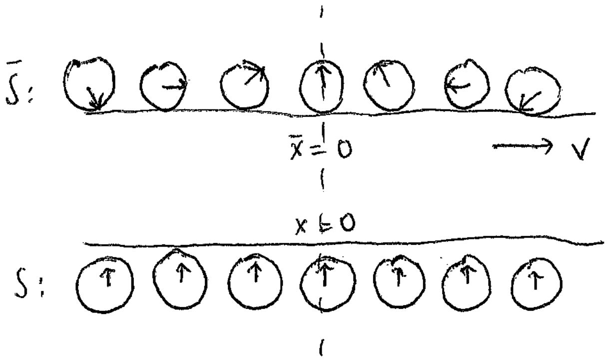
<u>Simultaneity, Synchronization and time dilation (revisited)</u>
Lorentz contraction

A occurs at $x_A = 0$ $t_A = 0$
B occurs at $x_B = b$ $t_B = 0$
} They are simultaneous in S

But they aren't simultaneous in $\bar{S}$:

$\bar{x}_A = 0$ $\bar{x}_B = \gamma b$ Acc. to $\bar{S}$ clocks B occured before A.

$\bar{t}_A = 0$ $\bar{t}_B = -\gamma\left(\frac{v}{c^2}\right)b$



Suppose that at time $t=0$ observer S decides to examine all clocks in $\bar{S}$. He finds that they read different times depending on their location

$$\bar{t} = -\gamma \frac{v}{c^2} x$$

Those to the left of origin (negative x) are ahead, and those to the right are behind by an amount that increases in proportion to their distance. Only master clock at origin reads $\bar{t}=0$. Thus, the nonsynchronization of moving clocks follows directly from the Lorentz transformations

Of course from $\bar{S}$, S clocks are out of sync.

- Suppose S focuses his attention on a single clock in $\bar{S}$ frame (say one at $\bar{x}=a$) and watches it over some interval Δt. How much time elapses on the moving clock? Because $\bar{x}$ is fixed now from

$$\left.\begin{aligned} t_1 &= \gamma\, \bar{t}_1 + \frac{v}{c^2}\bar{x} \\ t_2 &= \gamma\, \bar{t}_2 + \frac{v}{c^2}\bar{x} \end{aligned}\right\} \Delta t = t_2 - t_1 = \gamma(\bar{t}_2 - \bar{t}_1)$$

$$\Delta t = \gamma\, \Delta \bar{t}$$

$$\boxed{\Delta \bar{t} = \frac{\Delta t}{\gamma}}$$

* Note that we keep $\bar{x}$ fixed here because we are watching a single clock. If ~~we~~ we hold x fixed and watching a whole series of different $\bar{S}$ clocks as they pass by, ~~as~~ that won't tell us whether any of them is running slow

- Imagine a stick moving to the right at speed v. Its rest length (i.e., the length measured in $\bar{S}$) is $\Delta\bar{x} = \bar{x}_r - \bar{x}_\ell$ r, ℓ denote the right and left ends of the stick. If an observer in S were to measure the stick $\Delta x = x_r - x_\ell$ (two ends of the stick at one instant of his time t:

$$\bar{x}_r = \gamma(x_r - vt)$$
$$\bar{x}_\ell = \gamma(x_\ell - vt)$$

$$\bar{x}_r - \bar{x}_\ell = \Delta\bar{x} = \gamma\, \Delta x$$

Einstein's velocity addition rule:

Suppose a particle moves a distance dx (in S) in a time dt. Its velocity u is then

$$u = \frac{dx}{dt}$$

In $\bar{S}$, meanwhile, it has moved a distance

$$d\bar{x} = \gamma(dx - v\,dt)$$

in a time given by

$$d\bar{t} = \gamma\left(dt - \frac{v}{c^2}dx\right)$$

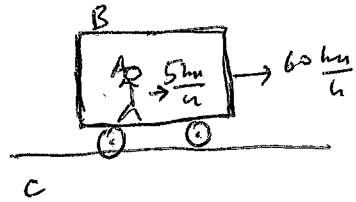
The velocity in $\bar{S}$ is therefore

$$\bar{u} = \frac{d\bar{x}}{d\bar{t}} = \frac{\cancel{\gamma}(dx - v\,dt)}{\cancel{\gamma}\left(dt - \frac{v}{c^2}dx\right)} = \frac{\cancel{dt}\left(\frac{dx}{dt} - v\right)}{\cancel{dt}\left(1 - \frac{v}{c^2}\frac{dx}{dt}\right)} = \frac{u - v}{1 - \frac{uv}{c^2}}$$

$$\boxed{\bar{u} = \frac{u - v}{1 - \frac{uv}{c^2}}}$$ Einstein's velocity addition rule.

Conventional ~~than~~ idea:

If we walk $5\,km/h$ down the corridor of a train going $60\,\frac{km}{h}$ our net speed relative to the ground is obviously $65\,\frac{km}{h}$



$$V_{AC} = V_{AB} + V_{BC}$$

V_{AC}: speed of me with respect to ground

V_{AB}: speed of me with respect to train

V_{BC}: speed of train with respect to ground

If we use EVAR:

A : particle u : V_{AB}

B : S $\bar{u}$: V_{AC}

C : $\bar{S}$ V :

The Structure of Space time

i) Four-vectors

Lorentz transformations will look simpler with the following transformation

$$X^0 \equiv ct \quad , \quad \beta \equiv \frac{v}{c} \rightarrow \text{unitless}$$

unit of time in meters :)

Using x^0 instead of t and β instead of v amounts to changing the unit of time from second to the meter : 1 meter of x^0 corresponds to the time it takes light to travel 1 meter (in vacuum) Also,

$$x^1 \equiv x, \quad x^2 \equiv y, \quad x^3 \equiv z$$

Lorentz transformations read

$$\left.\begin{aligned} \bar{X}^0 &= \gamma(x^0 - \beta x^1) \\ \bar{x}^1 &= \gamma(x^1 - \beta x^0) \\ \bar{x}^2 &= x^2 \\ \bar{x}^3 &= x^3 \end{aligned}\right\} \text{in matrix form} \quad \begin{pmatrix} \bar{x}^0 \\ \bar{x}^1 \\ \bar{x}^2 \\ \bar{x}^3 \end{pmatrix} = \begin{pmatrix} \gamma & -\gamma\beta & 0 & 0 \\ -\gamma\beta & \gamma & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x^0 \\ x^1 \\ x^2 \\ x^3 \end{pmatrix}$$

In index notation $\bar{x}^\mu = \sum_{\nu=0}^{3} (\Lambda^\mu_{\ \nu}) x^\nu$

(ii) The Invariant Interval

Event A occurs $(x_A^0, x_A^1, x_A^2, x_A^3)$

Event B occurs $(x_B^0, x_B^1, x_B^2, x_B^3)$

The difference $\Delta x^\mu \equiv x_A^\mu - x_B^\mu$

is the displacement 4-vector. The scalar product of Δx^μ with itself is a quantity of special importance; it is called the interval between two events

$$I \equiv (\Delta x)_\mu (\Delta x)^\mu = -(\Delta x^0)^2 + (\Delta x^1)^2 + (\Delta x^2)^2 + (\Delta x^3)^2 = -c^2t^2 + d^2$$

where t is the time difference btw two events and d is their spatial separation

(iii) Space - time diagrams :

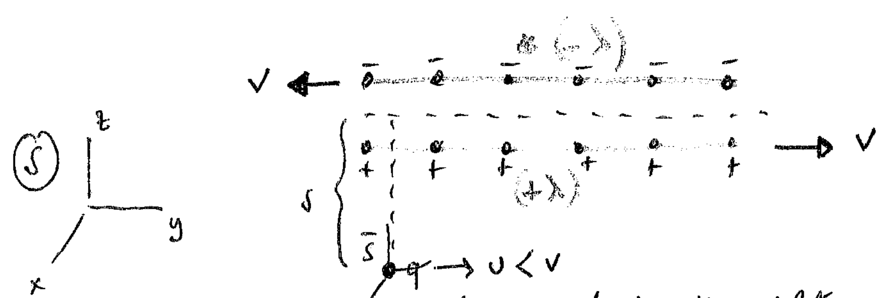
Relativistic Electrodynamics

Magnetism as a relativistic phenomenon

* Classical electrodynamics is already consistent with special relativity. Maxwell's eqs and the Lorentz force can be applied in any inertial ref. frame.
  Of course, what one observer ~~will~~ interprets as an electrical process another may regard as magnetic, but the actual particle motions they predict will be identical.

* What we do right now will not change the ~~rule~~ rules of electrodynamics at all. But we will be expressing these rules in a notation that exposes and illuminates their relativistic character

Why there had to be such a thing as magnetism given electrostatics on relativity?



The charges close enough to each other so that we can regard them as a continuous line charge

$\Rightarrow$ we have a net current to the right $I = 2\lambda V$

* Meanwhile a distance s away there is a point charge q travelling to the right at speed $u < V$. Because these 2 line charge distributions cancel, there is no electrical force on q in this system S

* Let us examine the same situation from the point of view of system $\bar{S}$, which moves right with speed u. In this frame q is at ~~res~~ rest. By the Einstein velocity addition rule, the velocities of the positive and negative lines are now

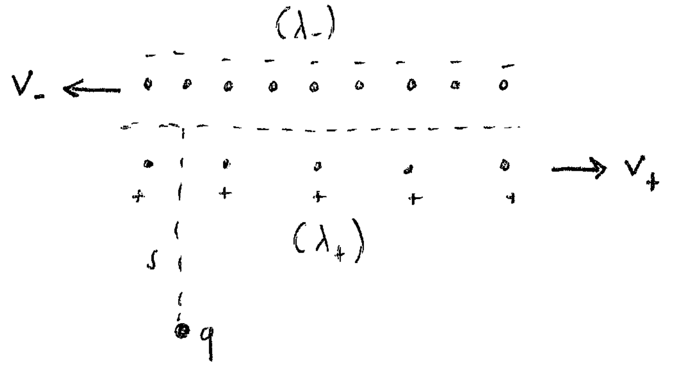
$$V_{\pm} = \frac{V \mp u}{1 \mp V\frac{u}{c^2}}$$

$$\boxed{\bar{u} = \frac{u - V}{1 - \frac{uV}{c^2}}}$$

velocity wrt S; $V \to$ velocity of $\bar{S}$ wrt S

remember.

Since $|V_-| > |V_+|$, the Lorentz contraction of spacing between negative charges is more severe than between positive charges; in this frame, therefore, the wire carries a net negative charge!



In fact,

$$\lambda_{\pm} = \pm(\gamma_{\pm})\lambda_0$$

$$\gamma_{\pm} = \frac{1}{\sqrt{1-\frac{V_{\pm}^2}{c^2}}}$$

Let us assume that λ_0 is the charge density of the positive line in its own rest system. That is not the same as λ, in S since they are moving at speed V. So

In S system:
$$\lambda = \gamma\lambda_0 \qquad \gamma = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}}$$

And similarly for $\bar{S}$ system

$$\lambda_{\pm} = \pm(\gamma_{\pm})\,\lambda_0 \qquad \text{where} \qquad \gamma_{\pm} = \frac{1}{\sqrt{1-\frac{V_{\pm}^2}{c^2}}}$$

Let us rewrite $\gamma_{\pm}$ in a simpler form:

a^2-b^2
$(a-b)(a+b)$

$$\gamma_{\pm} = \frac{1}{\sqrt{1-\frac{1}{c^2}\frac{(v\mp u)^2}{\left(1\mp\frac{vu}{c^2}\right)^2}}} = \frac{c\left(1\mp\frac{vu}{c^2}\right)}{\sqrt{c^2\left(1\mp\frac{vu}{c^2}\right)^2-(v\mp u)^2}}$$

$$= \frac{c^2\mp vu}{\sqrt{c^4\left(1\mp\frac{vu}{c^2}\right)^2-c^2(v\mp u)^2}} = \frac{c^2\mp vu}{\sqrt{(c^2\mp vu)^2-c^2(v\mp u)^2}} \overset{?}{\Rightarrow} \frac{c^2\mp uv}{(c^2-v^2)(c^2-u^2)}$$

$(c-v)(c+v)(c-u)(c+u)$
a^2-b^2
$(a-b)(a+b)$

$$(c^2\mp vu)^2-[c(v\mp u)]^2 = [c^2\pm vu - c(v\mp u)][c^2\mp vu + c(v\mp u)]$$
$$= [c^2\pm vu - cv\pm cu][c^2\mp vu + cv\mp cu]$$

$c(c-v)\pm u(c+v)$

$c^2+cv\mp u(v+c)$
$c(c+v)\mp u(v+c)$
$(v+c)(c\mp u)$

$$\boxed{\gamma_{\pm} = \gamma\,\frac{1\mp\frac{uv}{c^2}}{\sqrt{1-\frac{u^2}{c^2}}}}$$

Evidently, the net line charge in $\bar{S}$ is

$$\lambda_{tot} = \lambda_+ + \lambda_-$$

$$= \gamma_+ \lambda_0 - \gamma_- \lambda_0 = \lambda_0 (\gamma_+ - \gamma_-)$$

$$= \lambda_0 \left[\gamma \frac{1 - \frac{uv}{c^2}}{\sqrt{1 - \frac{u^2}{c^2}}} - \gamma \frac{1 + \frac{uv}{c^2}}{\sqrt{1 - \frac{u^2}{c^2}}} \right] = \left[\frac{\gamma}{\sqrt{1 - \frac{u^2}{c^2}}} \left[\cancel{1} - \frac{uv}{c^2} - \cancel{1} - \frac{uv}{c^2} \right] \right] \lambda_0$$

$$= \frac{\gamma}{\sqrt{1 - \frac{u^2}{c^2}}} \left(- \frac{2uv}{c^2} \right) \lambda_0 = \frac{-2\lambda uv}{c^2 \sqrt{1 - \frac{u^2}{c^2}}}$$

In ~~moving~~ $\bar{S}$ frame

$$\lambda_{tot} = - \frac{2\lambda uv}{c^2 \sqrt{1 - \frac{u^2}{c^2}}}$$

~~Result~~:

Conclusion: As a result of unequal Lorentz contraction of positive and negative lines a current-carrying wire that is electrically neutral in one inertial system will be charged in another

A line charge λ_{tot} sets up an electric field.

$$E = \frac{\lambda_{tot}}{2\pi \varepsilon_0 s}$$

So there is an electrical force on q in $\bar{S}$

$$\bar{F} = qE = - \frac{\lambda v}{\pi \varepsilon_0 c^2 s} \frac{qu}{\sqrt{1 - \frac{u^2}{c^2}}}$$

If there is a force on q in $\bar{S}$, there must be one in S; in fact we can calculate it by using transformation rules for forces. Since q is at rest in $\bar{S}$, and $\bar{F}$ is perpendicular to u, the force in S given by

$$F = \sqrt{1 - \frac{u^2}{c^2}} \bar{F} = - \frac{\lambda v}{\pi \varepsilon_0 c^2} \frac{qu}{s}$$

The charge is attracted toward the wire by a force that is purely electrical is $\bar{S}$ (where the wire is charged an q is at rest) but distinctly nonelectrical in S (where the wire is neutral)

Taken together, then, electrostatics and relativity imply the existence of another force. This "other force" is of course magnetic. If we consider $c^2 = \cancel{\epsilon_0} \cdot \frac{1}{\mu_0 \epsilon_0}$

and $I = 2\lambda v$:

$$F = -\frac{I}{\pi \cancel{\epsilon_0} \frac{1}{\mu_0 \cancel{\epsilon_0}}} \frac{qv}{s} = -qv \underbrace{\left(\frac{\mu_0 I}{2\pi s} \right)}$$

magnetic field of a long straight wire and the force is nothing but the Lorentz force.