02ELMA - Homework 1

Assigned for the week of Feb 17, 2025

Questions

1. Compute the gradient of the scalar field $f(x, y, z) = xyz$. After calculating the gradient, describe its geometric interpretation at the point $(1, 1, 1)$ and explain how it relates to the rate of change of f in various directions from this point.

2. Calculate the divergence of the vector field $\vec{F}(x, y, z) = (x^2, -y^2, z^2)$. Interpret the physical meaning of your result. Discuss how the divergence at the point $(1, -1, 1)$ reflects the behavior of the vector field in terms of sources, sinks, or neutrality.

3. Determine the curl of the vector field $\vec{F}(x, y, z) = (y, z, x)$. Explain the significance of your result in terms of the field's rotation. Illustrate what the curl at the point $(0, 0, 1)$ suggests about the rotational behavior of the vector field around this point.

4. Prove the following vector identities:
   - $\vec{\nabla} \times (\vec{\nabla} T) = 0$
   - $\vec{\nabla} \cdot (\vec{\nabla} \times \vec{T}) = 0$

5. Are $(\vec{A} \times \vec{B}) \times \vec{C}$ and $\vec{A} \times (\vec{B} \times \vec{C})$ equal? Justify your answer.