02ELMA - Homework 11

Assigned for the week of Apr 28, 2025

Questions

1. We know that if electric fields $\vec{E}$ and magnetic fields $\vec{B}$ exist in vacuum (in absence of charges and currents), they behave like waves according to Maxwell's equations. For now, let us focus only on electric fields. Consider that $\vec{E}$ propagates in the z direction and behaves like plane waves given in the form $\tilde{\vec{E}}(z,t) = \tilde{\vec{E}}_0 e^{i(kz-\omega t)}$ where the phase δ is absorbed in the constant $\tilde{\vec{E}}_0 \equiv \vec{E}_0 e^{i\delta}$ and $\vec{E}_0$ is the real amplitude of the electric field. Note that the real amplitude can be written in general (in 3 dimensions) as $\vec{E}_0 = (E_0)_x\hat{x} + (E_0)_y\hat{y} + (E_0)_z\hat{z}$. By considering $\vec{\nabla}.\vec{E} = 0$ (or equivalently, $\vec{\nabla}.\tilde{\vec{E}} = 0$), show that the component of electric field in the propogation direction is zero, i.e., $(E_0)_z = 0$. Is this also valid for magnetic fields?

2. Consider Faraday's law $\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$ and $\tilde{\vec{E}}(z,t) = \tilde{\vec{E}}_0 e^{i(kz-\omega t)}$ to show that $(\tilde{B}_0)_y(\omega/k) = (\tilde{E}_0)_x$ and $(\tilde{B}_0)_x(\omega/k) = -(\tilde{E}_0)_y$. If we also consider $(E_0)_z = (B_0)_z = 0$, explain how we can summarize these four equations with a single equation:
$$(\tilde{B}_0) = \frac{k}{\omega}(\hat{z} \times \tilde{\vec{E}}_0).$$
Using the equation above as guidance, briefly describe how electric and magnetic fields behave in vacuum.