02ELMA - Homework 12

Assigned for the week of May 5, 2025

Questions

1. Consider a train moving at a speed of 50 km / h with respect to the ground and a kid inside the train throwing a ball at a speed of 10 km / h with respect to the train. We want to calculate the speed of the ball with respect to the ground. What is the percentage of error introduced when we calculate the speed of the ball with respect to the ground by simply adding the speeds of the train with respect to the ground and the ball with respect to the train?

2. **Background:** Muons are elementary particles similar to electrons but with a larger mass. Muons have a mean lifetime of about 2.2×10^{-6} seconds in their rest frame. They are created in the upper atmosphere, approximately 15 kilometers above Earth's surface, because of cosmic-ray interactions.

   Question: Consider muons created at an altitude of 15 kilometers above the Earth's surface, traveling directly downward at a speed very close to the speed of light ($0.998c$).

   (a) **Without Special Relativity:** Calculate the distance a muon would travel before decaying if its lifetime is 2.2×10^{-6} seconds. Based on this calculation, would the muons be able to reach the Earth's surface?

   (b) **With Special Relativity:** Use the time dilation formula from special relativity to calculate the dilated lifetime of the muons as observed from the Earth's frame of reference. Show your work step by step.

   $$t' = \frac{t}{\sqrt{1 - \left(\frac{v}{c}\right)^2}}$$

   where t' is the dilated lifetime (measured in the reference frame of the Earth), t is the lifetime in the reference frame of the muon (2.2×10^{-6} seconds), v is the speed of the muon ($0.998c$) and c is the speed of light.

   (c) **Comparison and Conclusion:** Calculate the distance muons travel before decaying using the dilated lifetime. Compare this distance to the thickness of the atmosphere (15 kilometers). Based on this comparison, explain why special relativity is necessary to understand how muons created in the upper atmosphere can reach the Earth's surface.