

02ELMA - Homework 2

Assigned for the week of Feb 24, 2025

Questions

1. Evaluate the line integral of the vector field $\vec{F} = x\hat{x} + y\hat{y}$ along the closed path defined by the points $A(0, 1) \rightarrow B(1, 1) \rightarrow C(0, 0) \rightarrow A(0, 1)$.
2. Evaluate the surface integral of the vector field $\vec{F} = x\hat{x} + y\hat{y} + z\hat{z}$ over a sphere with radius 1 centered at $(0,0,0)$. If \vec{F} describes the flux of a quantity, describe what information does the integral give us? (Hint: Consider spherical coordinates)
3. A cylinder is positioned on the xy plane and centered on the z-axis, with a height of 3 units and a radius of 1 unit. The mass density of the cylinder is determined by the function $\rho(x, y, z) = x + y + z$. Calculate the total mass of the cylinder using the given mass density function. (Hint: Consider cylindrical coordinates)
4. Use the Divergence Theorem to evaluate the flux of the vector field $\vec{F} = x^2\hat{i} + y^2\hat{j} + z^2\hat{k}$ outward through the surface of a cube with any vertex at the origin and with side length 1.
5. Use Stokes' Theorem to find the circulation of the vector field $\vec{F} = -y\hat{i} + x\hat{j} + z\hat{k}$ around the circle $x^2 + y^2 = 1$ in the plane $z = 1$.