02ELMA - Homework 6

Assigned for the week of Mar 24, 2025

Questions

1. Two charges are placed in two different arrangements: (a) $3q$ at $(0, 0, a)$, and $-q$ at $(0, 0, 0)$, and (b) $3q$ at $(0, 0, 0)$, and $-q$ at $(0, 0, -a)$. For each arrangement, determine the monopole moment (in other words, the monopole contribution), the dipole moment, and the approximate potential (in spherical coordinates) using these moments.

2. A pure dipole p is located at the origin, oriented in the $-z$ direction. What is the force experienced by a point charge located at the coordinates $(a, 0, 0)$?

3. Two pure electric dipoles $\vec{p}_1$ and $\vec{p}_2$ are a distance r apart along the x-axis. What is the torque experienced by $\vec{p}_2$ due to the presence of $\vec{p}_1$? ($\vec{p}_1$ and $\vec{p}_2$ are oriented along the $\hat{z}$ and $\hat{x}$ directions, respectively.)

4. A sphere of radius R has a polarization $\vec{P}(\vec{r}) = k\vec{r}$ where k is a constant and $\vec{r}$ is the vector from the center. Calculate the bound charges σ_b and ρ_b, and find the field inside and outside the sphere.

Given for 2 and 3: The electric field created by a pure dipole:

$$\vec{E}_{dip}(r, \theta) = \frac{p}{4\pi\epsilon_0 r^3}\left[2\cos(\theta)\hat{r} + \sin(\theta)\hat{\theta}\right] \tag{1}$$